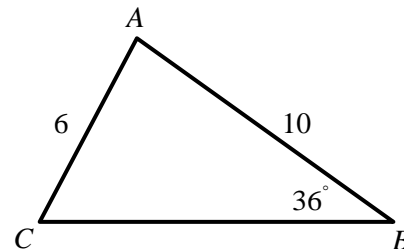


## THE AMBIGUOUS NATURE OF SINE ALGEBRA 2 WITH TRIGONOMETRY

The Law of Sines is known as **ambiguous** because it does not always result in a unique angle solution for a triangle. In fact, the Law of Sines can result in zero, one or two possible angles, and hence triangles, in a given scenario. Which case exists is easily determined by simply solving the trigonometric equation that results from the Law of Sines.

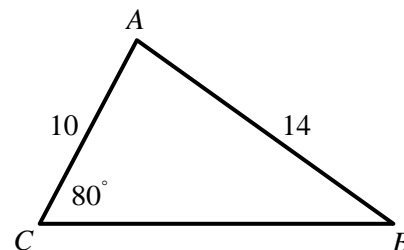
**Exercise #1:** In triangle  $ABC$ , which is shown below but not drawn to scale, it is known that  $AC = 6$ ,  $AB = 10$ , and  $m\angle B = 36^\circ$ . Determine *all possible* values for  $m\angle C$  to the nearest *tenth*.



Sine is **ambiguous** because it is positive for all angles on the interval  $(0^\circ, 180^\circ)$ . In most sine equations two values will be possible, an acute solution and an obtuse solution. But, as we will see in *Exercise #2*, both solutions are not always realistic.

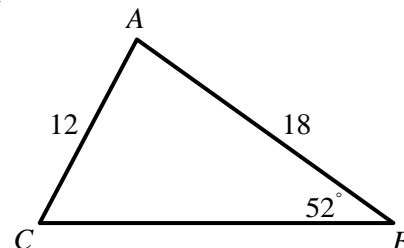
**Exercise #2:** In triangle  $ABC$ , which is shown below but not drawn to scale, it is known that  $AC = 10$ ,  $AB = 14$ , and  $m\angle C = 80^\circ$ .

- (a) Solve an equation to find all possible values for the measure of  $B$  to the nearest *tenth*.



- (b) Considering the measures the angles of any triangle must sum to be  $180^\circ$ , why must we reject the obtuse solution from part (a)?

**Exercise #3:** Explain why the triangle shown below cannot exist by finding all possible values for  $m\angle C$ .



**Exercise #4:** In  $\triangle DEF$ ,  $m\angle E = 72^\circ$ ,  $DE = 12$ , and  $DF = 15$ . How many triangles are possible given this information?

- (1) 1                                      (3) 3  
 (2) 2                                      (4) 0

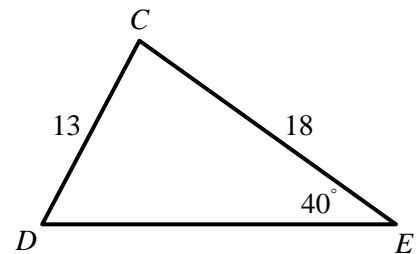
\_\_\_\_\_

**Exercise #5:** In  $\triangle ABC$ ,  $AB = 10$ ,  $BC = 10\sqrt{3}$ , and  $m\angle A = 60^\circ$ . Based on this angle  $B$  is

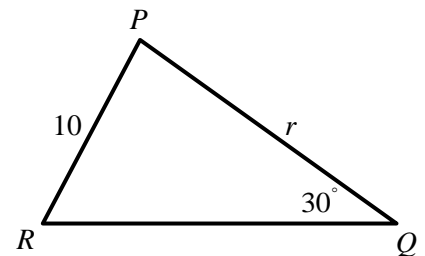
- (1) acute only                              (3) right only  
 (2) acute or obtuse                      (4) obtuse only

\_\_\_\_\_

**Exercise #6:** For  $\triangle CDE$ , which is shown below not drawn to scale, it is known that  $CD = 13$  inches and  $CE = 18$  inches. If  $m\angle E = 40^\circ$ , find all possible *areas* for  $CDE$ . Round your final answer(s) to the nearest square inch.



