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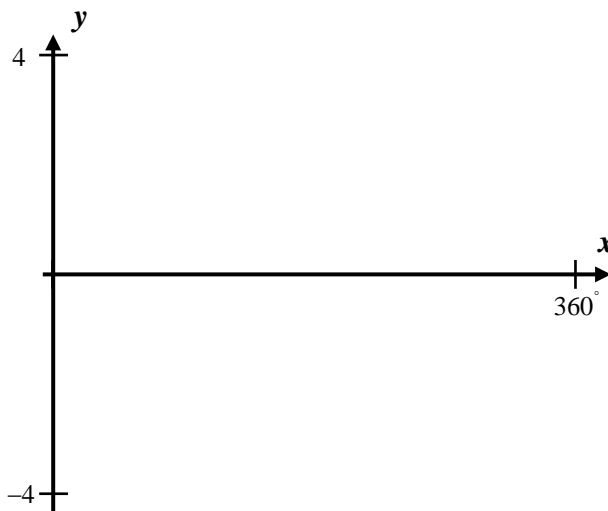
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## SOLVING TRIGONOMETRIC EQUATIONS GRAPHICALLY

### ALGEBRA 2 WITH TRIGONOMETRY

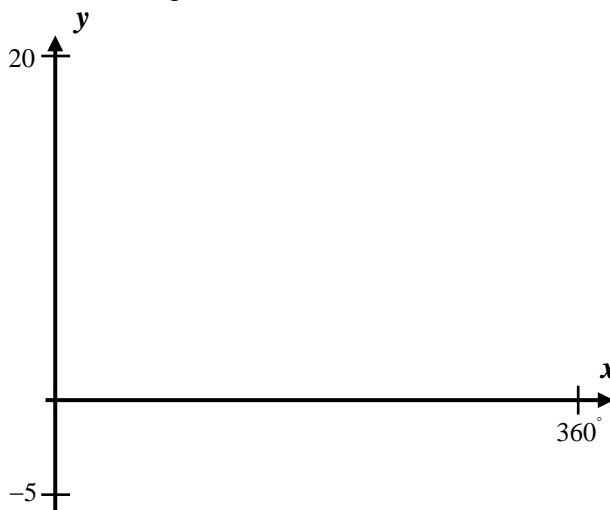
Although algebraic techniques can be used to solve trigonometric equations, they can also be solved graphically using our calculators. Intelligent windows and proper sketches are necessary when employing this technique. We will start with a simple example where the graph window and axes are provided.

**Exercise #1:** Graphically solve the equation  $2\cos x + 1 = 0$  for all values of  $x$  on the interval  $0^\circ \leq x \leq 360^\circ$ . Sketch and label a graph to justify your answers.



Graphical techniques are most useful to solve equations that are more difficult, such as the quadratic equations and especially those involving trigonometric substitutions. The next exercise again provides you with an appropriate window and axes.

**Exercise #2:** Graphically solve the equation  $15\cos^2 x - 7\cos x - 2 = 0$  for all values of  $x$  on the interval  $0^\circ \leq x \leq 360^\circ$ . Express all answers accurate to the nearest *tenth* of a degree.



Most often, when a student is solving an equation graphically, it will be up to the student to draw the axes and decide on an appropriate window. The next few exercises provide practice with these important skills.

**Exercise #3:** Graphically solve the equation  $3\cos^2 \alpha - 7\cos \alpha - 6 = 0$  for all values on  $0^\circ \leq \alpha \leq 360^\circ$ . Round all answers to the nearest *tenth* of a degree. Be sure to label all appropriate portions of your graph (your window, your curve, your intercepts, etc.)

**Exercise #4:** Graphically find all solutions to  $3\sin(2\theta) - \cos \theta = 0$  over the interval  $[0^\circ, 360^\circ]$ . Round all non-integer answers to the nearest *tenth* of a degree.

Some graphing windows can be a challenge and might require multiple adjustments to find all of the roots of an equation.

**Exercise #5:** Graphically solve  $2\cos^2 x - 3\sin x - 3 = 0$  for all values of  $x$  on  $0^\circ \leq x \leq 360^\circ$ .



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**SOLVING TRIGONOMETRIC EQUATIONS GRAPHICALLY**  
**ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK**

**SKILLS**

1. Which of the following represents the largest solution, on the interval  $0^\circ \leq x \leq 360^\circ$ , of the trigonometric equation  $3\sin^2 x - 1 = 0$  to the nearest *degree*?
- (1)  $215^\circ$                       (3)  $341^\circ$   
(2)  $325^\circ$                       (4)  $285^\circ$                       \_\_\_\_\_
2. The solution set of  $2\sin^2 x + 3\sin x - 2 = 0$  on the interval  $[0^\circ, 360^\circ]$ ?
- (1)  $\{45^\circ, 135^\circ\}$                 (3)  $\{30^\circ, 150^\circ\}$   
(2)  $\{30^\circ, 330^\circ\}$                 (4)  $\{60^\circ, 240^\circ\}$                 \_\_\_\_\_
3. Which of the following sets represents all solutions to  $\cos^2 x = \cos x$  on the interval  $[0^\circ, 360^\circ]$ ?
- (1)  $\{0^\circ, 180^\circ, 270^\circ\}$             (3)  $\{90^\circ, 270^\circ\}$   
(2)  $\{0^\circ, 90^\circ, 270^\circ\}$             (4)  $\{90^\circ, 360^\circ\}$                 \_\_\_\_\_
4. The smallest solution to  $\cos(2x) - \cos x + 1 = 0$  on the interval  $0^\circ \leq x \leq 360^\circ$  is
- (1)  $60^\circ$                               (3)  $15^\circ$   
(2)  $30^\circ$                               (4)  $90^\circ$                               \_\_\_\_\_
5. How many solutions does the equation  $\sin(2\alpha) + 2\sin \alpha = 0$  have on the interval  $0^\circ \leq \alpha \leq 360^\circ$ ?
- (1) 1                                      (3) 3  
(2) 2                                      (4) 4                                      \_\_\_\_\_
6. Which of the following represents the number of solutions to  $2\cos^2 x - \cos x - 1 = 0$  on the half-closed interval  $[0, 360^\circ)$ ?
- (1) 1                                      (3) 3  
(2) 2                                      (4) 4                                      \_\_\_\_\_



7. Solve each of the following trigonometric equations graphically for all values of  $x$  on the interval  $0^\circ \leq x \leq 360^\circ$ . Round all non-integer answers to the nearest *tenth* of a degree. In each case provide a graph that justifies your solution. Include on each of your graphs the equation of any curves sketched, the window you used, and any intersections or intercepts needed for solution. **Don't forget to state the solution set.**

(a)  $2 \sin^2 x - 1 = 0$

(b)  $12 \cos^2 x - 5 \cos x - 2 = 0$

(c)  $5 \sin^2 x + 2 \sin x = 0$

(d)  $2 \sin(2x) = 1$

8. Solve the following equation over the interval  $0^\circ \leq x \leq 360^\circ$  graphically. Show an appropriate graph that justifies your solution. Round your final answers to the nearest *tenth* of a degree.

$$5 \cos(2x) + 2 = 4 \sin x + 3$$

