Algebra 2 with Trigonometry

Practice Exam #1 – Answer Key

Answer all 39 questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. No partial credit will be allowed on the multiple-choice section.

For Parts II, III, and IV, clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in these parts, a correct numerical answer with no work shown will receive only 1 credit.

A reference sheet that you may need to answer some questions in this examination is included.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this examination as scrap paper. Scrap graph paper is provided at the end of this examination for any question for which graphing may be helpful but is not required. Any work done on this sheet of scrap graph paper will *not* be scored. Write all your work in pen, except graphs and drawings, which should be done in pencil.

Note: A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

Part I

Answer all 27 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [54]

1. If $\sin\beta = \frac{2}{5}$ then $\cos(2\beta) =$

(1) $\frac{1}{5}$ (3) $\frac{11}{25}$

(2) $\frac{17}{25}$ (4) $-\frac{3}{5}$

$\cos(2\beta) = 1 - 2\sin^2\beta$

$= 1 - 2\left(\frac{2}{5}\right)^2 = 1 - 2\left(\frac{4}{25}\right)$

$= 1 - \frac{8}{25} = \frac{17}{25}$

(2)

2. If the sum of $4 - 2i$ and $-7 + 8i$ were plotted in the complex plane, the result would fall in which of the following quadrants?

(1) I (3) III

(2) II (4) IV

$4 - 2i + -7 + 8i = -3 + 6i$

$(-3, 6) \Rightarrow$ Quadrant II

(2)

3. The solution set to the quadratic inequality $4x^2 + 7x - 2 \le 0$ is

(1) $\left\{x \mid -2 \le x \le \frac{1}{4}\right\}$ (3) $\{x \mid x < -3 \text{ or } x > 1\}$

(2) $\{x \mid -3 < x < 1\}$ (4) $\left\{x \mid x \le -2 \text{ or } x \ge \frac{1}{4}\right\}$

$(4x - 1)(x + 2) = 0$

$x = \frac{1}{4}$ or $x = -2$

Now test values to finish.

(1)

4. The largest solution, to the nearest degree, of the equation $5\sin^2 x + 9\sin x - 2 = 0$ on the interval $0^\circ \le x \le 360^\circ$ is which of the following?

(1) 12° (3) 168°

(2) 278° (4) 212°

This problem is most easily solved by graphing $y = 5\sin^2 x + 9\sin x - 2$ and then finding its largest x-intercept using your graphing calculator.

(3)

5. The smallest root of $x^4 + 2x^3 - 8 = 0$ is approximately

(1) –2.5 (3) –3.8

(2) 1.7 (4) 4.6

Graph the function $y = x^4 + 2x^3 - 8$ and use your calculator to find the smallest zero of this function, which is approximately –2.507...

(1)

6. A quadratic function of the form $y = ax^2 + bx + c$ has a leading coefficient, *a*, that is positive and a turning point at $(-6, -2)$. Which of the following represents the range of the quadratic function?

(1) $[-2, \infty)$ (3) $(-\infty, -2)$

(2) $[-6, \infty)$ (4) $(-6, \infty)$

A leading coefficient that is positive indicates that the parabola opens upwards and has a minimum *y*-value of –2 but no maximum *y*-value.

(1)

7. Eight finalists for the New York State math championship are competing. Different prizes are awarded for the top four finishers. Assuming no ties are possible, in how many ways can these prizes be given out?

