

VERTICAL DILATION OF FUNCTIONS

ALGEBRA 2 WITH TRIGONOMETRY

We have now seen how to shift and reflect functions, specifically in the context of parabolas. In this lesson we will see how to stretch or compress a function in the vertical direction. This is known as **vertically dilating** a function. The first exercise will illustrate this concept with three related parabolas.

Exercise #1: Consider the quadratic function $f(x) = x^2 - 4x - 5$. The quadratic functions g and h are defined by the formulas $g(x) = 2f(x)$ and $h(x) = \frac{1}{2}f(x)$.

(a) Determine formulas for both g and h in simplest trinomial form.

(b) Using your calculator, sketch and label each curve on the set of axes below. Use the window indicated by the axes.

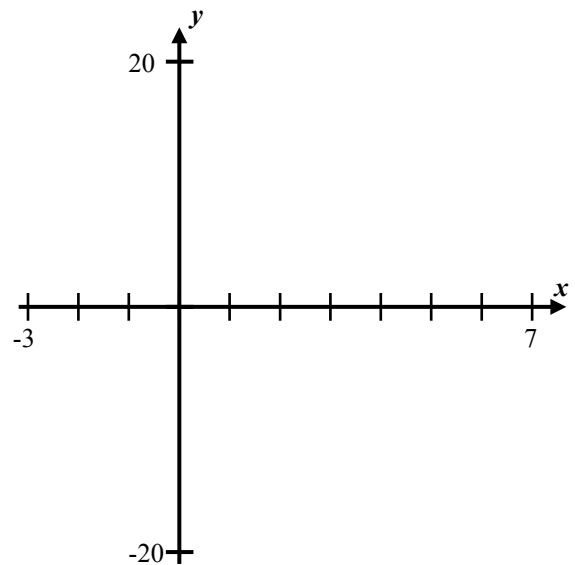
(c) Using the **MINIMUM** command on your calculator, determine the minimum value for each function.

$$f_{\min} =$$

$$g_{\min} =$$

$$h_{\min} =$$

(d) What points did not vary when f was vertically dilated by factors of 2 and $1/2$? Explain why this happened.



VERTICAL DILATION OF FUNCTIONS

The function $h(x) = k \cdot f(x)$ represents a vertical stretch of the function $f(x)$ if $k > 1$ and a vertical compression of the function $f(x)$ if $0 < k < 1$.

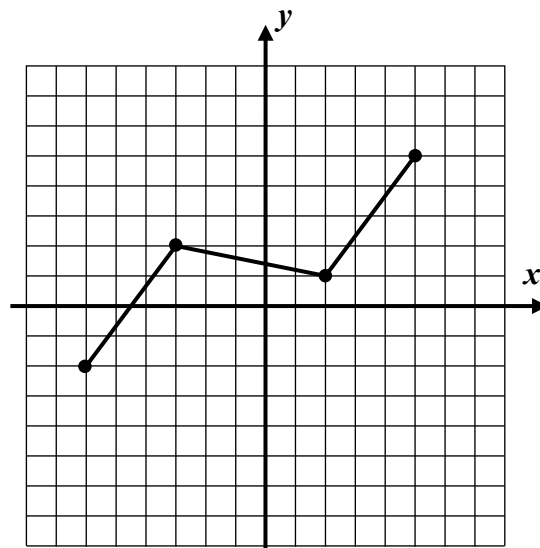


Exercise #2: If the point $(-3, 12)$ lies on the graph of the function $y = f(x)$, which of the following points *must* lie on the graph of $y = 3f(x)$?

- (1) $(-9, 36)$ (3) $(-3, 4)$
 (2) $(-3, 36)$ (4) $(-9, 12)$

Exercise #3: The graph of $y = f(x)$ is shown below. Consider the function $y = g(x)$ defined by $g(x) = 2f(x) - 3$.

(a) What two transformations have occurred to the graph of f in order to produce the graph of g ? Specify both the transformations and their order.



(b) Graph and label $y = g(x)$

Exercise #4: The function $h(x)$ has a domain given by the interval and a range given by the interval $[2, 10]$.

The function $f(x)$ is defined by $f(x) = \frac{3}{2}h(x) + 8$. Which of the following gives the range of $f(x)$?

- (1) $[10, 22]$ (3) $[15, 27]$
 (2) $[8, 12]$ (4) $[6, 32]$

Exercise #5: If the quadratic function $g(x)$ has a y -intercept of 12. Which of the following is true about the function $h(x) = 3g(x) - 5$?

