Name: ___________________________________ Date: _________________

TRIGONOMETRIC IDENTITIES – DAY #1
ALGEBRA 2 WITH TRIGONOMETRY

Equations that are true (check) for every value of their input variable(s) are known as **identities**. The most important trigonometric identity you have already seen, the **Pythagorean Identity:**

THE PYTHAGOREAN IDENTITY

$\sin^2\theta + \cos^2\theta = 1$ for any value of θ

So useful is this identity that it must be memorized. It primary use is to produce the value of either sine or cosine if the other is known.

***Exercise* #1:** The angle θ has its terminal ray in the second quadrant and $\sin\theta = \frac{5}{13}$. Algebraically determine the value of $\cos\theta$.

There are many, many other trigonometric identities. Their derivation and memorization are left for a future course. In the rest of this lesson, we will work with the sum and difference identities for sine and cosine.

THE SUM AND DIFFERENCE IDENTITIES

$\sin(A+B) = \sin(A)\cos(B) + \sin(B)\cos(A)$ $\qquad$ $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

$\sin(A-B) = \sin(A)\cos(B) - \sin(B)\cos(A)$ $\qquad$ $\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$

***Exercise* #2:** Find an expression for the exact value of $\sin(75^\circ)$ in simplest radical form.

***Exercise* #3:** Determine the value of $\cos(15^\circ)$ in simplest radical form.

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THE SUM AND DIFFERENCE IDENTITIES (AGAIN)

$\sin(A+B)=\sin(A)\cos(B)+\sin(B)\cos(A)$ $\quad$ $\cos(A+B)=\cos(A)\cos(B)-\sin(A)\sin(B)$

$\sin(A-B)=\sin(A)\cos(B)-\sin(B)\cos(A)$ $\quad$ $\cos(A-B)=\cos(A)\cos(B)+\sin(A)\sin(B)$

***Exercise* #4:** For two angles, α and β, it is known that $0<\alpha<90^\circ$ and $90^\circ<\beta<180^\circ$. If $\cos\alpha=\frac{3}{5}$ and $\sin\beta=\frac{7}{25}$ find the following values in simplest form.

(a) $\sin\alpha$

(b) $\cos\beta$

(c) $\sin(\alpha-\beta)$

(d) $\cos(\alpha+\beta)$

(e) $\sin(2\alpha)$

(f) $\cos(2\beta)$

***Exercise* #5:** Which of the following is equivalent to $\cos(100^\circ)\cos(20^\circ)+\sin(100^\circ)\sin(20^\circ)$?

(1) $\cos(120^\circ)$ $\quad$ (3) $\sin(120^\circ)$

(2) $\cos(80^\circ)$ $\quad$ (4) $\sin(80^\circ)$

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***Exercise* #6:** Which of the following is *not* equivalent to $\sin 70^\circ$?

(1) $\sqrt{1-\cos^2 70^\circ}$ $\quad$ (3) $\sin 40^\circ\cos 30^\circ+\sin 30^\circ\cos 40^\circ$

(2) $\sin 80^\circ\cos 10^\circ-\sin 10^\circ\cos 80^\circ$ $\quad$ (4) $\sin 60^\circ\cos 10^\circ-\sin 10^\circ\cos 60^\circ$

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TRIGONOMETRIC IDENTITIES – DAY #1
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

THE SUM AND DIFFERENCE IDENTITIES (YET AGAIN)

$\sin(A+B)=\sin(A)\cos(B)+\sin(B)\cos(A)$ $\cos(A+B)=\cos(A)\cos(B)-\sin(A)\sin(B)$

$\sin(A-B)=\sin(A)\cos(B)-\sin(B)\cos(A)$ $\cos(A-B)=\cos(A)\cos(B)+\sin(A)\sin(B)$

SKILLS

1. Match each of the trigonometric values in column A with an equivalent expression in column B.

Column A	Column B
$\sin 35^\circ$	$\cos 50^\circ \cos 15^\circ + \sin 50^\circ \sin 15^\circ$
$\cos 35^\circ$	$\sin 50^\circ \cos 15^\circ + \sin 15^\circ \cos 50^\circ$
$\sin 65^\circ$	$\sin 50^\circ \cos 15^\circ - \sin 15^\circ \cos 50^\circ$
$\cos 65^\circ$	$\cos 50^\circ \cos 15^\circ - \sin 50^\circ \sin 15^\circ$

2. Which of the following is equivalent to $\cos 75^\circ \cos 25^\circ - \sin 75^\circ \sin 25^\circ$?

(1) $\sin 100^\circ$ (3) $\cos 100^\circ$

(2) $\sin 50^\circ$ (4) $\cos 50^\circ$ ________

3. Which of the following gives the exact value of $\cos 75^\circ$?

(1) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$ (3) $\dfrac{\sqrt{3}-\sqrt{2}}{2}$

(2) $\dfrac{1-\sqrt{3}}{4}$ (4) $\dfrac{\sqrt{2}+\sqrt{6}}{2}$ ________

4. The value of $\sin 80^\circ$ can be expressed in terms of $\sin 40^\circ$ and $\cos 40^\circ$ as

(1) $\sin^2 40^\circ - \cos^2 40^\circ$ (3) $2\sin 40^\circ \cos 40^\circ$

(2) $1-\cos^2 40^\circ$ (4) $\sin^2 40^\circ + \cos^2 40^\circ$ ________

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THE SUM AND DIFFERENCE IDENTITIES (YET AGAIN)

$\sin(A+B)=\sin(A)\cos(B)+\sin(B)\cos(A)$

$\cos(A+B)=\cos(A)\cos(B)-\sin(A)\sin(B)$

$\sin(A-B)=\sin(A)\cos(B)-\sin(B)\cos(A)$

$\cos(A-B)=\cos(A)\cos(B)+\sin(A)\sin(B)$

5. For two angles, A and B, it is known that $90^\circ < A < 180^\circ$ and $270^\circ < B < 360^\circ$. If $\cos A = -\frac{12}{13}$ and $\cos B = \frac{4}{5}$ then find each of the following values in exact, simplest form.

(a) $\sin A$

(b) $\sin B$

(c) $\sin(A+B)$

(d) $\sin(A-B)$

(e) $\cos(A+B)$

(f) $\cos(A-B)$

6. For two angles, α and β, it is known that $0^\circ < \alpha < 90^\circ$ and $90^\circ < \beta < 180^\circ$. If $\sin\alpha = \frac{\sqrt{5}}{3}$ and $\sin\beta = \frac{1}{2}$ then find $\sin(\alpha+\beta)$ in simplest radical form.

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