(1) 8! (3) ${}_8C_4$

(2) 4! (4) ${}_8P_4$

$$n_T = n(1^{st}) \cdot n(2^{nd}) \cdot n(3^{rd}) \cdot n(4^{th})$$
$$= 8 \cdot 7 \cdot 6 \cdot 5 = {}_8P_4$$

(4)

8. Which of the following represents the inverse of $y = 5x - 30$?

(1) $y = -5x + 30$ (3) $y = \frac{1}{5}x + 30$

(2) $y = -\frac{1}{5}x - 6$ (4) $y = \frac{1}{5}x + 6$

$$x = 5y - 30 \Rightarrow x + 30 = 5y$$
$$y = \frac{1}{5}(x + 30) = \frac{1}{5}x + 6$$

(4)

9. The value of $\sin 100^\circ$ can be expressed equivalently as

(1) $2\sin 50^\circ$ (3) $2\sin 50^\circ \cos 50^\circ$

(2) $\sin 50^\circ + \cos 50^\circ$ (4) $\cos^2 50^\circ - \sin^2 50^\circ$

Using $\sin(2A) = 2\sin A \cos A$ we have:

$$\sin(100^\circ) = \sin(2 \cdot 50^\circ) = 2\sin 50^\circ \cos 50^\circ$$

(3)

10. The interquartile range of the data set $\{3, 7, 13, 14, 17, 20\}$ is

(1) 17 (3) 12

(2) 13.5 (4) 10

$Q_1 = 7$ found using the calculator

$Q_3 = 17$ found using calculator

$IQR = Q_3 - Q_1 = 10$

(4)

11. Which of the following quadratics would have roots that sum to -3?

(1) $3x^2 - 2x + 1 = 0$ (3) $5x^2 + 15x - 2 = 0$

(2) $x^2 - 3x + 8 = 0$ (4) $2x^2 + 8x - 1 = 0$

$$\textbf{sum} = \frac{-b}{a} = \frac{-15}{5} = -3$$

(3)

12. Which of the following represents the range of the function $y = -4\sin(x) + 6$?

(1) $(2, 10)$ (3) $[-10, 2]$

(2) $[2, 10]$ (4) $(-10, 2)$

$y_{min} = 6 - 4 = 2$

$y_{max} = 6 + 4 = 10$

Range: $[2, 10]$

(2)

13. A function, $y = f(x)$, has a *y*-intercept of 7. What is the *y*-intercept of the function $y = 3f(x) - 10$?

(1) 11 (3) –9

(2) –2 (4) –10

$f(0) = 7$

$y = 3f(0) - 10$

$y = 3(7) - 10 = 11$

(1)

14. An angle is drawn in standard position. If its terminal ray lies in the lies in the second quadrant and intersects the unit circle at an *x*-coordinate of $x = -\frac{1}{4}$, then the *y*-coordinate of intersection is

(1) $y = \frac{3}{4}$ (3) $y = \frac{\sqrt{15}}{4}$

(2) $y = -\frac{\sqrt{3}}{2}$ (4) $y = -\frac{1}{2}$

$$\left(-\frac{1}{4}\right)^2 + y^2 = 1 \Rightarrow \frac{1}{16} + y^2 = 1$$

$$y^2 = \frac{15}{16} \Rightarrow y = \pm\sqrt{\frac{15}{16}} = \pm\frac{\sqrt{15}}{4}$$

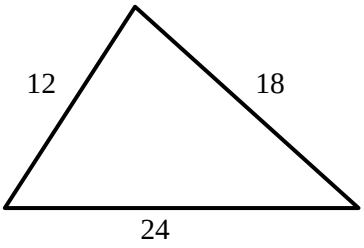
$y = \frac{\sqrt{15}}{4}$ **due to Quad. II**

(3)

15. In the triangle shown below, what is the measure of the *smallest* angle, to the nearest degree?

(1) 29° (3) 76°

(2) 47° (4) 22°



$$12^2 = 24^2 + 18^2 - 2(24)(18)\cos x$$

$$144 = 900 - 864\cos x$$

$$\cos x = \frac{-756}{-864} \Rightarrow x \approx 29^\circ$$

(1)

16. On a multiple choice test with 4 choices per question, which of the following gives the probability of getting exactly seven out of ten questions correct?

