

ARITHMETIC SERIES
ALGEBRA 2 WITH TRIGONOMETRY

A **series** is simply the **sum of the terms of a sequence**. The fundamental definition/notion of a series is below.

THE DEFINITION OF A SERIES

If the set $\{a_1, a_2, a_3, \dots\}$ represent the elements of a sequence then the series, S_n , is defined by:

$$S_n = \sum_{i=1}^n a_i$$

In truth, you have already worked extensively with series in previous lessons almost anytime you evaluated a summation problem.

Exercise #1: Given the arithmetic sequence defined by $a_1 = -2$ and $a_n = a_{n-1} + 5$, then which of the following is

the value of $S_5 = \sum_{i=1}^5 a_i$?

(1) 32

(3) 25

(2) 40

(4) 27

The sums associated with arithmetic sequences, known as **arithmetic series**, have interesting properties, many applications and values that can be predicted with what is commonly known as **rainbow addition**.

Exercise #2: Consider the arithmetic sequence defined by $a_1 = 3$ and $a_n = a_{n-1} + 2$. The series, based on the first eight terms of this sequence, is shown below. Terms have been paired off as shown.

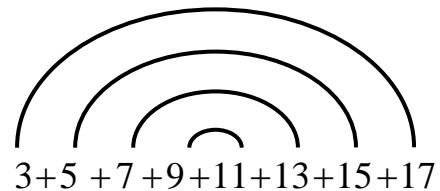
(a) What does each of the paired off sums equal?

(b) Why does it make sense that this sum is constant?

(c) How many of these pairs are there?

(d) Using your answers to (a) and (c) find the value of the sum using a multiplicative process.

(e) Generalize this now and create a formula for an arithmetic series sum based only on its first term, a_1 , its last term, a_n , and the number of terms, n .



SUM OF AN ARITHMETIC SERIES

Given an arithmetic series with n terms, $\{a_1, a_2, \dots, a_n\}$, then its sum is given by:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Exercise #3: Which of the following is the sum of the first 100 natural numbers?

- (1) 5,000 (3) 10,000
(2) 5,100 (4) 5,050
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Exercise #4: Find the sum of each arithmetic series described or shown below.

(a) The sum of the sixteen terms given by:

$$-10 + -6 + -2 + \dots + 46 + 50.$$

(b) The first term is -8 , the common difference, d , is 6 and there are 20 terms

(c) The last term is $a_{12} = -29$ and the common difference, d , is -3 .

(d) The sum $5 + 8 + 11 + \dots + 77$.

Exercise #5: Kirk has set up a college savings account for his son, Maxwell. If Kirk deposits \$100 per month in an account, increasing the amount he deposits by \$10 per month each month, then how much will be in the account after 10 years?



5. For an arithmetic series that sums to 1,485, it is known that the first term equals 6 and the last term equals 93. *Algebraically* determine the number of terms summed in this series.

APPLICATIONS

6. Arlington High School recently installed a new black-box theatre for local productions. They only had room for 14 rows of seats, where the number of seats in each row constitutes an arithmetic sequence starting with eight seats and increasing by two seats per row thereafter. How many seats are in the new black-box theatre? Show the calculations that lead to your answer.
7. Simeon starts a retirement account where he will place \$50 into the account on the first month and increasing his deposit by \$5 per month each month after. If he saves this way for the next 20 years, how much will the account contain in principal?

REASONING

8. For an arithmetic series it is known that the first term is 2, the common difference is 5, and the sum is 87. Determine the number of terms in this series. Hint – you will need to solve a system of two equations.

