Name: ___________________________________ Date: __________________

LINEAR RELATIONSHIPS
ALGEBRA 2 WITH TRIGONOMETRY

Linear relationships, which are all functions, are extremely important in practical applications. The definition of a linear relationship between two variables, *x* and *y*, is given below.

LINEAR RELATIONSHIPS

Two variables, *x* and *y*, have a **linear relationship** if any two arbitrary points, say (x_1, y_1) and (x_2, y_2), satisfy the relationship:

$$\frac{\Delta y}{\Delta x} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1} = \text{a constant (called the slope)}$$

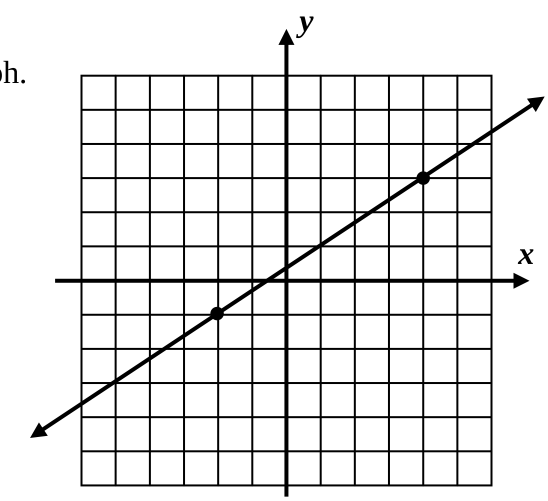
Thought of another way, **the change in *y* is always a constant multiple of the change in *x***.

***Exercise* #1:** If the three points $A(-2, 4)$, $B(2,10)$, and $C(8, 25)$ were all plotted, would they form a linear relationship? (In other words, would they fall in a straight line or be collinear?) Justify your answer.

In this last exercise, we used the slope to justify whether the rate at which *y* is changing compared to the rate at which *x* is changing remained constant. If we know this constant ratio, then we can use how much one variable has changed to determine how much another has changed.

***Exercise* #2:** Two variables, *x* and *y*, are linearly related. Two points on this linear relationship, $(-2, -1)$ and $(4, 3)$, are shown below on the graph connected with a line.

(a) What is the slope of this linear relationship? Illustrate it on the graph.



(b) If the *x* variable increases by 15 units, what would the *y* variable increase by?

(c) Use your answer from part (b) to determine the value of *k* such that the point $(19, k)$ lies on this line?

Linear relationships exist in real-life whenever the ratio of the change in one variable to the change in another remains constant. This constant will always represent the slope of the relationship when plotted on a graph.

***Exercise* #3:** The amount of gasoline in a person's tank is a linear relationship to how many miles they have driven. Abagail was trying to determine this relationship. She noticed that after driving for a total of 50 miles on a full tank of gas she had 12 gallons left. After driving for a total of 100 miles she had 10 gallons left.

(a) Calculated the $\frac{\Delta g}{\Delta m}$, i.e. the change in gallons over the change in miles. Express your answer as a decimal and include appropriate units.

(b) How many gallons would be left after Abagail drives a total of 200 miles?

(c) How many gallons of gas did Abagail start with (full tank)?

(d) How many total miles can Abagail drive if she starts with a full tank of gas?

It is important to understand that the slope calculation is always the calculation of a rate, or how fast one variable is changing compared to another.

***Exercise* #4:** Bien was driving north on an interstate highway and noted his distance away from New York City. After driving for 2 hours he was 209 miles away and after driving for 3.5 hours he was 302 miles away.

(a) Calculate $\frac{\Delta D}{\Delta t}$, the change in Bien's distance to the change in time. Include appropriate units.

(b) What does your answer in part (a) represents about Bien's motion?

(c) How far will Bien be from New York City after 6 hours of driving if he retains a constant speed?

(d) If Bien has been traveling at this constant speed his entire trip, how far from New York City did he start?

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LINEAR RELATIONSHIPS
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

SKILLS

1. Calculate $\frac{\Delta y}{\Delta x}$ for each of the following coordinate pairs. Express your answer in simplified fractional form.

(a) $(-2, 5)$ and $(4, 19)$ (b) $(4, 5)$ and $(12, 1)$ (c) $(-6, -4)$ and $(1, 17)$

2. Calculate $\frac{\Delta y}{\Delta x}$ for each of the following coordinate pairs. Express your answer in decimal form.

(a) $(4.5, 12.8)$ and $(7.0, 19.2)$ (b) $(20, 23)$ and $(60, 25)$ (c) $(2.2, -2.1)$ and $(3, -4.9)$

3. In each of the following a value for $\frac{\Delta y}{\Delta x}$ is given along with a change in x, Δx. Calculate the change in y, Δy, for each.

(a) $\frac{\Delta y}{\Delta x} = \frac{2}{3}$ and $\Delta x = 15$ (b) $\frac{\Delta y}{\Delta x} = -\frac{1}{2}$ and $\Delta x = 20$ (c) $\frac{\Delta y}{\Delta x} = 0.35$ and $\Delta x = 6$

4. For a linear relationship $\frac{\Delta y}{\Delta x} = \frac{5}{2}$. If the point $(5, 3)$ satisfies this relationship, then for which of the following values of y will the point $(13, y)$ also satisfy the relationship?

(1) $y = 8$ (3) $y = 20$

(2) $y = 16$ (4) $y = 23$

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APPLICATIONS

5. A rocket upon takeoff uses fuel at a constant rate with respect to time. The table below shows the amount of gallons of fuel, g, the rocket *still contains* as a function of the number of minutes, m, after it took off.

Time, m (minutes)	4	6	8	10
Fuel Remaining, g (gallons)	30	25	20	15

(a) Calculate $\frac{\Delta g}{\Delta m}$. Include appropriate units.

(b) Why is the slope you calculated in part (b) negative?

(c) What does the value of your answer in part (a) physically represent?

(d) After how many minutes, m, will the rocket run out of fuel?

6. Camille is driving west away from Albany, NY, on an interstate highway going an average speed of 64 miles per hour. After 2 hours of driving she sees a sign stating that she is 176 miles from Albany. How far from Albany is she after 5 hours of driving assuming she maintains this constant speed?

REASONING

7. A linear relationship between the variables x and y exists such that $\frac{\Delta y}{\Delta x} = -\frac{4}{5}$. If the point $(4, 32)$ satisfies this linear relationship, then does the point $(19, 22)$ also satisfy it? Justify your response.