

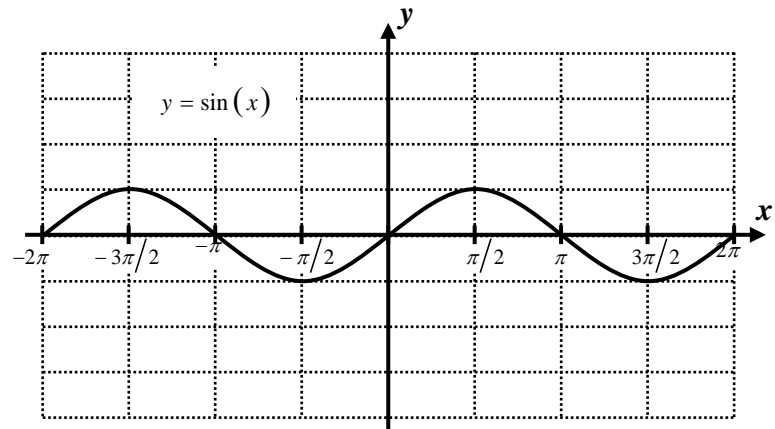
THE INVERSE TRIGONOMETRIC FUNCTIONS

ALGEBRA 2 WITH TRIGONOMETRY

This unit will be primarily devoted to the study of solving trigonometric equations. In order to do this it will be important to have a basic understanding of how these functions are inverted. In this lesson, we will explore the basic graphs and properties of the inverse sine, inverse cosine, and inverse tangent functions.

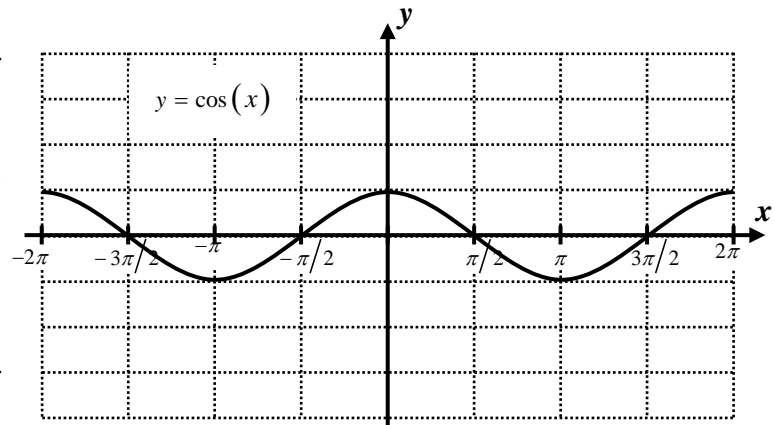
Exercise #1: Shown below are the basic graphs of sine, cosine, and tangent over the interval $-2\pi \leq x \leq 2\pi$.

- (a) Are any of these three functions one-to-one?
How can you tell?



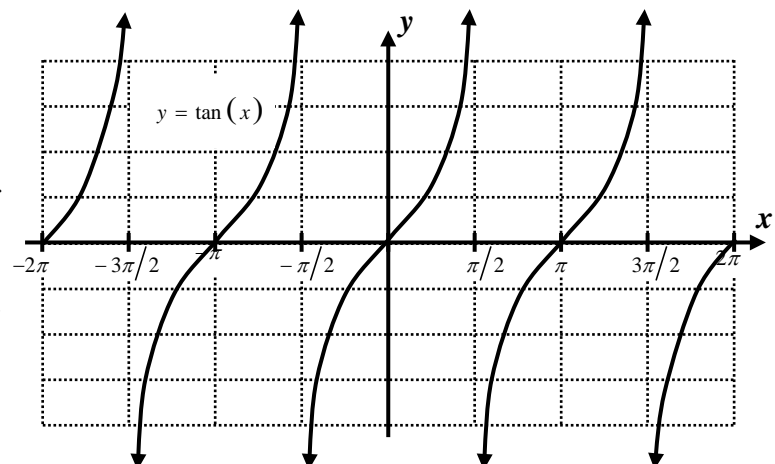
- (b) What does your answer to part (a) imply about the inverses of these three functions?

- (c) Give a restriction on the domain of $y = \sin(x)$ that would make it a one-to-one function and would include the first quadrant.



- (d) Give a restriction on the domain of $y = \cos(x)$ that would make it a one-to-one function and would include the first quadrant.

- (e) Give a restriction on the domain of $y = \tan(x)$ that would make it a one-to-one function and would include the first quadrant.



Now that we have **restricted the domains** of sine, cosine, and tangent, we can graph their inverses. Recall that if the point (a, b) lies on a function then the point (b, a) lies on its inverse.

Exercise #2: Consider the functions $y = \sin(x)$ and $y = \sin^{-1}(x)$.