(1) ${}_{10}P_7\left(\frac{1}{4}\right)^7\left(\frac{3}{4}\right)^3$ (3) ${}_{10}C_7\left(\frac{7}{10}\right)^1\left(\frac{3}{10}\right)^3$

(2) ${}_{10}C_7\left(\frac{1}{4}\right)^7\left(\frac{3}{4}\right)^3$ (4) ${}_{10}P_7\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^7$

Binomial probability with:

$p = \frac{1}{4}, q = \frac{3}{4}, n = 10$, and $r = 7$

${}_{10}C_7\left(\frac{1}{4}\right)^7\left(\frac{3}{4}\right)^3$

(2)

17. For an angle A where $90^\circ < A < 180^\circ$ it is given that $\sin A = \frac{\sqrt{5}}{3}$. Which of the following is the value of $\cos A$?

(1) $\frac{4}{9}$ (3) $-\frac{\sqrt{3}}{2}$

(2) $-\frac{2}{3}$ (4) $\frac{\sqrt{3}}{3}$

$\cos^2 A + \left(\frac{\sqrt{5}}{3}\right)^2 = 1$

$\cos^2 A + \frac{5}{9} = 1 \Rightarrow \cos^2 A = \frac{4}{9}$

$\cos A = \pm\sqrt{\frac{4}{9}} = \pm\frac{2}{3}$

Quad II $\Rightarrow \cos A = -\frac{2}{3}$

(2)

18. Which of the following sets gives the x-coordinates where the parabola $y = x^2 + 4x - 50$ and the line $y = 7x - 10$ intersect when drawn in the coordinate plane?

(1) $\{-5, 8\}$ (3) $\{-10, 4\}$

(2) $\{-2, 12\}$ (4) $\{0, 25\}$

$x^2 + 4x - 50 = 7x - 10$

$x^2 - 3x - 40 = 0$

$(x - 8)(x + 5) = 0$

$x = 8$ or $x = -5$

(1)

19. In which of the following situations would it be most appropriate to use an entire population instead of a sample?

(1) when determining the average age of teachers in a school district

(2) when determining the average weight of freshmen in American high schools

(3) when determining the average speed of drivers on an interstate highway

(4) when determining the average thickness of the ozone layer surrounding the planet

(1)

It is only appropriate to use the entire population when we can be certain that every member of the population will be surveyed. In all other choices, surveying the entire population is not possible because of its size.

20. In ΔMNP, $m\angle N = 30^\circ$, $MN = 10$ and $MP = 6$. Given this information angle P could be

(1) acute only (3) acute or obtuse

(2) right only (4) obtuse only

$\frac{\sin P}{10} = \frac{\sin 30^\circ}{6} \Rightarrow \sin P = 0.833...$

$P \approx 56.4^\circ$ or 123.6°

Both work with 30°.

(3)

21. The complex fraction $\dfrac{\frac{1}{2}+\frac{3}{x}}{\frac{1}{2}-\frac{1}{2x}}$ can be simplified as

(1) $\dfrac{6x+1}{x}$ (3) $\dfrac{x+6}{x-1}$

(2) $\dfrac{6+x}{x}$ (4) $\dfrac{x+3}{x}$

$$= \frac{\left(\frac{1}{2}+\frac{3}{x}\right)(2x)}{\left(\frac{1}{2}-\frac{1}{2x}\right)(2x)} = \frac{x+6}{x-1}$$

(3)

22. For an arithmetic sequence whose third term is 14 and whose common difference is four, which of the following gives the value of the 20^{th} term?

(1) 90 (3) 86

(2) 76 (4) 82

$$a_2 = 14-4 = 10 \Rightarrow a_1 = 10-4 = 6$$
$$a_{20} = a_1 + 19d = 6+(19)(4) = 82$$

(4)

23. Which of the following is the value of $\sum_{j=-1}^{3} 2^j$?

(1) 12 (3) $8\frac{3}{4}$

(2) $15\frac{1}{2}$ (4) $5\frac{1}{2}$

$$= 2^{-1}+2^0+2^1+2^2+2^3$$
$$= \tfrac{1}{2}+1+2+4+8 = 15\tfrac{1}{2}$$

(2)

24. If the graph of $y = 5\sin(3x-\pi)$ was compared to the graph of $y = 5\sin(3x)$ then the phase shift would be which of the following?

