Name: ___________________________________ Date: _________________

SEQUENCES
ALGEBRA 2 WITH TRIGONOMETRY

Sequences, or an ordered list of numbers, are extremely important in mathematics, both theoretical and applied. A **sequence** is formally defined as **a function that has as its domain the set** $\{1, 2, 3, ..., n\}$.

***Exercise* #1:** A sequence is defined by the equation $a(n) = 2n - 1$.

(a) Find the first three terms of this sequence, denoted by a_1, a_2, and a_3.

(b) Evaluate the sum given by $\sum_{i=1}^{5} a_i$.

Sequences can also be described by using **recursive definitions**. When a sequence is defined recursively, terms are found by operations on previous terms.

***Exercise* #2:** A sequence is defined by the recursive formula: $a_{n+1} = a_n + 5$ with $a_1 = -2$.

(a) Generate the first five terms of this sequence. Label each term with proper subscript notation.

(b) Determine the value of a_{20}. Hint – think about how many times you have added 5 to -2.

***Exercise* #3:** Determine a recursive definition for the sequence shown below. Be sure to include a starting value.

5, 10, 20, 40, 80, 160, …

***Exercise* #4:** For the recursively defined sequence $t_n = (t_{n-1})^2 + 2$ and $t_1 = 2$, the value of t_4 is

(1) 18 (3) 456

(2) 38 (4) 1446

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***Exercise* #3:** One of the most well-known sequences is the Fibonacci, which is defined recursively using two previous terms. Its definition is given below.

$$a_{n+1} = a_n + a_{n-1} \text{ and } a_1 = 1 \text{ and } a_2 = 1$$

Generate values for a_3, a_4, a_5, and a_6.

It is possible to find algebraic formulas for simple series, and this skill should be practiced.

***Exercise* #4:** Find an algebraic formula $a(n)$, similar to that in *Exercise* #1, for each of the following sequences. Recall that the domain that you map from will be the set $\{1, 2, 3, ..., n\}$.

(a) $4, 5, 6, 7, ...$

(b) $2, 4, 8, 16, ...$

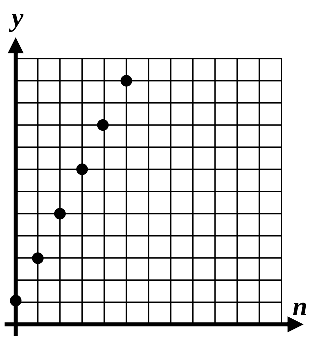
(c) $5, \frac{5}{2}, \frac{5}{3}, \frac{5}{4}, ...$

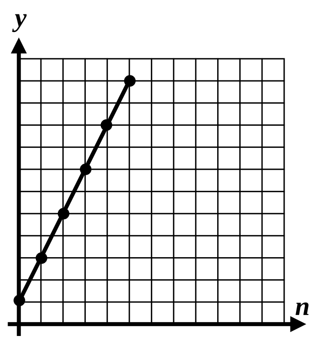
(d) $-1, 1, -1, 1, ...$

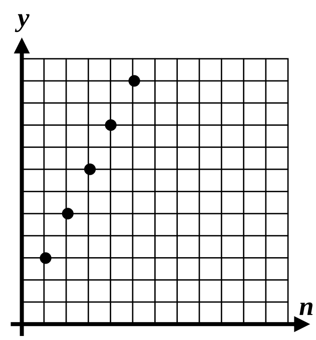
(e) $10, 15, 20, 25, \ldots$

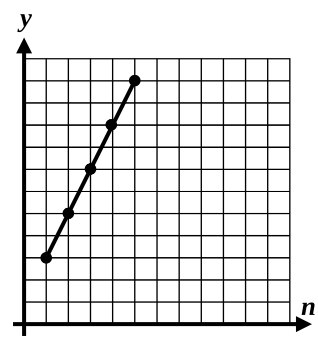
(f) $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, ...$

***Exercise* #5:** Which of the following would represent the graph of the sequence $a_n = 2n + 1$?

(1)


(2)


(3)


(4)


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SEQUENCES
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

SKILLS

1. Given each of the following sequences defined by formulas, determine and label the first 4 terms. A variety of different notations is used below for practice purposes.

(a) $a(n)=7n+2$ (b) $a_n=n^2-5$ (c) $t(n)=\left(\frac{2}{3}\right)^n$ (d) $t_n=\frac{1}{n+1}$

2. Sequences below are defined recursively. Determine and label the **next** four terms of the sequence.

(a) $a_1=4$ and $a_n=a_{n-1}+8$ (b) $t_n=t_{n-1}\cdot\frac{1}{2}$ and $t_1=24$ (c) $b_n=b_{n-1}+2n$ with $b_1=5$

3. Given the sequence defined by $a_n=3n-1$, the value of $\sum_{i=1}^{3}a_i=$

(1) 15 (3) 8

(2) 6 (4) 22 __________

4. A recursive sequence is defined by $a_{n+1}=2a_n-a_{n-1}$ with $a_1=0$ and $a_2=1$. Which of the following represents the value of a_5?

(1) 8 (3) 3

(2) -7 (4) 4 __________

5. Which of the following formulas would represent the sequence 10, 20, 40, 80, 160, …

(1) $a_n=10^n$ (3) $a_n=5(2)^n$

(2) $a_n=10(2)^n$ (4) $a_n=2n+10$ __________

6. For each of the following sequences, determine an algebraic formula, similar to *Exercise* #4, that defines the sequence.

(a) 5, 10, 15, 20, …

(b) 3, 9, 27, 81, …

(c) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, ...$

7. For each of the following sequences, state a recursive definition. Be sure to include a starting value or values.

(a) 8, 6, 4, 2, …

(b) 2, 6, 18, 54, …

(c) $2, -2, 2, -2, ...$

APPLICATIONS

8. **Fibonacci's Rabbits** – Rabbits reproduce quickly. Let's say that we model an idealized population of rabbits using the fact that a pair of rabbits can reproduce when they are two months old and will produce one pair of rabbits of opposite gender each month thereafter. If we start with one pair of newborn rabbits in the first month, determine how many pairs of rabbits will exist in months 2, 3, 4, and 5.

REASONING

9. Consider a sequence defined similarly to the Fibonacci, but with a slight twist:

$$a_{n+1} = a_n - a_{n-1} \text{ with } a_1 = 2 \text{ and } a_2 = 5$$

Generate terms $a_3, a_4, a_5, a_6, a_7,$ and a_8. Then, determine the value of a_{25}.