Name: ______________________________ Date: ______________

GRAPHS AND ROOTS OF A POLYNOMIAL
ALGEBRA 2 WITH TRIGONOMETRY

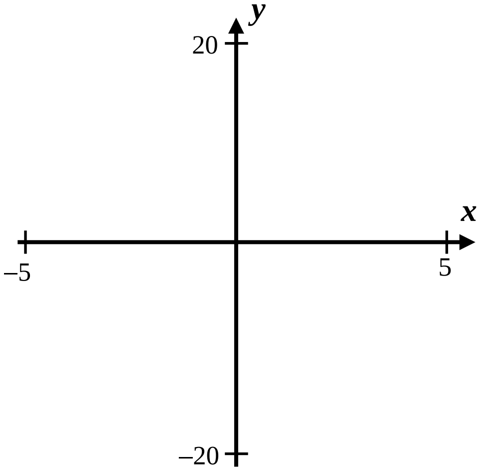
A polynomial is a function consisting of terms that all have whole number powers. In its most general form, a polynomial can be written as:

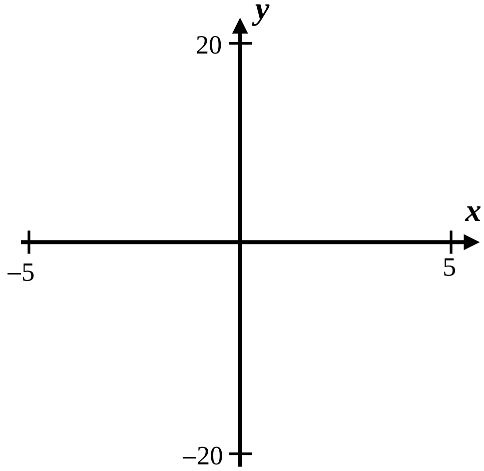
$$y = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

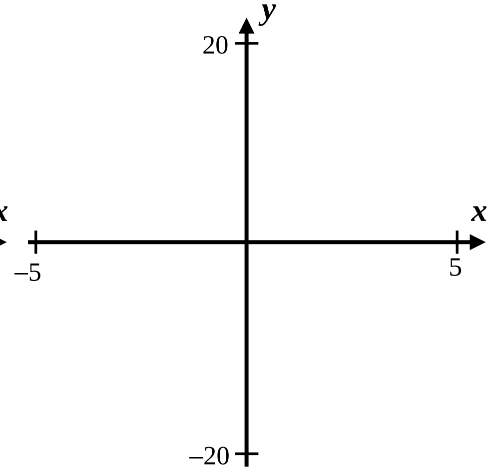
Quadratic and linear functions are the simplest of all polynomials. In this lesson we will explore cubic and quartic functions, those whose highest powers are x^3 and x^4 respectively.

***Exercise* #1:** For each of the following cubic functions, sketch the graph and circle its x-intercepts.

(a) $y = x^3 - 3x^2 - 6x + 8$ (b) $y = 2x^3 - 8x + 9$ (c) $y = 2x^3 - 12x^2 + 18x$





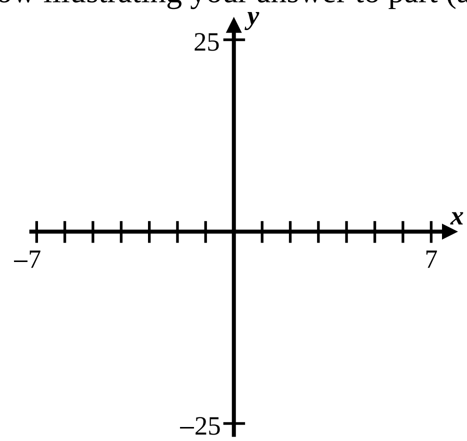


Clearly, a cubic may have one, two or three real roots and can have two turning points. Just as with parabolas, there exists a tie between a cubic's factors and its x-intercepts.

***Exercise* #2:** Consider the cubic whose equation is $y = x^3 - x^2 - 12x$.

(a) *Algebraically* determine the zeros of this function.

(b) Sketch a graph of this function on the axes below illustrating your answer to part (a).