(1) π (3) $\dfrac{\pi}{3}$

(2) 5π (4) $-\dfrac{\pi}{6}$

$$y = 5\sin(3x-\pi)$$
$$= 5\sin\left(3\left(x-\frac{\pi}{3}\right)\right)$$

(3)

25. Which of the following values of *x* solves: $6^{2x-1} = 36^{-x}$?

(1) –4 (3) $\frac{1}{4}$

(2) $\frac{1}{3}$ (4) 4

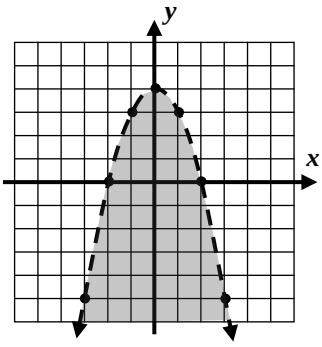
$$6^{2x-1} = \left(6^2\right)^{-x} \Rightarrow 6^{2x-1} = 6^{-2x}$$
$$2x-1 = -2x \Rightarrow 4x = 1 \Rightarrow x = \tfrac{1}{4}$$

(3)

26. Which of the following inequalities represents the graph shown below?

(1) $y > x^2 - 4$ (3) $y \le 4 - x^2$

(2) $y < 4 - x^2$ (4) $y \ge x^2 + 4$



Since the parabola opens downward $a < 0$. As well, since the curve is dashed, the equality is not included.

(2)

27. The solution set to the equation $x - \frac{10}{x} = 3$ is

(1) $\{-2, 5\}$ (3) $\{0, 6\}$

(2) $\{-1, 10\}$ (4) $\{-4, 2\}$

$$x\left(x - \frac{10}{x}\right) = x(3)$$

$$x^2 - 10 = 3x \Rightarrow x^2 - 3x - 10 = 0$$

$$(x+2)(x-5) = 0 \Rightarrow x = -2 \text{ or } x = 5$$

(1)

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [16]

28. Benjamin is choosing three jelly filled donut holes out of a box. If the box contains eight strawberry and six blueberry filled donuts, find the probability that Benjamin will choose two strawberry and one blueberry by randomly choosing his three donuts.

$$P(\text{two strawberry and one blueberry}) = \frac{n(\text{two strawberry}) \cdot n(\text{one blueberry})}{n(\text{total sets of 3 donuts})} = \frac{{}_8C_2 \cdot {}_6C_1}{{}_{14}C_3} = \frac{28 \cdot 6}{364} = \frac{168}{364} \text{ or } \frac{6}{13}$$

29. Find the following product in simplest form:

$$(x+5)(x-5)(x+2)(x-2)$$

$$= (x^2 - 25)(x^2 - 4)$$

$$= x^4 - 4x^2 - 25x^2 + 100$$

$$= x^4 - 29x^2 + 100$$

30. Solve the equation shown below for all values of x in simplest radical form.

$$x^2 - 2x - 6 = 0$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-6)}}{2(1)} = \frac{2 \pm \sqrt{28}}{2}$$

$$= \frac{2 \pm \sqrt{4} \cdot \sqrt{7}}{2} = \frac{2 \pm 2\sqrt{7}}{2} = \frac{\cancel{2}\left(1 \pm \sqrt{7}\right)}{\cancel{2}}$$

