

QUADRATIC INEQUALITIES IN TWO VARIABLES

ALGEBRA 2 WITH TRIGONOMETRY

Solving quadratic inequalities of one variable, typically x , involved finding the set of all values of that variable that made a particular inequality true. Likewise, solving a quadratic inequality of two variables simply implies finding, graphically, all (x, y) coordinate pairs that make a particular inequality true.

Exercise #1: For the quadratic inequality $y > x^2 - 4$ answer each of the following questions.

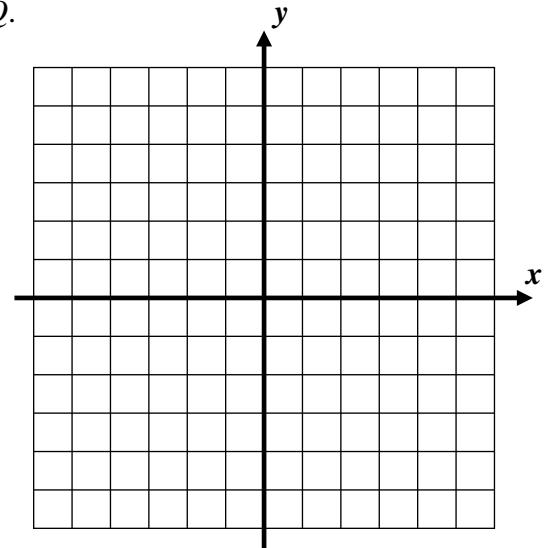
(a) Specify whether each of the following points lies in the solution set.

$(1, 4)$

$(2, 0)$

$(-2, 1)$

(c) Sketch the inequality below. Label its solution set Q .



(b) Do the points on the parabola $y = x^2 - 4$ fall in the solution set of this inequality? Explain.

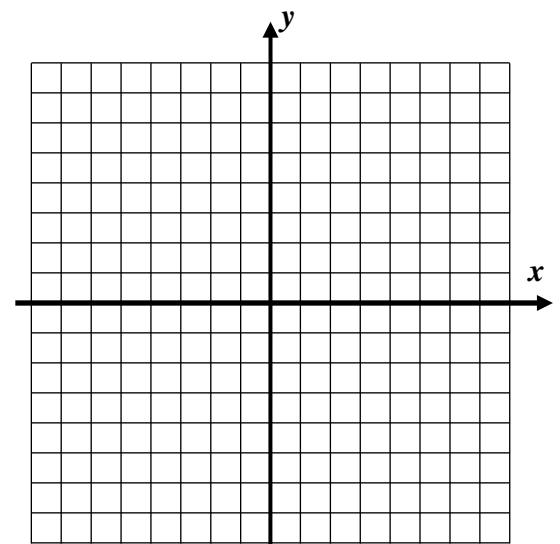
Just as in Algebra 1, we can graph a system of inequalities by finding where the solution set of two inequalities intersects.

Exercise #2: Sketch the solution set of the system of quadratic inequalities given below. Label the solution set

