Name: _____ Date: _____

MODELING WITH QUADRATIC FUNCTIONS
ALGEBRA 2 WITH TRIGONOMETRY

We have already seen quadratic functions used to model a wide variety of real-world applications. In this lesson we will focus on using quadratic models to predict and optimize. Almost always our solution will depend on either the intercepts or the turning point of the quadratic model.

***Exercise* #1:** An object is fired upwards with an initial velocity of 112 feet per second. Its height, in feet above the ground, as a function of time, in seconds since it was fired, is given by the equation $h(t) = -16t^2 + 112t$.

(a) At what height was the object fired?

(b) Sketch a general curve of this equation below.

(c) Algebraically, find the time that the rocket reaches its greatest height and the maximum height. Label these on the graph that you drew in part (b).

(d) Algebraically, determine the time when the rocket reaches the ground. Label this on your graph in (b).

***Exercise* #2:** A skateboard half-pipe ramp has a shape in the form of a parabola whose equation is $y = 0.06x^2 - 1.2x + 7$ where x represents the horizontal distance across the 20-foot wide half-pipe and y represents the ramp's height above the ground in feet. With the help of your calculator, sketch a graph of the half-pipe below Label its height at its endpoints and its minimum point.

***Exercise* #3:** The Crazy Carmel Corn company has determined that the percentage of kernels that pop rises and then falls as the temperature of the oil the kernels are cooked in increases. It modeled this trend using the equation

$$P = -\frac{1}{250}T^2 + 2.8T - 394$$

Where P represents the percent of the kernels that pop and T represents the temperature of the oil in degrees Fahrenheit.

(a) Algebraically determine the temperature at which the highest percentage of kernels pop. Also, determine the percent of kernels that pop at this temperature.

(b) Using your calculator, sketch a curve below for $P \geq 0$. Label your window.

(c) Using the **ZEROS** command on your calculator, determine, to the nearest degree, the two temperatures at which $P = 0$. Label them on your graph drawn in part (b).

(d) If a typical batch of popcorn consists of 800 kernels, how many does the Crazy Carmel Corn company expect to pop at the optimal temperature?

(e) For a batch of popcorn to be successful, the company wants at least 85% of its kernels to pop. Write an inequality whose solution represents all temperatures that would ensure a successful batch. Solve this inequality graphically, to the nearest degree, and show your graph to below, labeling all relevant points.

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MODELING WITH QUADRATIC FUNCTIONS
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

APPLICATIONS

1. The height of a missile *t* seconds after it has been fired is given by $h = -4.9t^2 + 44.1t$. Which of the following represents the number of seconds it will take for the rocket to reach its greatest height?

   (1) 108 (3) 99

   (2) 4.5 (4) 7.5

   _____

2. The daily cost per car manufactured at a certain automotive plant decreases as the number of cars increase and then increases again due to overtime production costs. The cost *C*, per car, is given by $C(n) = 0.3n^2 - 90n + 12,450$ where *n* represents the number of cars produced. Which of the following is the lowest per car cost?

   (1) \$5,700 (3) $\$12,450$

   (2) \$150 (4) \$2,150

   _____

3. A decathlete at the Olympics throws a javelin such that its height, *h*, above the ground can be modeled as a quadratic function of the horizontal distance, *d*, that it has traveled. Which of the following is a realistic quadratic function for this scenario?

   (1) $h = \frac{1}{100}d^2 + 75d + 3$ (3) $h = -\frac{1}{100}d^2 + 75d + 3$

   (2) $h = \frac{1}{100}d^2 + 75d - 3$ (4) $h = -\frac{1}{100}d^2 + 75d - 3$

   _____

4. A ball thrown vertically in the air reaches its peak height after 3.5 seconds. If its height, as a function of time, is given by $h = -16t^2 + bt + 4$, then which of the following is the value of *b*?

   (1) 56 (3) -112

   (2) -56 (4) 112

   _____

5. A tour company has a ticket price that goes down \$2 for every additional person who signs up for a group trip. They charge, per person, $52 - 2n$ where *n* is the number of people that go on the trip. Their total revenue, *R*, as a function of the number of people who go on the trip is $R = 52n - 2n^2$. How many people maximize the revenue for the tour company?

   (1) 13 (3) 26

   (2) 39 (4) 22

   _____

6. Bacteria tend to grow very fast in a Petri dish at first because of unlimited food and then begin to die out due to competition. In a certain culture, the number of bacteria is given by $N(t) = -2t^2 + 92t + 625$, where t represents the hours since 625 bacteria were introduced to the Petri dish. Determine the maximum number of bacteria that occur in the dish.

7. A tennis ball is thrown upwards from the top of a 30-foot high building. Its height, in feet above the ground, t-seconds after it is thrown is given by $h = -16t^2 + 80t + 30$.

(a) Algebraically determine the time when the tennis ball reaches its greatest height? What is that height?

(b) Using your calculator, sketch a general graph showing the ball's height for all times where $t \geq 0$ and $h \geq 0$. Label the information you found in part (a).

(c) Using the **ZERO** command on your calculator, determine the amount of time the ball stays in the air. Round your answer to the nearest tenth of a second and label this on your graph drawn in part (a).

(d) The ball can be seen from the ground whenever it is at a height of at least 100 feet. Graphically determine the interval of time that the ball can be seen. Show the work on your graph from in part (b).

8. The area of a rectangle whose perimeter is a fixed 80 feet is given by $A = 40w - w^2$, where w is the width of the rectangle. Determine the width of the rectangle that gives the maximum area. What type of special rectangle is necessary to produce this maximum area? Justify.