

## PIECEWISE LINEAR FUNCTIONS COMMON CORE ALGEBRA I



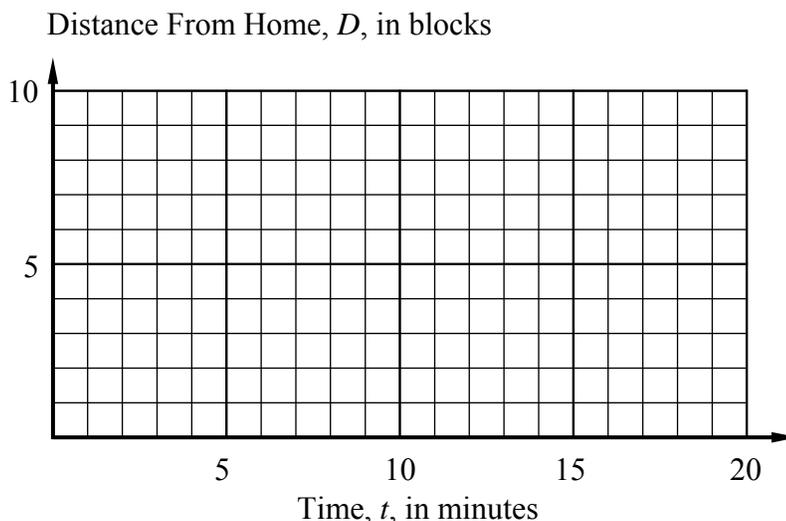
We modeled with **piecewise functions** back in Unit #3. In today's lesson we will work specifically with **piecewise linear functions**, or those that are comprised of **linear segments**. These are particularly helpful in modeling certain situations, especially with **motion**.

**Exercise #1:** Mateo is walking to school. It's a nice morning, so he is moving at a comfortable pace. After walking for 9 minutes, he is 6 blocks from home. He stops to answer a text on his phone from his mother. After 5 minutes standing still, he walks home quickly in 6 minutes to get a paper he forgot for school. We are going to model Mateo's distance from home,  $D$ , in blocks as a function of the time,  $t$ , in minutes since he left.

(a) Draw a graph of Mateo's distance from home on the grid provided.

(b) Determine a formula for the distance he is from home,  $D$ , over the time interval  $0 \leq t \leq 9$ .

(c) Determine a formula for the distance he is from home,  $D$ , over the time interval  $9 \leq t \leq 14$ .



(d) The trickiest part of this modeling will be to determine the linear equation for the distance,  $D$ , on the time interval  $14 \leq t \leq 20$ . Pick two points on this line and form an equation in the form  $D = mt + b$ .

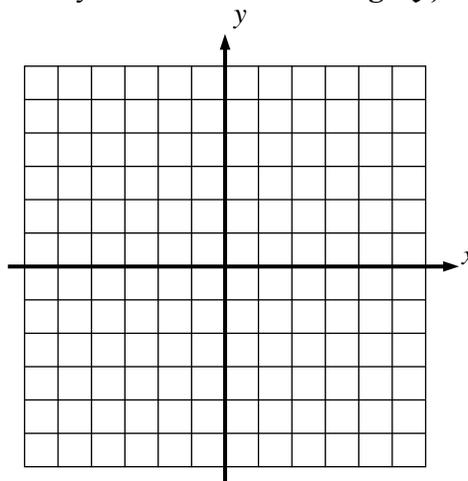
Piecewise linear functions are more **complex function rules**. One way or another, though, they fit the standard definition of a function, i.e. for **every value in the domain ( $x$ )** there is only **one value in the range ( $y$ )**.

**Exercise #2:** Consider the function defined by:

$$f(x) = \begin{cases} 2x + 4 & -4 \leq x \leq 1 \\ 6 - x & 1 < x \leq 5 \end{cases}$$

(a) Graph the function  $f(x)$  by graphing each of the two lines.

(b) State the range of the function  $f(x)$ .



Piecewise linear functions can often have horizontal components as well as slanted components. They will obviously never have vertical components (or they wouldn't be functions). Let's see if we can translate from a graph to a piecewise equation.

**Exercise #3:** The piecewise linear function  $f(x)$  is shown graphed below.

(a) Find the slope of each of the line segments:

$\overline{AB}$ :

$\overline{BC}$ :

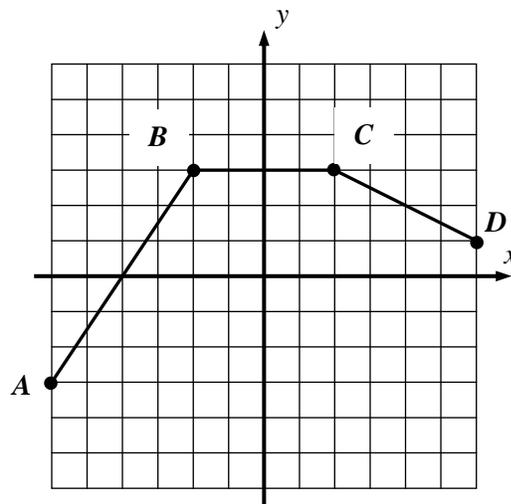
$\overline{CD}$ :

(b) Now find the equation of the line that passes through each of the following pairs of points in  $y = mx + b$  form where applicable. How can you find the  $y$ -intercepts by using the graph?

