Name: _____ Date: _____

LIMITS TO ACCURACY OF OUR MODELS
COMMON CORE ALGEBRA I

Most mathematical models of **real world phenomena** contain errors. It is rare that we can **predict** the outcome to almost any event with 100% confidence (more on that in the statistics of Algebra II). Knowing what introduced error is important to consider as we conclude the lessons of this course. Let's start by investigating a very simple mathematical model that you should feel comfortable with.

***Exercise* #1:** Mia is trying to calculate the area of her closet so she can purchase wood flooring. She measures the width and length and rounds to the nearest tenth of a meter. She found the length to be 2.7 meters and the width to be 1.4 meters.

(a) Calculate the area of the rectangular floor. Include proper units.

(b) Why does it *not* make sense to leave this answer accurate to the nearest *hundredth*? Write down a proper level of precision for the area. Include units.

Generally speaking, without getting too deep into what scientists refer to as **significant figures**, the limitations on any **calculation** or **prediction** will be limited by the **least precise** input to the model.

CHOOSING YOUR PRECISION LEVEL

The calculation of an **output** to a **model** should be **rounded** to the **level of accuracy** of the **least accurate input** to the model.

***Exercise* #2:** The weight of newborns for a day were recorded at a local hospital. The weights were rounded to the nearest *tenth* of a pound. They are as follows:

6.2, 8.4, 5.6, 10.1, 7.4, 8.7, 9.3, 6.8, 7.5

Calculate the mean and the standard deviation of this data set. Include appropriate units for both results and round to appropriate levels of accuracy.

***Exercise* #3:** Jonathan knows that if a projectile is fired from a height of exactly 3 meters above the ground at an initial speed of 24 meters per second, then its height, *h*, in meters above the ground after *t* seconds will be given by the formula:

$$h = -4.9t^2 + 24t + 3$$

Jonathan starts a timer and takes a picture at a time when the ball is in the air. The picture records the time-signature, i.e. when the picture was taken, to be 1.7 seconds after it was launched, rounded to the nearest tenth of a second.

(a) Use the function above to determine the height of the projectile at $t = 1.7$ seconds. Do not round your answer.

(b) Why should Jonathan not report the height of the projectile to the level of accuracy given in (a)? What should be the proper answer (with units)?

***Exercise* #4:** Water control engineers are keeping track of the volume of water in a local storage facility. They measure the initial amount of water to be 362 gallons, to the nearest gallon. Water is being withdrawn at a rate of 12.8 gallons per minute, to the nearest tenth of a gallon per minute.

(a) Write a formula for the volume of water, *V*, left in the reservoir as a function of time, *t*, in minutes that the water has been draining.

(b) Engineers would like to know how much water is in the reservoir at 7 minutes. Determine the volume and use an appropriate level of precision.

***Exercise* #5:** A radioactive material decays such that 5% of it is lost every hour. Scientists take a small portion of the material and weigh it to be 24.8 grams, to the nearest *tenth* of a gram. Develop an exponential formula, of the form $A = a(b)^t$ for the amount of material still radioactive after *t*-hours. Use your model to determine the amount still radioactive after 10.0 hours.

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LIMITS TO ACCURACY OF OUR MODELS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Pick the best choice below to fill in the blank: The precision of any calculation based on inputs should be

   as precise as its _____ input.

   most precise least precise

2. Given that each value in the data set below has been rounded, which of the following choices should we make for the mean of the data set.

   6.1, 8.6, 4.35, 7.8, 2.71

   (1) 5.912 (3) 5.9

   (2) 6 (4) 5.91

APPLICATIONS

3. Jonathan is driving at 62 miles per hour, rounded to the nearest integer, away from Ashmore, Illinois. He started at $h = 0$ in Ashmore.

   (a) Write an equation for Jonathan's distance from Ashmore, *d*, as a function of the number of hours he has been driving, *h*.

   (b) Determine Jonathan's distance from Ashmore after driving for 2.7 hours, given that time has been rounded to the nearest tenth of an hour. Include units in your answer.

4. To be classified as a Large Egg, eggs must weigh between 2 and 2.25 ounces. A local hen farm selected 10 eggs they considered to be Large and weighed them to the nearest *tenth* of an ounce. Here is their data:

   2.1, 2.3, 2.0, 2.1, 2.2, 2.5, 2.2, 2.3, 2.1, 1.9

   (a) Determine the mean egg weight for this sample and the sample standard deviation. Include units and round to the correct precision level.

   (b) What percent of this data set should *not* have been classified as Large? Show the work that leads to your answer.

5. Engineers modeled the depth of water, in feet, in a reservoir as it is being drained by the equation:

$$d(t) = 26(0.62)^t + 6 \qquad \text{where } t \text{ is the number of hours it has been draining}$$

(a) According to the engineer's model, what was the depth of the water at $t = 0$ hours?

(b) The engineers wanted to record the depth every tenth of an hour, but missed it at 3.7 hours, rounded to the nearest tenth of an hour. What was the depth according to their model?

6. Forrest biologists are trying to find a correlation between the height of maple trees and their diameters at ground height. They find the following data which has been rounded:

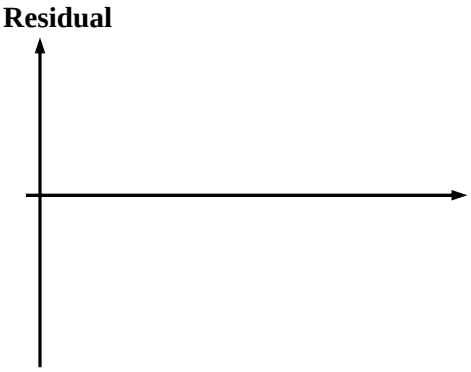
Diameter, *x*, in inches	4	10	13	20	24	32
Height, *y*, in feet	18	28	32	40	44	51

(a) Find the equation of the best fit line for this data set. Round all parameters to the nearest *hundredth*.

(b) Using your model, predict the height of a tree that has a diameter, rounded to the nearest inch, of 22 inches.

(c) The linear correlation coefficient for this data set is 0.99, rounded to the nearest hundredth. Does this indicate a strong, moderate, or weak positive association? Explain.

(d) Produce a rough plot of the residuals below. Does this indicate that a linear model is appropriate for this data set? Why or why not?



(e) In this model, we have a high *r*-value, but a residual graph that shows a pattern. In the statement below, circle one of the two words underlined (each time) to complete the statement.

A model with a high *r*-value may be very accurate appropriate but if its residual graph shows a definitive pattern then the model may not be very accurate appropriate.