Name: ___________________________ Date: _______________

THE NUMBER e AND THE NATURAL LOGARITHM
ALGEBRA 2 WITH TRIGONOMETRY

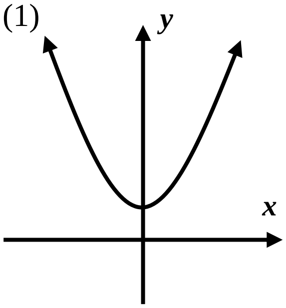
There are many numbers in mathematics that are more important than others because they find so many uses in either mathematics or science. Good examples of important numbers are 0, 1, i, and π. In this lesson you will be introduced to an important number given the letter **e** for its "inventor" Leonhard Euler (1707-1783). This number plays a crucial role in Calculus and more generally in modeling exponential phenomena.

THE NUMBER e

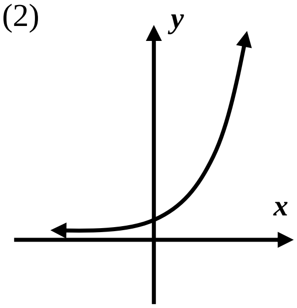
1. Like π, e is irrational. 2. e ≈ 2.72 3. Used in Exponential Modeling

***Exercise* #1:** Which of the graphs below shows $y = e^x$? Explain your choice. Check on your calculator.

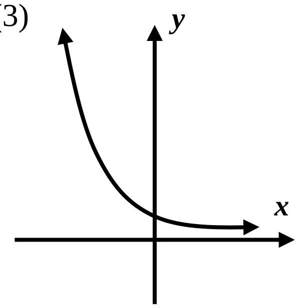
(1)



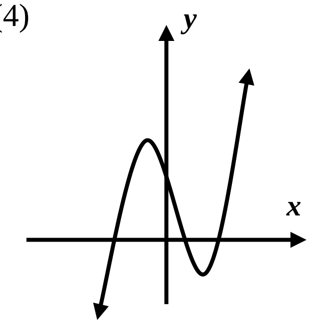
(2)



(3)



(4)



Explanation: __________

Very often e is involved in exponential modeling of both increasing and decreasing quantities. The creation of these models is beyond the scope of this course, but we can still work with them.

***Exercise* #2:** A population of llamas on a tropical island can be modeled by the equation $P = 500e^{0.035t}$, where t represents the number of years since the llamas were first introduced to the island.

(a) How many llamas were initially introduced at $t = 0$. Show the calculation that leads to your answer.

(b) Algebraically determine the number of years for the population to reach 600. Round your answer to the nearest *tenth* of a year.

Because of the importance of $y = e^x$, its **inverse**, known as the **natural logarithm**, is also important.

THE NATURAL LOGARITHM

The inverse of $y = e^x$: $y = \ln x \quad \left(y = \log_e x\right)$

The natural logarithm, like all logarithms, gives an exponent as its output. In fact, it gives the power that we must raise **e** to in order to get the input.

***Exercise* #3:** *Without* the use of your calculator, determine the values of each of the following.

(a) $\ln(e)$ (b) $\ln(1)$ (c) $\ln\left(e^5\right)$ (d) $\ln\sqrt{e}$

The natural logarithm follows the three basic logarithm laws that all logarithms follow. The following problems give additional practice with these laws.

***Exercise* #4:** Which of the following is equivalent to $\ln\left(\frac{x^3}{e^2}\right)$?

(1) $\ln x + 6$ (3) $3\ln x - 6$

(2) $3\ln x - 2$ (4) $\ln x - 9$

____

***Exercise* #5:** Solve the following equation. Express your answer in exact terms of e.

$$2\ln(x+3) - 5 = 3$$

***Exercise* #6:** Find the equation of the inverse of the function $y = \ln(x-8) + 2$.

Name: ____________________________________ Date: _________________

THE NUMBER e AND THE NATURAL LOGARITHM
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

SKILLS

1. Which of the following is closest to the y-intercept of the function whose equation is $y = 10e^{x+1}$?

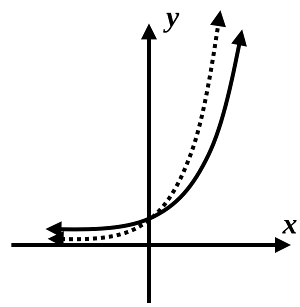
(1) 10 (3) 27

(2) 18 (4) 52 __________

2. On the grid below, the solid curve represents $y = e^x$. Which of the following exponential functions could describe the dashed curve?

(1) $y = \left(\frac{1}{2}\right)^x$ (3) $y = 2^x$

(2) $y = e^{-x}$ (4) $y = 4^x$ __________



3. The logarithmic expression $\ln\left(\frac{\sqrt{e}}{y^3}\right)$ can be rewritten as

(1) $3\ln y - 2$ (3) $\frac{\ln y - 6}{2}$

(2) $\frac{1 - 6\ln y}{2}$ (4) $\sqrt{\ln y} - 3$ __________

4. Which of the following values of x solves the equation $3\ln x = 1$?

(1) $\sqrt[3]{e}$ (3) e^3

(2) $\frac{1}{3}e$ (4) $3e$ __________

5. The inverse of $y = \ln(x+5)$ is

(1) $y = e^{x-5}$ (3) $y = -\ln(x-5)$

(2) $y = \frac{1}{\ln(x+5)}$ (4) $y = e^x - 5$ __________

6. Solve each of the following logarithmic equations for the value of x. Express your answers in terms of **e**.

(a) $2\ln x - 1 = 15$ (b) $5\ln(x-2)+2=17$ (c) $\frac{2}{3}\ln\left(\frac{1}{2}x\right)+3=11$

7. Find the equation of the inverse for each of the following logarithmic functions shown below.

(a) $y=\ln(x+6)-3$ (b) $y=\ln\left(\frac{1}{3}x-1\right)$

APPLICATIONS

8. Flu is spreading exponentially at a school. The number of new flu patients can be modeled using the equation $F=10e^{.12d}$, where d represents the number of days since 10 students had the flu. How many days will it take for the number of new flu patients to equal 50? Round your answer to the nearest day.

9. The savings in a bank account can be modeled using $S=1250e^{.045t}$, where t is the number of years the money has been in the account. Determine, to the nearest *tenth* of a year, how long it will take for the amount of savings to double from the initial amount deposited of $1250.