A. State one point that lies in the solution set.

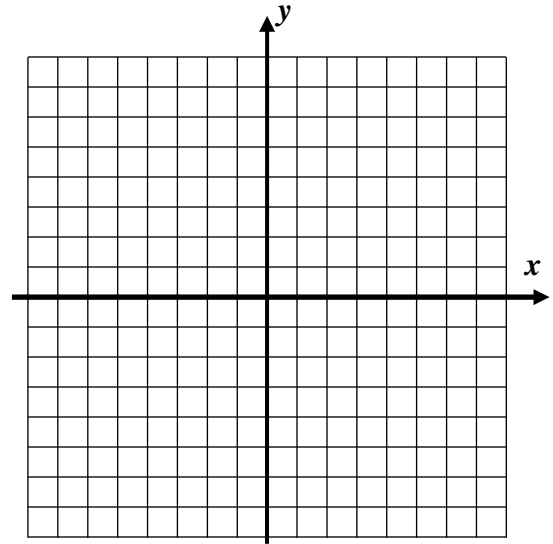
$$y < 6 - x^2$$

$$y \geq x^2 - 4x + 3$$



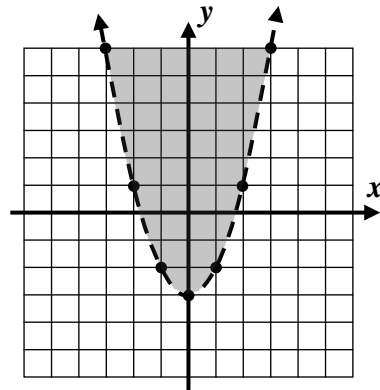
Exercise #3: Graph the region described by the inequality given below.

$$x^2 - 4 < y \leq 5$$



Exercise #4: Which of the following inequalities is represented by the graph shown below?

- (1) $y < x^2 - 3$ (3) $y > x^2 - 3$
 (2) $y \leq x^2 - 3$ (4) $y \geq x^2 - 3$



Exercise #5: An arch bridge crosses a 600-foot span above a river. Its structure, as viewed from the side, occupies the two-dimensional space given by the set of inequalities shown below. Produce a sketch showing the side-view of the bridge. Label all relevant features of the graph.

$$0 \leq x \leq 600 \text{ and } \frac{600x - x^2}{360} \leq y \leq 300$$



QUADRATIC INEQUALITIES IN TWO VARIABLES
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

SKILLS

1. Which of the following points lies in the solution set of $y < 2x^2 - 10x$?

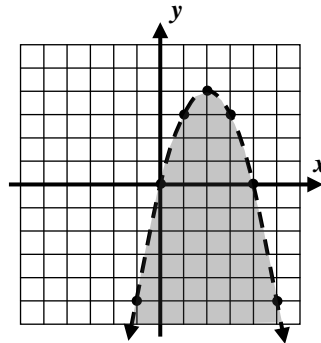
(1) the origin (3) (4, 2)

(2) (1, -6) (4) (-1, 4)

2. Which of the inequalities describes the shaded region shown below?

(1) $y < 4x - x^2$ (3) $y > 4x - x^2$

(2) $y < x^2 - 4x$ (4) $y \geq x^2 - 4x$



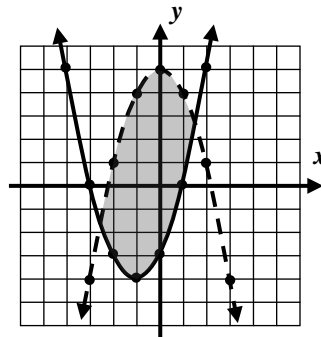
3. Which of the inequalities describes the shaded area below?

(1) $5 - x^2 < y \leq x^2 - 2x - 3$

(2) $x^2 - 2x - 3 \leq y < 5 - x^2$

(3) $5 - x^2 \leq y < x^2 - 2x - 3$

(4) $x^2 - 2x - 3 < y \leq 5 - x^2$



4. Which of the following points does not lie in the solution set of $y + 4 \geq x^2 - 2$?

(1) (-3, 7) (3) (3, 3)

(2) (4, 8) (4) (-6, 40)

5. Which of the following points does *not* lie in the solution set of the inequality $x^2 - 2x - 1 < y \leq -x^2 + 2x + 5$?

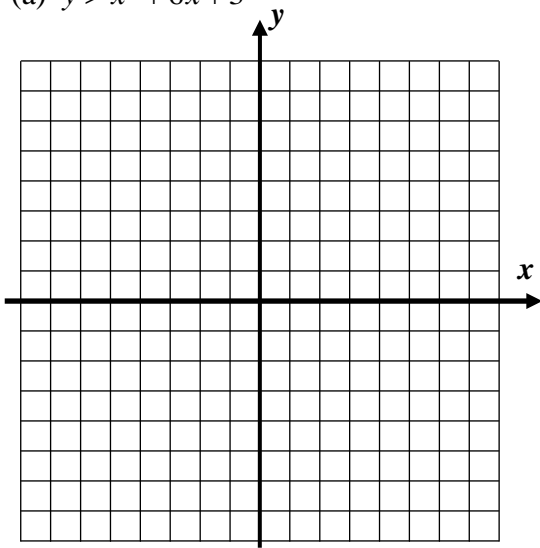
(1) (0, 0) (3) (-1, 2)

