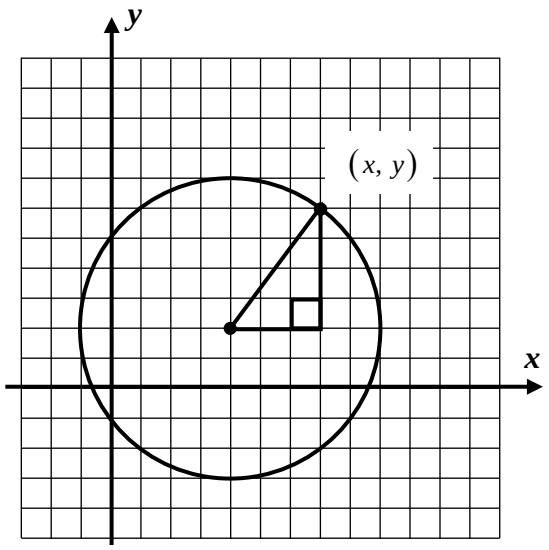
Name: _____ Date: _____

EQUATIONS OF CIRCLES
ALGEBRA 2 WITH TRIGONOMETRY

Not all quadratic relationships, those involving squared quantities, are functions. One such relationship is the equation of a circle. The first exercise will show how its equation is developed by a very simple application of the Pythagorean Theorem.

***Exercise* #1:** The circle shown below has a radius of 5 and a center at the point $(4, 2)$. An arbitrary point on the circle, (x, y), is shown marked.



(a) What length does the binomial $(x-4)$ represent on the right triangle shown? Mark it on the graph.

(b) What length does the binomial $(y-2)$ represent on the right triangle shown? Mark it on the graph.

(c) Using the Pythagorean Theorem, write an equation that related the two binomials from (a) and (b). This is the **equation of this circle**.

(d) Show algebraically that the point $(1, -2)$ must also lie on the circle.

THE EQUATION OF A CIRCLE

A circle whose center is at (h, k) and whose radius is r is given by

$$(x-h)^2+(y-k)^2=r^2$$

***Exercise* #2:** Which of the following equations would have a center of $(-3, 6)$ and a radius of 3?

(1) $(x-3)^2+(y+6)^2=9$ (3) $(x-3)^2+(y-6)^2=3$

(2) $(x+3)^2+(y-6)^2=9$ (4) $(x+3)^2+(y+6)^2=3$ _____

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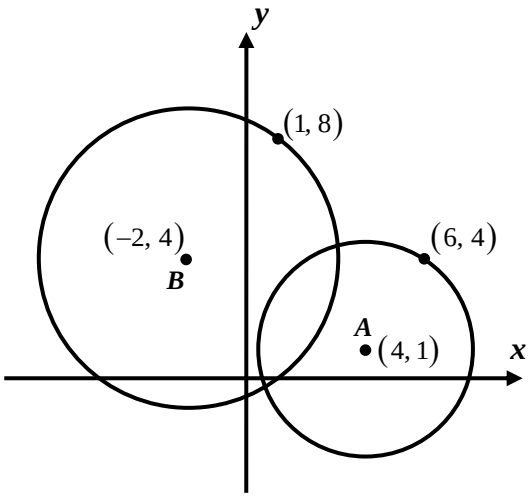
***Exercise* #3:** For each of the following equations of circles, determine both the circle's center and its radius. If its radius is not an integer, express it in decimal form rounded to the nearest *tenth*.

(a) $(x-2)^2+(y-7)^2=100$ (b) $(x-5)^2+(y+8)^2=4$ (c) $x^2+y^2=121$

(d) $(x+1)^2+(y+2)^2=1$ (e) $x^2+(y-3)^2=49$ (f) $(x+6)^2+(y-5)^2=18$

(g) $x^2+y^2=64$ (h) $(x-4)^2+(y-2)^2=20$ (i) $x^2+y^2=57$

***Exercise* #4:** Write equations for circles *A* and *B* shown below. Show how you arrive at your answers.



***Exercise* #5:** By completing the square on both quadratic expressions in *x* and *y* determine the center and radius of a circle whose equation is

$$x^2+10x+y^2-2y=10$$

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EQUATIONS OF CIRCLES
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

SKILLS

1. Each of the following is an equation of a circle. State the circle's center and radius. In the cases where the radius is not an integer, give its value rounded to the nearest tenth.