- (1) It has a y -intercept of -5.
 (2) It has a y -intercept of 21.
 (3) It has a y -intercept of -15.
 (4) It has a y -intercept of 31.



Name: _____

Date: _____

VERTICAL DILATION OF FUNCTIONS
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

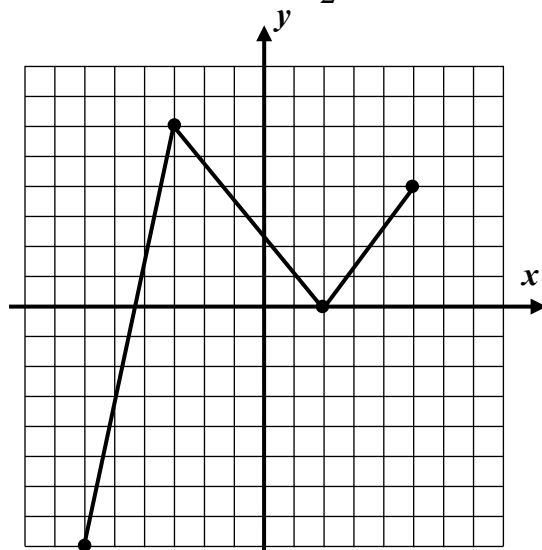
SKILLS

1. If the point $(-6, 10)$ lies on the graph of $y = f(x)$ then which of the following points *must* lie on the graph of $y = \frac{1}{2}f(x)$?
- (1) $(-3, 5)$ (3) $(-6, 5)$
- (2) $(-3, 10)$ (4) $(-12, 20)$ _____
2. If the function $h(x)$ is defined as vertical stretch by a factor of 2 followed by a reflection in the x -axis of the function $f(x)$ then $h(x) =$
- (1) $2f(-x)$ (3) $-\frac{1}{2}f(x)$
- (2) $\frac{1}{2}f(x)$ (4) $-2f(x)$ _____
3. If the graph of $y = x^2$ is compressed by a factor of 3 in the y -direction and then shifted 4 units down, the resulting graph would have an equation of
- (1) $y = \frac{1}{3}x^2 - 4$ (3) $y = -4x^2 - 3$
- (2) $y = -3x^2 - 4$ (4) $y = -\frac{1}{3}x^2 + 4$ _____
4. The quadratic function $f(x)$ has a turning point at $(-3, 6)$. The quadratic $y = \frac{2}{3}f(x) + 3$ would have a turning point of
- (1) $(-2, 9)$ (3) $(-3, 7)$
- (2) $(1, 7)$ (4) $(-1, 9)$ _____
5. The function $g(x)$ is defined by $g(x) = -5f(x) + 4$. What three transformation have occurred to the graph of f to produce the graph of g ? Specify both the transformations and their order.



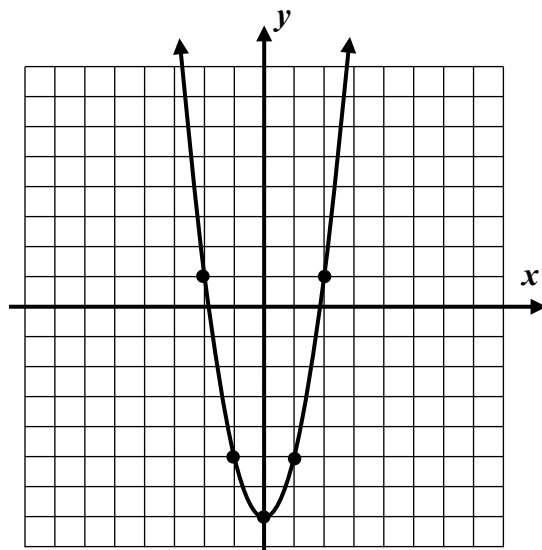
6. The graph of $y = h(x)$ is shown below. The function $f(x)$ is defined by $f(x) = -\frac{1}{2}h(x) + 3$.

(a) What three transformations have occurred to the graph of h to produce the graph of f ? Specify the transformations and the order they occurred in.



(b) Graph and label the function $f(x)$ on the grid below that contains $h(x)$.

7. A parabola is shown graphed to the right that is a transformation of $y = x^2$. The transformation includes a vertical stretch and a vertical shift. What are the stretch and shift? Based on your answer, write an equation for this parabola.



REASONING

8. The function $h(x)$ is defined by the equation $h(x) = 4f(x) - 12$. Determine two different *sets* of transformations that could produce the graph of $h(x)$ from the graph of $f(x)$. For each, specify two transformations and the order in which they occurred. As a hint, write $h(x)$ in its factored form.