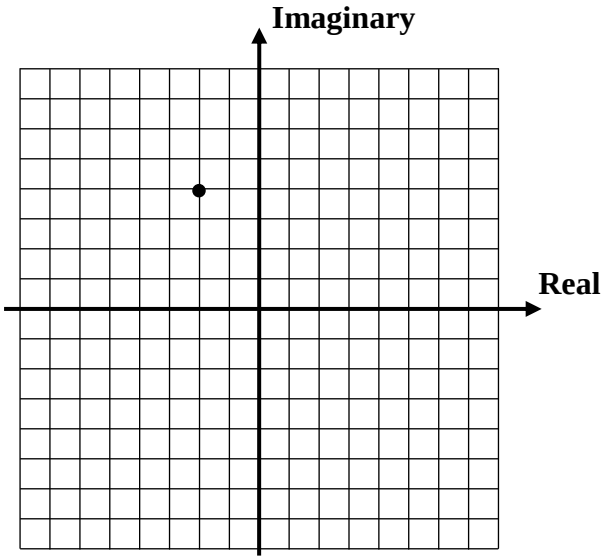
$$= 1 \pm \sqrt{7}$$

31. Find the sum of the complex numbers $-4 + i$ and $2 + 3i$. Express your answer in $a + bi$ form and then plot the result on the complex plane below.

$$= -4 + i + 2 + 3i$$

$$= -2 + 4i$$

$$(-2, 4)$$



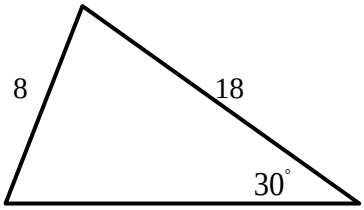
32. Explain why the triangle shown below cannot exist.

$$\frac{\sin x}{18} = \frac{\sin 30^\circ}{8}$$

$$\sin x = \frac{18 \sin 30^\circ}{8} = 1.125$$

But $-1 \le \sin x \le 1$.

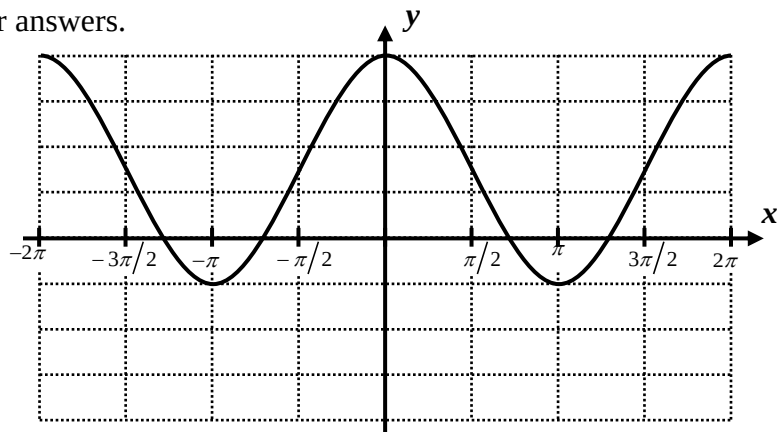
Thus this triangle cannot exist.



33. For a particular real number a and base b it is known that $\log_b a = 2.75$. Determine the value of $\log_b\left(a^3\right)$.

$$\log_b\left(a^3\right) = 3\log_b a = 3(2.75) = 8.25$$

34. The graph below can be described by an equation of the form $y = A\cos(x) + B$. Determine the values of *A* and *B* and show the calculations that lead to your answers.



$y_{min} = -1$ and $y_{max} = 4$

$$A = \frac{y_{max} - y_{min}}{2} = \frac{4-(-1)}{2} = 2.5$$

$$B = \frac{y_{min} + y_{max}}{2} = \frac{-1+4}{2} = 1.5$$

35. A drug company found that a sample of 140 people who took their new diet pill lost an average of 5.75 pounds over a two week period. If their results were normally distributed with a standard deviation of 2 pounds, then approximately how many people lost more than 8.75 pounds?

