Name: ___________________________________ Date: _________________

EXPONENTIAL MODELING – DAY 2
ALGEBRA 2 WITH TRIGONOMETRY

Exponential models can both increase and decrease depending the whether their base is greater than one (increasing) or between zero and one (decreasing). There are many scenarios in the real world that can be described by decreasing exponential functions. Their development will be similar to those that increase.

***Exercise* #1:** A radioactive substance is decreasing such that 2% of it is lost every year. This is known as radioactive decay. Originally there were 200 kilograms of this substance.

(a) How much of the material remains after one year?

(b) By what single number could have your multiplied the 200 kg by in order to calculate your answer in part (b)?

(c) Determine an exponential function for the amount of radioactive substance left $A(t)$ as a function of the number of years it has been decaying, *t*.

(d) Use tables on your calculator to determine, to the nearest year, the half-life of this substance, i.e. the time it takes for only half the amount to remain. Provide numerical evidence to support your answer.

Similar to the case of increasing exponentials, ones that decrease follow a very predictable pattern:

DECREASING EXPONENTIAL MODELS

If quantity *Q* is known to decrease by a fixed percentage *p*, in decimal form, then *Q* can be modeled by

$$Q(t)=Q_0(1-p)^t$$

where Q_0 represents the amount of *Q* present at $t=0$ and *t* represents time.

***Exercise* #2:** If the population of a town is decreasing by 4% per year and started with 12,500 residents, which of the following is its projected population in 10 years?

(1) 9,230 (3) 18,503

(2) 76 (4) 8,310

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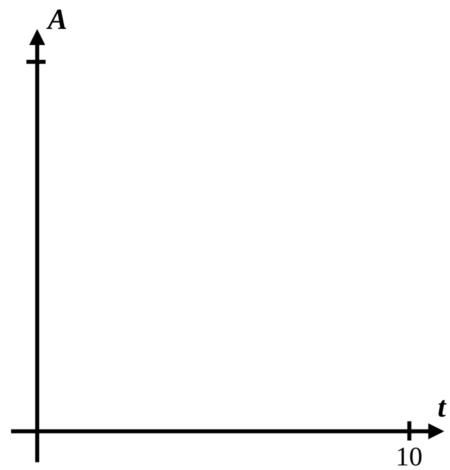
***Exercise* #3:** Hydrologists have found that the amount of water that a soil can absorb, in inches per hour, decreases by 30% for each hour that the soil is flooded with water. A particular dry soil is initially able to absorb 4 inches of water per hour.

(a) Write an equation for the amount of water, $A(t)$, in inches per hour, that this soil can absorb, if t represents the number of hours since the soil was flooded.

(b) Sketch a graph of the function that you wrote in part (a) on the axes below for all times on the interval $[0,10]$. Label your *y*-intercept.



(c) A steady rainfall of 0.5 inches per hour is flooding this soil. Water will runoff this soil when the rainfall rate exceeds the rate at which the soil can absorb the rain. At what time, to the nearest *tenth* of an hour, will runoff start? Show your solution graphically on the graph in part (b).

***Exercise* #4:** The stock price of WindpowerInc is increasing at a rate of 4% per week. Its initial value was $20 per share. On the other hand, the stock price in GerbilEnergy is crashing (losing value) at a rate of 11% per week. If its price was $120 per share when Windpower was at $20, after how many weeks will the stock prices be the same? Show the equation you are solving and then solve it graphically. Round your answer to the nearest week.

Name: ___________________________________ Date: _________________

EXPONENTIAL MODELING – DAY 2
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

APPLICATIONS

1. If a radioactive substance is quickly decaying at a rate of 13% per hour approximately how much of a 200 pound sample remains after one day?

(1) 7.1 pounds (3) 25.6 pounds

(2) 2.3 pounds (4) 15.6 pounds _________

2. A population of llamas stranded on a dessert island is decreasing due to a food shortage by 6% per year. If the population of llamas started out at 350, how many are left on the island 10 years later?

(1) 257 (3) 102

(2) 58 (4) 189 _________

3. When the world market is glutted with oil, its price per barrel can drop by as much as 5% per day. If the per barrel price of oil starts the beginning of the week with a value of $120 and loses 5% per day, how much value will it *lose* over the span of 5 days?

(1) $92.85 (3) $27.15

(2) $54.75 (4) $32.80 _________

4. The acceleration of an object falling through the air will decrease at a rate of 15% per second due to air resistance. If the initial acceleration due to gravity is 9.8 meters per second per second, which of the following equations best models the acceleration t seconds after the object begins falling?

(1) $a = 15 - 9.8t^2$ (3) $a = 9.8(1.15)^t$

(2) $a = \frac{9.8}{15t}$ (4) $a = 9.8(0.85)^t$ _________

5. A radioactive substance loses 1.5% of its mass each year that it decays. Which of the following is closest to its half-life in years?

(1) 58 (3) 215

(2) 46 (4) 3,176 _________

6. A warm glass of water, initially at 120 degrees Fahrenheit, is placed in a refrigerator at 34 degrees Fahrenheit and its temperature is seen to decrease according to the exponential function

$$T(h) = 86(0.83)^h + 34$$

(a) Verify that the temperature starts at 120 degrees Fahrenheit by evaluating $T(0)$.

(b) Using your calculator, sketch a graph of T below for all values of h on the interval $0 \le h \le 24$. Be sure to label your *y*-axis and *y*-intercept.

(c) After how many hours will the temperature be at 50 degrees Fahrenheit? State your answer to the nearest *hundredth* of an hour. Illustrate your answer on the graph your drew in (b).

7. The pressure of the atmosphere decreases as one increases one's altitude. For every kilometer one rises in the atmosphere 14% of the pressure is lost. The atmospheric pressure at sea-level is approximately 15 pounds per square inch.

(a) Write an equation for the atmospheric pressure, $P(z)$, as a function of the number of kilometers, *z*, above sea-level.

(b) How much does the air pressure need to be increased in a plane that is flying at an altitude of 9800 meters above sea-level in order for passengers to be experiencing 15 pounds per square inch of pressure? Round your answer to the nearest *tenth* of a pound per square inch.

(c) Humans have difficulty breathing when the air pressure drops below 10 pounds per square inch. At what altitude, to the nearest *meter*, will a person have difficulty breathing? Produce a graph to justify your answer.