Name: _____ Date: _____

PERMUTATIONS AND THE FUNDAMENTAL COUNTING PRINCIPLE
ALGEBRA 2 WITH TRIGONOMETRY

Because many probability problems involve counting the number of outcomes to an experiment and event, it is important to develop techniques that will allow us to count the outcomes in sample spaces that are very large. The primary tool for this counting is known as the **Fundamental Counting Principle** and will be illustrated in *Exercise* #1.

***Exercise* #1:** Consider flipping a standard coin and rolling a standard six-sided die.

(a) Draw a tree diagram to determine the total number of outcomes for this experiment.

(b) How could you have determined the total number of outcomes by considering the outcomes just for the coin and the outcomes just for the die?

THE FUNDAMENTAL COUNTING PRINCIPLE

Suppose and event, E, can be broken into a sequence of n sub-events, $E_1, E_2, ..., E_n$. Then the total number of ways that E can occur is given by:

$$n(E) = n(E_1) \cdot n(E_2) \cdot ... \cdot n(E_n)$$

The advantage of the Counting Principle is that the sub-events are often **easier** to count than the overall, more complex event.

***Exercise* #2:** At a cafeteria, a student can order the daily special for $4.99 consisting of one drink, one salad, and one entree. If students have five drinks to choose from, four salads, and three entrees, how many different choices to do they have for the daily special?

***Exercise* #3:** Letters are to be chosen from the word HYPERBOLA without replacement to create a four-letter code. Determine the number of these four-letter codes that begin with two vowels (here Y is a vowel).

A special application of the Counting Principle occurs quite often when we count the number of ways to rearrange items from a given set. These arrangements are known as **permutations**. A permutation is simply any ordering of objects from a set when those objects have been chosen **without replacement**.

***Exercise* #4:** Consider the letters in the word CAT. Two-letter codes are to be made out of these three letters without repetition of letters.

(a) List all of the different two-letter codes.

(b) Calculate the number of codes using the Counting Principle.

Permutations are used so often in counting that they have their own notation.

PERMUTATION NOTATION

The number of ways to permute *n* objects taken *r* at a time is notated as:

$${}_nP_r$$

***Exercise* #5:** Write out each of the following, given in permutation notation, as an equivalent product. Evaluate the product using multiplication and using your calculator's permutation function.

(a) ${}_8P_3$

(b) ${}_{15}P_2$

(c) ${}_4P_4$

***Exercise* #6:** Ten runners are entered in the 100-yard dash. How many different ways could the runners finish first, second and third? Express your answer in permutation notation and as an integer.

***Exercise* #7:** Darla chose 12 songs for her workout playlist. If she puts her mp3 player on a random, no-repeat setting, in how many different orders could Darla listen to all 12 songs? Express your answer in permutation notation, using factorial, and as an integer.

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PERMUTATIONS AND THE FUNDAMENTAL COUNTING PRINCIPLE
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

APPLICATIONS

1. From a club of 28 students, a president, vice-president, and treasurer must be chosen. In how many different ways can this be done if no student can serve in more than one position?

2. License plates in a certain state consist of six characters, each of which could be either a letter or digit. If any of the 10 digits or the 26 letters can be chosen, determine how many license plates can be created that fit the following criteria.

   (a) Each character can be either a digit or letter, but no letter or digit can be repeated.

   (b) The first three characters must be letters, the last three digits and repetition of characters is allowed.

   (c) The first four characters must be letters and the last two digits and repetition is allowed for numbers, but not letters.

   (d) The first character must be a letter, the last a number, and no repetition is allowed.

3. Consider a true-false quiz that has five total questions on it.

   (a) How many different ways are there of filling out the answers on this quiz? Use the Fundamental Counting Principle based on the fact that there are two ways to answer each individual question.

   (b) Since there is only one way to answer all of the questions correctly, what is the probability that random guessing of all questions will result in a perfect? Express your answer as a fraction and as a percent.

4. Which of the following would be equivalent to ${}_{12}P_3$?

(1) $10+11+12$ (3) $12\cdot 3$

(2) $12\cdot 11\cdot 10$ (4) $12\cdot 11\cdot ...\cdot 4\cdot 3$ _____

5. Eight contestant are awarded prizes for finishing first through fourth in a swimming race, where the prizes vary by finishing rank. Which of the following represents the number of ways these prizes can be awarded?

(1) 1,680 (3) 384

(2) 4,560 (4) 32

_____

6. Which of the following would calculate the number of 3-letter codes that can be made from the letters in the word TORQUE if no letter can be repeated.

(1) $6\cdot 3$ (3) $6!$

(2) ${}_6P_3$ (4) $6+3$

_____

7. Six runners compete in a track race. Three are from Red Hook and three are from Rhinebeck.

(a) In how many different orders can these six runners finish?

(b) In how many different orders can these runners finish if the first three are from Red Hook and the last three are from Rhinebeck?

(c) What is the probability, if all of the runners are equally talented, that the three from Red Hook would finish in first through third by chance alone?

(d) What is the probability, if all of the runners are equally talented, that the first three finishers would be from the same school?

8. A computer service recently added a new six-character password to their login system. The first character of the password has to be a letter and must be uppercase. The last character had to be a digit (0 to 9). The other four characters could be either digits or letters, and those letters could be either uppercase or lowercase, in other words the password is "case sensitive." How many different passwords could be created if letters and digits may be repeated?