***Exercise* #3:** The largest root of $x^3 - 9x^2 + 12x + 22 = 0$ falls between what two consecutive integers?

(1) 4 and 5 (3) 10 and 11

(2) 6 and 7 (4) 8 and 9

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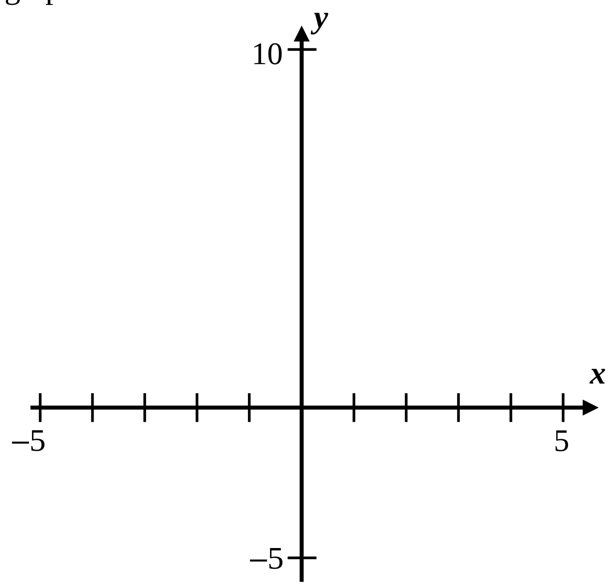
***Exercise* #4:** Consider the quartic function $y = x^4 - 5x^2 + 4$.

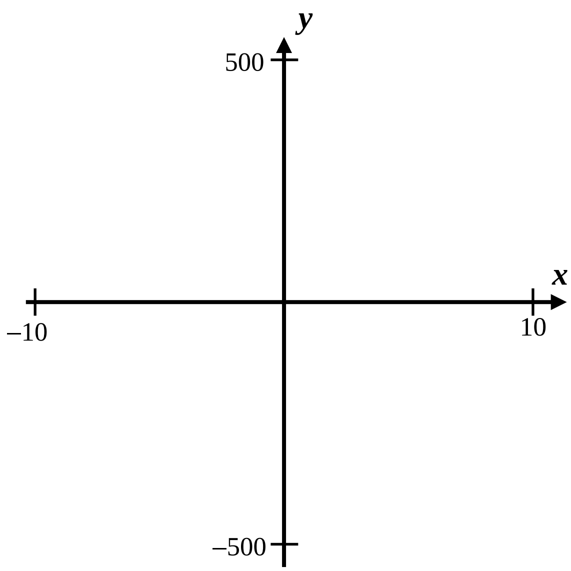
(a) *Algebraically* determine the x-intercepts of this function.

(b) Verify your answer to part (a) by sketching a graph of the function on the axes below.



***Exercise* #5:** Consider the quartic whose equation is $y = x^4 + 3x^3 - 35x^2 - 39x + 70$.

(a) Sketch a graph of this quartic on the axes below. Label its x-intercepts.



(b) Based on your graph from part (a), write the expression $x^4 + 3x^3 - 35x^2 - 39x + 70$ in its factored form.

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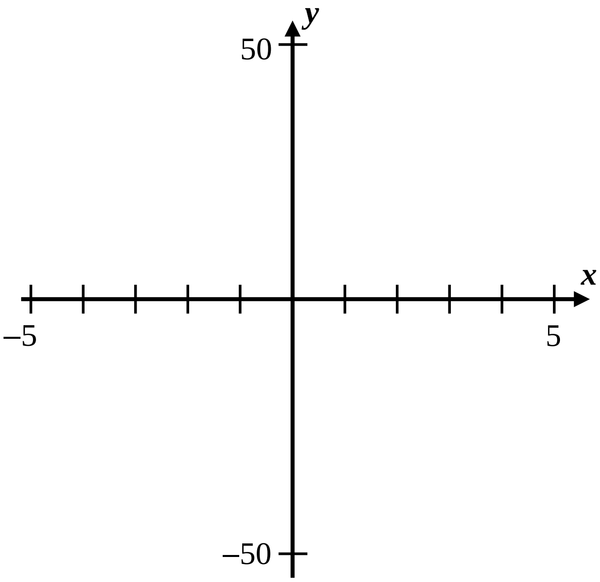
GRAPHS AND ROOTS OF A POLYNOMIAL
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

SKILLS

1. Consider the function $y = x^3 + 3x^2 - 6x - 8$.

(a) Sketch the function on the axes given. Clearly plot and label each x-intercept.