$\frac{8.75 - 5.75}{2} = 1.5\ \sigma$'s above the mean

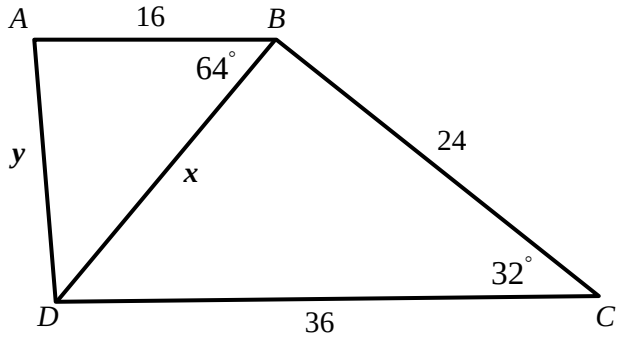
Percent greater $= 4.4 + 1.7 + 0.5 + 0.1 = 6.7\%$

Number of people $= (0.067)(140) = 9.38 \approx 9$

Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

36. In quadrilateral *ABCD* it is known that $AB = 16$, $BC = 24$, $CD = 36$, $m\angle C = 32^\circ$ and $m\angle ABD = 64^\circ$. Determine, to the nearest *tenth* the length of $\overline{AD}$.



$x^2 = 36^2 + 24^2 - 2(36)(24)\cos 32^\circ$

$x^2 = 406.572... \Rightarrow x = \sqrt{406.572...} = 20.163...$

$y^2 = x^2 + 16^2 - 2(x)(16)\cos 64^\circ$

$y^2 = 379.719... \Rightarrow y = \sqrt{379.719...} = 19.486... \approx 19.5$

37. Combine and simplify the following subtraction.

$$\frac{8}{x^2+2x-3}-\frac{6}{x^2+3x}$$

$$=\frac{8}{(x+3)(x-1)}-\frac{6}{x(x+3)}$$

$$=\frac{x}{x}\cdot\frac{8}{(x+3)(x-1)}-\frac{x-1}{x-1}\cdot\frac{6}{x(x+3)}$$

$$=\frac{8x}{x(x+3)(x-1)}-\frac{6x-6}{x(x+3)(x-1)}$$

$$=\frac{2x+6}{x(x+3)(x-1)}=\frac{2\cancel{(x+3)}}{x\cancel{(x+3)}(x-1)}=\frac{2}{x(x-1)}$$

38. Solve the following equation for all value(s) of x: $|x+3|-9=2x$.

$$|x+3|=2x+9$$

$$x+3=\pm(2x+9)$$

$$x+3=-2x-9 \text{ or } x+3=2x+9$$

$$3x=-12 \text{ or } -x=6$$

$$x=-4 \text{ or } \cancel{x=-6}$$

Be sure to check and reject!

Part IV

Answer the question from this part. The correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. A correct numerical answer with no work shown will receive only 1 credit. [6]

39. Algebraically determine the intersection point(s) of the two logarithmic functions given below.

$$y=\log_3(x-6) \text{ and } y=3-\log_3 x$$

$$\log_3(x-6)=3-\log_3 x \Rightarrow \log_3(x-6)+\log_3 x=3$$

$$\log_3\left(x(x-6)\right)=3 \Rightarrow x(x-6)=3^3$$

$$x^2-6x=27 \Rightarrow x^2-6x-27=0$$

$$(x+3)(x-9)=0 \Rightarrow \cancel{x=-3} \text{ or } x=9$$

$x=-3$ is an extraneous root and must be rejected

⇒

We still need the $y-$coordinate of intersection:

$$y=\log_3(9-6)=\log_3(3)=1$$

Intersection at: $(9,1)$

ALGEBRA 2 WITH TRIGONOMETRY PRACTICE EXAM

ANSWER SHEET

Answer Key

Student …….

Teacher ……. School ……

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer all 27 questions in this part.

1 (2)	8 (4)	15 (1)	22 (4)
2 (2)	9 (3)	16 (2)	23 (2)
3 (1)	10 (4)	17 (2)	24 (3)
4 (3)	11 (3)	18 (1)	25 (3)
5 (1)	12 (2)	19 (1)	26 (2)
6 (1)	13 (1)	20 (3)	27 (1)
7 (4)	14 (3)	21 (3)	

Your answers for Parts II, III, and IV should be written in the test booklet.