(2) (2, 5) (4) (1, -1)

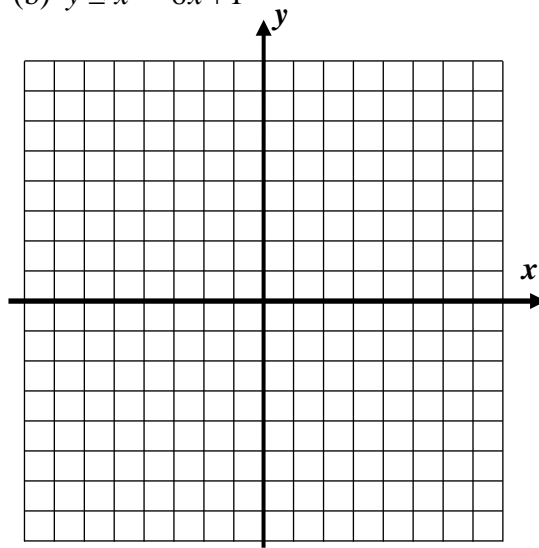


6. Sketch the solution set of each inequality shown below. Label the solution set A.

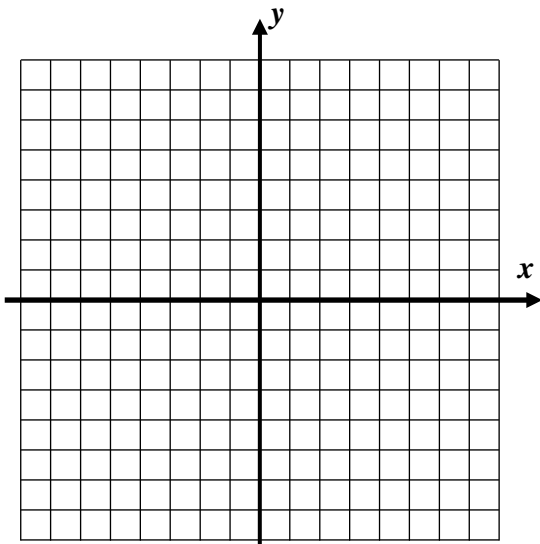
(a) $y > x^2 + 6x + 3$



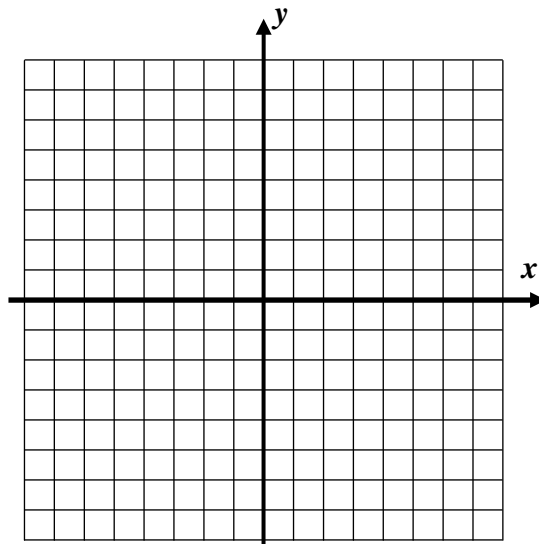
(b) $y \leq x^2 - 6x + 1$



(c) $x^2 - 5 < y \leq 3 - x^2$



(d) $y \leq -x^2 - 6x - 7$ or $y \geq x^2 - 4x + 8$



APPLICATIONS

7. The entrance to a tunnel through a mountain is shaped like a parabola whose equation is

$$y = \frac{120x - 4x^2}{75}, \text{ where } x \text{ represents the}$$

horizontal distance across the roadway and y represents the height of the top of the opening.

To the right, sketch the region given by

$$0 \leq y \leq \frac{120x - 4x^2}{75}. \text{ Show the width of the}$$

region as well as its height.