**Exercise #7:** In  $\triangle PQR$  it is known that  $PR = 10$  and  $m\angle Q = 30^\circ$ . Find all possible values of  $r$  such that these two measures are not possible. State your answer as an inequality in interval notation.



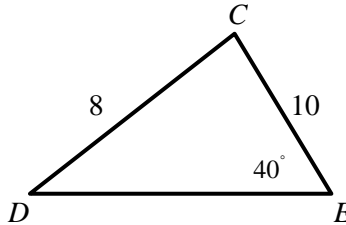
**THE AMBIGUOUS NATURE OF SINE**  
**ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK**

**SKILLS**

Diagrams given in problems are not drawn to scale and angles that appear acute may in fact be obtuse and vice versa.

1. In  $\triangle CDE$  shown below  $CE = 10$ ,  $CD = 8$ , and  $m\angle E = 40^\circ$ . How many possible values exist for  $m\angle D$ ?

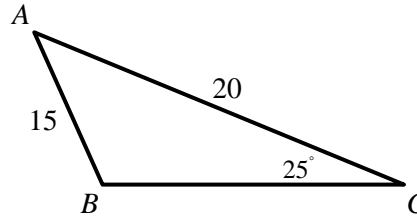
- (1) 1  
(2) 2  
(3) 3  
(4) 0



\_\_\_\_\_

2. In  $\triangle ABC$  shown  $m\angle C = 25^\circ$ ,  $c = 15$ , and  $b = 20$ . Angle B is

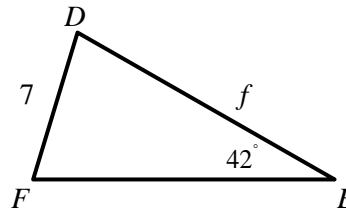
- (1) acute only  
(2) right only  
(3) obtuse only  
(4) acute or obtuse



\_\_\_\_\_

3. For which value of  $f$  shown below will there be no solution for angle E in triangle DEF?

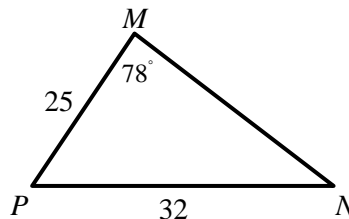
- (1) 9  
(2) 10  
(3) 11  
(4) 5



\_\_\_\_\_

4. Accurate to the nearest *tenth* the *largest* possible value of  $m\angle N$  in the diagram below is

- (1)  $49.8^\circ$   
(2)  $130.2^\circ$   
(3)  $37.3^\circ$   
(4)  $105.6^\circ$



\_\_\_\_\_

5. How many triangles can be formed in which the shortest side measures 9 inches, the longest side measures 14 inches and the smallest angle measures  $48^\circ$ ?

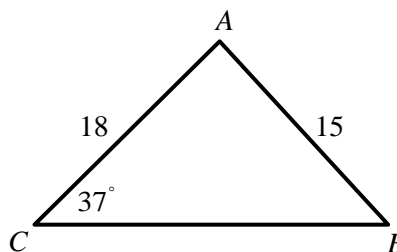
- (1) 1  
(2) 2  
(3) 3  
(4) 0

\_\_\_\_\_



6. In  $\triangle QRS$ ,  $QR = 8$ ,  $QS = 7$ , and  $m\angle R = 52^\circ$ . Find all possible values for  $m\angle Q$ . Round your answers to the nearest *tenth*. Be sure to draw a diagram of  $\triangle QRS$ .

7. In  $\triangle ABC$ ,  $AC = 18$ ,  $AB = 15$ , and  $m\angle C = 37^\circ$ . Find all possible areas of  $\triangle ABC$  to the nearest *tenth*.



### REASONING

8. In  $\triangle ABC$  below,  $AB = 7\sqrt{2}$  and  $m\angle C = 45^\circ$ . Determine all values of  $b$  such that there exists no solution for  $m\angle B$ . Clearly justify your answer.