$\overline{AB}$ :

$\overline{BC}$ :

$\overline{CD}$ :



(c) Write the formal piecewise definition for this function.

(d) Find the one zero of the function algebraically by setting the formula for this function that applies from  $-6 \leq x \leq -2$  equal to zero and solving.

(e) Why does setting the formula for this function that applies from  $2 \leq x \leq 6$  equal to zero not produce a viable zero of the function?

(f) What parameter in the piecewise linear model indicates that the function is decreasing between  $x = 2$  and  $x = 6$ ? Explain your choice.



**PIECEWISE LINEAR FUNCTIONS**  
**COMMON CORE ALGEBRA I HOMEWORK**

**FLUENCY**

1. Given the function  $f(x) = \begin{cases} 2x+6 & x < -1 \\ -4x+8 & x \geq -1 \end{cases}$ , answer the following questions.

(a) Evaluate each of the following function values. Carefully pay attention to which of the formulas applies.

$f(2) =$

$f(0) =$

$f(-5) =$

$f(-1) =$

(b) Algebraically determine the zeroes of this function.

2. Given the piecewise function  $g(x) = \begin{cases} \frac{1}{2}x+6 & x < 0 \\ 4x+1 & x \geq 0 \end{cases}$ , what is the average rate of change over the interval  $-2 \leq x \leq 1$ ?

(1)  $\frac{1}{2}$

(3)  $-3$

(2)  $0$

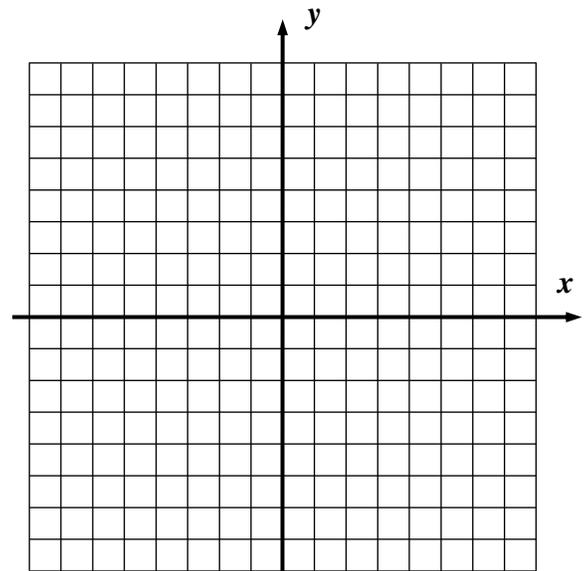
(4)  $4$

3. Consider the piecewise function  $h(x) = \begin{cases} -2x-6 & -6 \leq x < 0 \\ \frac{1}{2}x-6 & 0 \leq x \leq 4 \end{cases}$ .

(a) Graph  $h(x)$  on the grid.

(b) State the range of  $h(x)$ .

(c) What values of  $x$  solve  $h(x) = 0$ ?



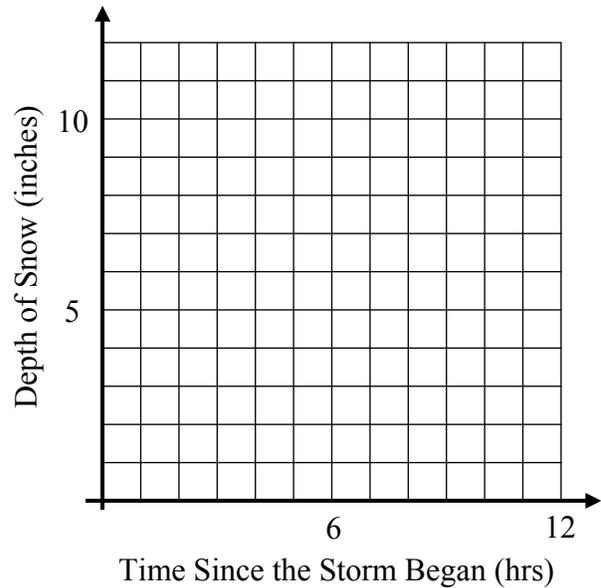
## APPLICATIONS

4. A substantial snowstorm is hitting the Northeast region and is predicted to snow at a rate of 2 inches per hour for the first three hours of the storm. The storm is supposed to pause for three hours and then resume at a rate of one-half inch per hour for the next four hours. The depth,  $D$ , of the storm is the total number of inches of snow that has fallen at a given time.

(a) How many hours is the snow storm?

(b) How many total inches of snow fell? Show the calculations that lead to your answer.

(c) Graph the snow depth as a function of time since the storm began for the length of the storm.



(d) Determine a piecewise linear function for  $D$  as a function of the number of hours,  $t$ , since the storm began. There should be three formulas. The first two should be relatively simple, while the third might take some additional thinking.

## REASONING

5. The function  $f(x) = \begin{cases} 2x-8 & x < 0 \\ \frac{x}{2}+5 & x \geq 0 \end{cases}$  has no zeroes even though each individual line, i.e.  $y = 2x - 8$  and

$y = \frac{x}{2} + 5$  each have a zero. Why does  $f(x)$  lack zeroes?

