Name: ____________________________________ Date: _________________

EXPONENTIAL MODELING – DAY 1
ALGEBRA 2 WITH TRIGONOMETRY

Exponential functions are important primarily because of their usefulness modeling a variety of real world phenomena where the percent that a quantity changes by over time stays constant. In this lesson we will explore how to model phenomena that **increase** as a function of time.

***Exercise* #1:** Suppose that you deposit money into a savings account that receives 5% interest per year on the amount of money that is in the account for that year. Assume that you deposit $400 into the account initially.

(a) How much will the savings account increase by over the course of the year?

(b) How much money is in the account at the end of the year?

(c) By what single number could you have multiplied the $400 by in order to calculate your answer in part (b)?

(d) Using your answer from part (c), determine the amount of money in the account after 2 and 10 years. Round all answers to the nearest cent when needed.

(e) Give an equation for the amount in the savings account $S(t)$ as a function of the number of years since the $400 was invested.

(f) Using a table on your calculator determine, to the nearest year, how long it will take for the initial investment of $400 to double. Provide evidence to support your answer.

The thinking process from *Exercise* #1 can be generalized to any situation where a quantity is increased by a fixed percentage over a fixed interval of time. This pattern is summarized below:

INCREASING EXPONENTIAL MODELS

If quantity *Q* is known to increase by a fixed percentage *p*, in decimal form, then *Q* can be modeled by

$$Q(t) = Q_0(1+p)^t$$

where Q_0 represents the amount of *Q* present at $t=0$ and *t* represents time.

***Exercise* #2:** Which of the following gives the savings *S* in an account if $250 was invested at an interest rate of 3% per year?

(1) $S = 250(4)^t$

(3) $S = (1.03)^t + 250$

(2) $S = 250(1.03)^t$

(4) $S = 250(1.3)^t$

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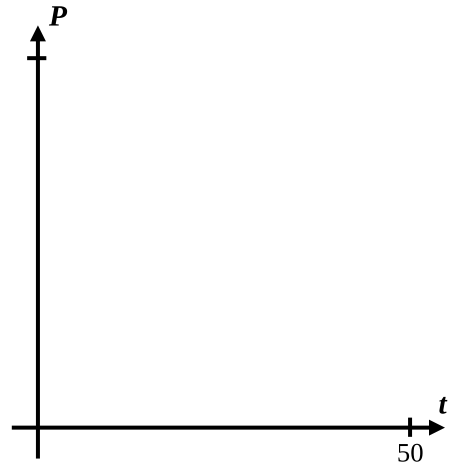
There are many other real world phenomena that follow this exponentially increasing pattern. A good example is that of unrestricted population growth. If a population has no constraints on its growth then it typically increases at a fixed rate.

***Exercise* #3:** The population of Fruit Bud, NY, is increasing at a steady pace of 2.5% per year. In order to plan for school growth, the town board would like to mathematically model the future population of the town. At the end of 2005, Fruit Bud had a population of 6,500 residents.

(a) Determine an equation for Fruit Bud's population, P, as a function of the number of years, t, since the end of 2005.

(b) Sketch a graph of Fruit Bud's population as a function of time on the axes below for the 50 years following 2005. Be sure to label your y-intercept and your graphing window. Mark Fruit Bud's population at the end of 2055.



(c) Use your model from (a) to predict the number of residents at the end of 2010. Round your answer to the nearest person.

(d) Graphically determine the year that the population reaches the 10,000 mark. Show work on your graph from part (c).

***Exercise* #4:** The number of students at a particular school who have the flu is increasing at a rate of 12% per day. If at the beginning of Monday there were 56 students with the flu, then which of the following is closest to the number of students who have the flu on Friday morning?

(1) 104 (3) 88

(2) 74 (4) 128 ________

***Exercise* #5:** If a population of 45 rabbits on an island is increasing at a rate of 8% per year, which of the followings is closest to the time it will take for the population to double?

(1) 14 years (3) 6 years

(2) 22 years (4) 9 years ________

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EXPONENTIAL MODELING – DAY 1
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

APPLICATIONS

1. If \$130 is invested in a savings account that earns 4% interest per year, which of the following is closest to the amount in the account at the end of 10 years?

   (1) \$218 (3) \$168

   (2) \$192 (4) \$324 _________

2. A population of 50 fruit flies is increasing at a rate of 6% per day. Which of the following is closest to the number of days it will take for the fruit fly population to double?

   (1) 18 (3) 12

   (2) 6 (4) 28 _________

3. The population of Arlington High School is increasing at an annual rate of 3.5% per year. If the population this year is 3,500 students, which of the following represents the population in 10 years?

   (1) 4,937 (3) 3,793

   (2) 5,129 (4) 70,373 _________

4. Which of the following equations would model a population with an initial size of 625 that is growing at an annual rate of 8.5%?

   (1) $P = 625(8.5)^t$ (3) $P = 1.085^t + 625$

   (2) $P = 625(1.085)^t$ (4) $P = 8.5t^2 + 625$ _________

5. Given that population can be modeled by the equation $P = 185(1.065)^t$ which of the following is true?

   (1) The initial population was 185 and the population is growing at a rate of 6.5%.

   (2) The initial population was 152 and the population is growing at a rate of 65%.

   (3) The initial population was 185 and the population is growing at a rate of 65%.

   (4) The initial population was 152 and the population is growing at a rate of 6.5%. _________

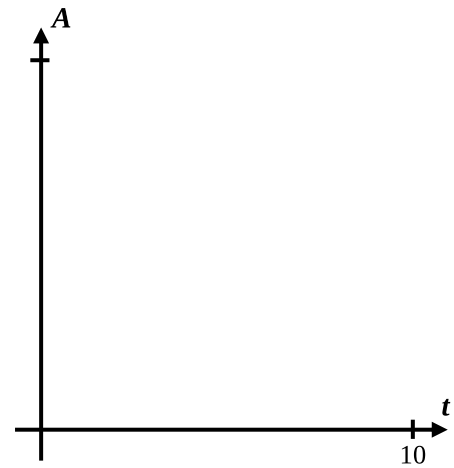
6. A bank customer would like to keep track a 10-year investment. She initially deposits \$750 into an account that earns a constant 4.5% interest compounded annually.

(a) How much money does the customer have in the bank at the end of a single year? Show the calculation that leads to your answer.

(b) Write an equation for the amount of money $A(t)$ in the account as a function of the number of years, t, that have passed.

(d) Between what two consecutive years will the account reach \$1,000? Show numerical evidence to support your answer.

(c) Sketch a graph of your function on the axes below for $0 \le t \le 10$. Be sure to label your window, the y-intercept and the amount of money in the account at the end of 10 years.



7. Red Hook has a population of 6,200 people and is growing at a rate of 8% per year. Rhinebeck has a population of 8,750 and is growing at a rate of 6% per year. In how many years, to the nearest year, will Red Hook have a greater population than Rhinebeck? Show the equation or inequality you are solving and solve it graphically.

REASONING

8. Does the initial amount invested make a difference on the amount of time it takes for an investment to double? Experiment with an investment that returns 5% per year. Try initial investments of \$100, \$300, and \$1000.