(a) $x^2+y^2=144$ (b) $(x-3)^2+(x+7)^2=36$ (c) $(x+5)^2+(y+1)^2=64$

(d) $(x-2)^2+(y-9)^2=100$ (e) $x^2+y^2=1$ (f) $x^2+(y+5)^2=25$

(g) $x^2+y^2=50$ (h) $(x-3)^2+y^2=200$ (i) $(x-6)^2+(y+6)^2=20$

2. Which of the following is true about a circles whose equation is $(x+5)^2+(y-3)^2=36$?

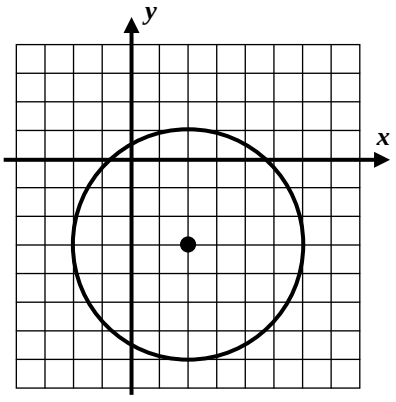
(1) It has a center of $(5,-3)$ and an area of 12π.

(2) It has a center of $(-5,3)$ and a diameter of 6.

(3) It has a center of $(-5,3)$ and an area of 36π.

(4) It has a center of $(5,-3)$ and a circumference of 12π. _____

3. Which of the following represents the equation of the circle shown graphed below?



(1) $(x-2)^2+(y+3)^2=16$

(2) $(x+2)^2+(y-3)^2=4$

(3) $(x-2)^2+(y+3)^2=4$

(4) $(x+2)^2+(y-3)^2=16$ _____

4. By completing the square on each of the quadratic expressions, determine the center and radius of a circle whose equation is shown below.

$x^2-6x+y^2+10y=66$

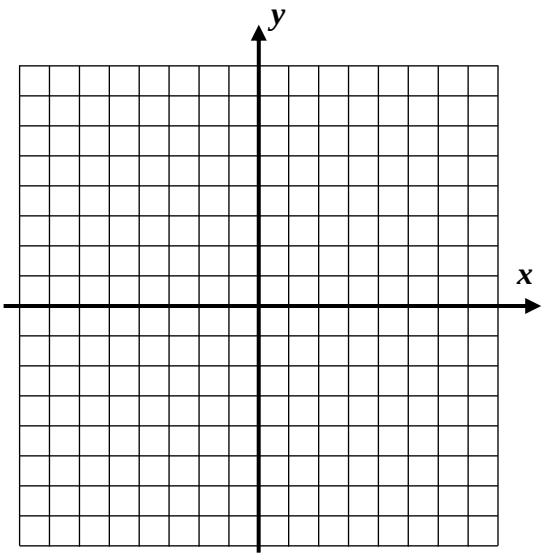
5. Circles are described below by the coordinates of their centers, C, and one point on their circumference, A. Determine an equation for each circle in center-radius form.

   (a) $C(5, 2)$ and $A(11, 10)$ (b) $C(-2, -5)$ and $A(3, -17)$ (c) $C(5, -1)$ and $A(-2, -5)$

6. Solve the following system of equations *graphically*.

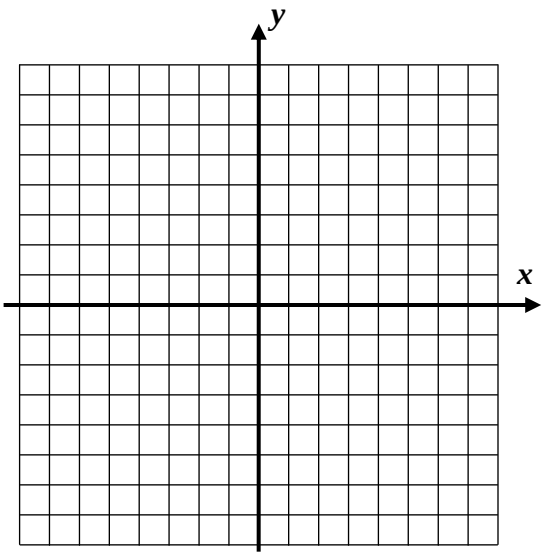
   $x^2 + y^2 = 25$

   $y = 5 - x^2$



APPLICATIONS

7. Jonas is designing a circular garden whose equation is $x^2 + y^2 = 49$. He wishes to place a walkway within the garden at all points within the circle that satisfy the inequality $-2 \le y \le 2$. Graph the circle on the grid to the right and shade in all points that represent the walkway.



8. By counting squares, estimate the area of the walkway. (The actual area, easily calculable using techniques in calculus, is 55.228, accurate to three decimal places.)