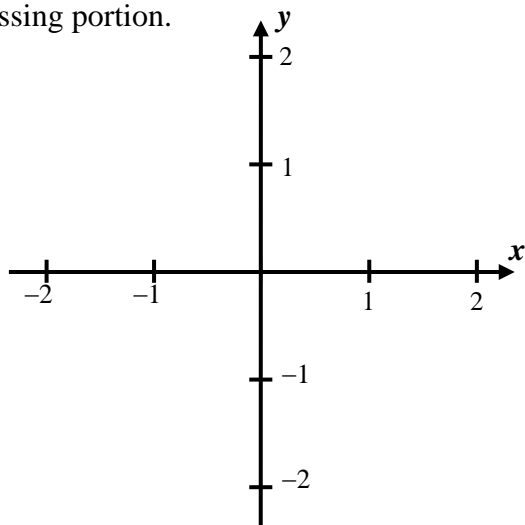
(a) Fill in the table below for the sine function on the restricted interval $-\pi/2 \leq x \leq \pi/2$.

x	$\sin(x)$
$-\pi/2$	
0	
$\pi/2$	

(b) Using your table in part (a) construct a table for $y = \sin^{-1}(x)$.

x	$\sin^{-1}(x)$

(c) Plot and label these points on the axes below and then use your calculator to fill in the missing portion.



(d) Solve the equation $\sin^{-1}(x) = \pi/6$ graphically.

Illustrate your solution with a rough sketch below.

(e) How do you interpret your answer to part (d) in terms of the original sine function?

(f) State, in exact form, the value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$.



THE INVERSE TRIGONOMETRIC FUNCTIONS
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

SKILLS

In this homework, you will now explore the graphs of $y = \cos^{-1}(x)$ and $y = \tan^{-1}(x)$.

1. Consider the functions $y = \cos(x)$ and $y = \cos^{-1}(x)$.

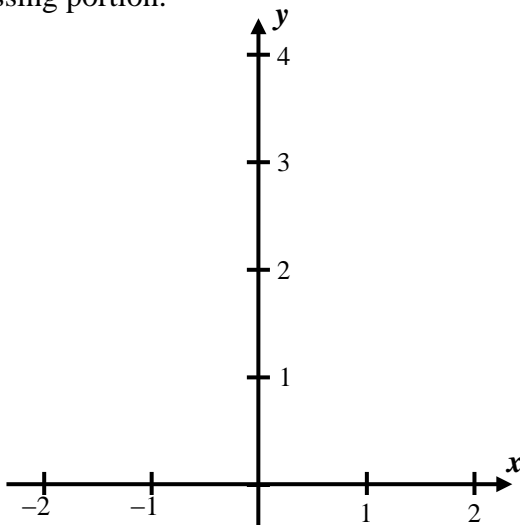
(a) Fill in the table below for the cosine function on the restricted interval $0 \leq x \leq \pi$.

x	$\cos(x)$
0	
$\pi/2$	
π	

(b) Using your table in part (a) construct a table for $y = \cos^{-1}(x)$.

x	$\cos^{-1}(x)$

(c) Plot and label these points on the axes below and then use your calculator to fill in the missing portion.



(d) State in units of radians the exact value for each of the following:

$$\cos^{-1}\left(\frac{1}{2}\right) = \quad \cos^{-1}\left(-\frac{1}{2}\right) =$$

$$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \quad \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) =$$

$$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \quad \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) =$$

2. The range, in degrees, of $y = \cos^{-1}(x)$ is which of the following.

(1) $-90^\circ \leq y \leq 90^\circ$ (3) $0^\circ \leq y \leq 90^\circ$

(2) $-180^\circ \leq y \leq 180^\circ$ (4) $0^\circ \leq y \leq 180^\circ$



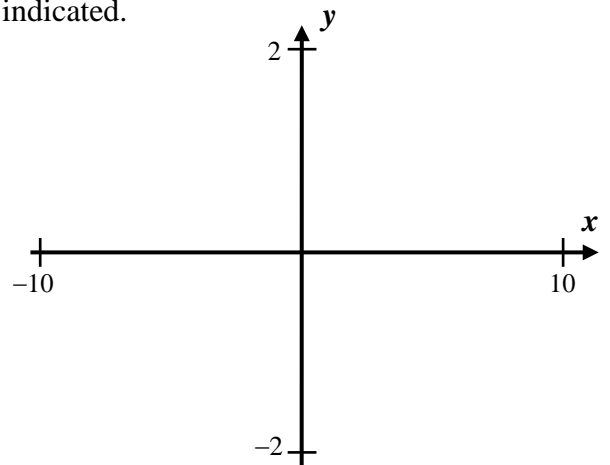
3. Consider the function $y = \tan(x)$ and its inverse $y = \tan^{-1}(x)$.

(a) What is the standard domain restriction for $y = \tan(x)$ that forces it to be one-to-one?

(b) What is the range of $y = \tan(x)$?

(c) Based on your answers to parts (a) and (b), what are the domain and range of $y = \tan^{-1}(x)$?

(d) Use your calculator to sketch a graph of $y = \tan^{-1}(x)$ using the graphing window indicated.



4. The inverse tangent function has a limiting value at x gets larger in both a negative and positive sense.

(a) Fill in the two tables below for $y = \tan^{-1}(x)$. Express the values to the nearest hundredth.

x	$y = \tan^{-1}(x)$
0	
10	
100	
1000	

x	$y = \tan^{-1}(x)$
0	
-10	
-100	
-1000	

(b) Considering your answer to #3 part (b), what irrational number is the inverse tangent becoming closer to as x gets larger?

(c) Which of the following lines would not intersect $y = \tan^{-1}(x)$?

(1) $y = -1.3$

(3) $y = 0.8$

(2) $y = 1.9$

(4) $y = -0.25$ _____