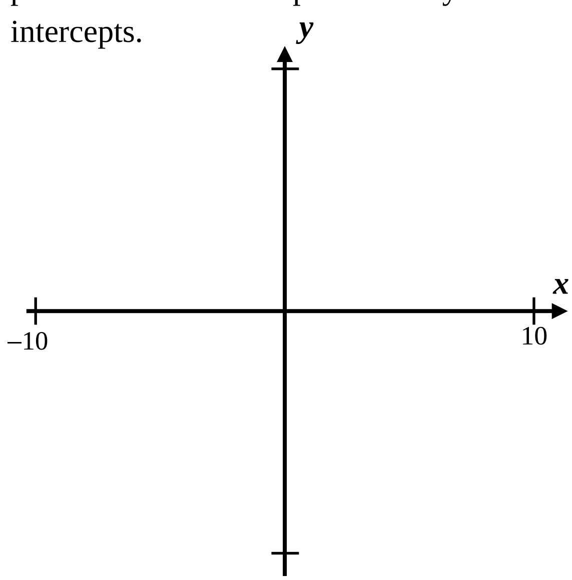


(b) Considering your answer to part (a), what values of x are solutions to the equation $x^3 + 3x^2 - 6x - 8 = 0$.

(c) Based on your answer to part (b), how must the expression $x^3 + 3x^2 - 6x - 8$ factor?

2. Consider the cubic function $y = x^3 + 2x^2 - 36x - 72$.

(a) Find an appropriate y-window for the x-window shown on the axes and sketch the graph. Make the sure the window is sufficiently large to show the two turning points and all intercepts. Clearly label all x-intercepts.

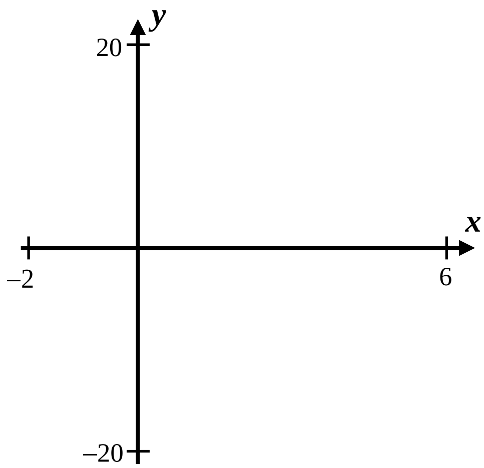


(b) What are the solutions to the equation $x^3 + 2x^2 - 36x - 72 = 0$?

(c) How does the expression $x^3 + 2x^2 - 36x - 72$ factor?

3. Consider the cubic function given by $y = x^3 - 6x^2 + 12x - 5$.

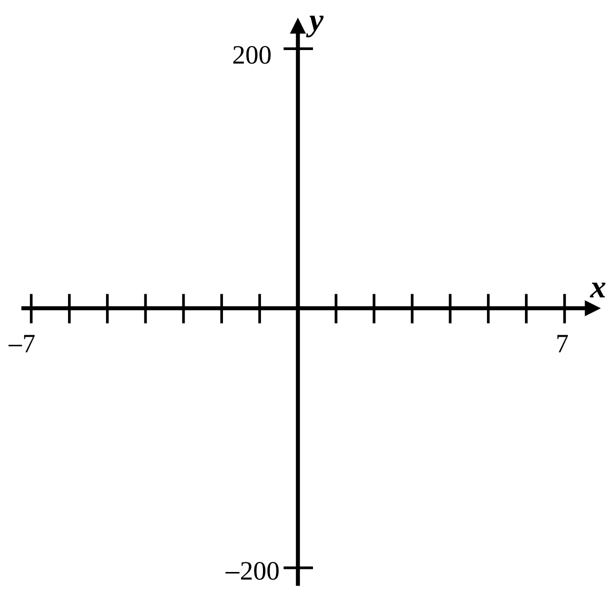
(a) Sketch a graph of this function on the axes given below.



(b) Considering the graphs of cubics you saw in class and those in problems 1 and 2, what is different about the way this cubic's graph looks compared to the others?

4. Consider the quartic function $y = x^4 - x^3 - 27x^2 + 25x + 50$.

(a) Sketch the graph of this function on the axes given below. Clearly mark all *x*-intercepts.



(b) Use your graph from part (a) to solve the equation $x^4 - x^3 - 27x^2 + 25x + 50 = 0$.

(c) Considering your answer to (b), how does the expression $x^4 - x^3 - 27x^2 + 25x + 50$ factor?

5. In general, how does the number of zeros (or *x*-intercepts) relate to the highest power of a polynomial? Be specific. Can you make a statement about the minimum number of zeros as well as the maximum?