

Name: _____

Date: _____

SEEING STRUCTURE IN EXPRESSIONS COMMON CORE ALGEBRA I



Many times the techniques of algebra can seem like mindless moving of symbols from here to there without any obvious purpose. In the Common Core, we seek to challenge students to do **mindful manipulations**. In other words, always have a reason for the manipulation you are doing.

MINDFUL MANIPULATION

A **mindful manipulation** will be an algebraic technique applied with a purpose in mind, even if we are unsure if the manipulation will result in success. In this sense, we want to give ourselves permission to do manipulations even if they fail to reach our purpose.

The exercises in this lesson are about problem solving and using the properties we have learned about in **mindful** ways to try to solve problems that are puzzle like in nature. But, we will start with a problem that illustrates the idea.

Exercise #1: Consider the expressions $2x+1$ and $6x+3$.

(a) Find the value of both expressions when $x = 2$.

(b) What is the ratio of the larger outcome to the smaller? Why did the ratio turn out this way? What property can you use to justify this?

O.k. Now for some problems that are a bit more challenging. You are going to be asked for the value of an expression **without knowing the value of x** . Let's do a warm up.

Exercise #2: The expression $3x+2$ is equal to 7 for some value of x (don't solve for it!). Determine the values of each of the following expressions for the same value of x . Show your reasoning.

(a) $6x+4$

(b) $3x+5$



The last exercise forces us to do mindful manipulations because we have to think about how expressions relate to each other and how to write equivalent expressions. Let's do some more of these types of puzzles.

Exercise #3: The expression $2x+5$ has a value of 10 for some value of x (don't solve for it). Do mindful manipulations on each of the following to find the values of these expressions for this same value of x .

(a) $4x+10$

(b) $2x+20$

(c) $2x+1$

(d) $-2x-5$

(e) $10x+25$

(f) $2x-5$

(g) Challenge: Find the value of $6x+20$. Hint: just try to come close and see what else you have to do in terms of adding or subtracting.

Exercise #4: If the expression $3x-4$ has a value of -3 for some value of x , then what is the value of the expression $(3x-4)^2 + 6x - 8$ for the same value of x ? Show the calculations that lead to your choice below.

(1) 11

(3) 3

(2) -15

(4) -6



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SEEING STRUCTURE
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Get a warm-up with the following. Evaluate each expression for the given value of x . Do these without the aid of a calculator to practice your mental arithmetic.

(a) $3x - 8$ for $x = 5$

(b) $5(x + 7) - 1$ for $x = -3$

(c) $\frac{x-8}{4} + 5$ for $x = 4$

2. If the expression $x - 3$ has a value of -5 , then which of the following represents the value of $3x - 9$? Explain how you arrived at your choice.

(1) -2

(3) -15

(2) -9

(4) -42

3. The expression $2x + 6$ is equal to 9 for some value of x . Without finding the value of x , determine the values for each of the following expressions. Show how you arrived at each answer.

(a) $4x + 12$

(b) $2x + 9$

(c) $x + 3$

(d) $-6x - 18$

(e) $2x + 1$

(f) $10x + 32$

4. The expression $x - 2$ has a value of -5 for some value of x . For the same value of x , what is the value of the expression $(x - 2)^2 + 5x - 10$? Show your reasoning for this problem in the space provided.

(1) 0

(3) 14

(2) 15

(4) -10



APPLICATIONS

5. The number of feet that Jennifer can run in a given time period t is given by the expression $8t + 2$. Her friend Erika can run a distance given by the expression $4t + 3$. Erika claims that she can only run half of what Jennifer can plus an additional 2 feet. Is she correct?

(a) Let's build up some evidence by playing around with various values of t . Fill out the following chart for both Jennifer and Erika's distances given the value of t .

Time, t	Jennifer's Distance $8t + 2$	Erika's Distance $4t + 3$	Is Erika Correct?
1			
3			
5			
10			

(b) The table provides good numerical evidence that what Erika says is true. Show by using mindful manipulations of the expression $8t + 2$ that Erika's distance is always 2 feet more than half of Jennifer's.

REASONING

We can use the same sort of reasoning to help solve equations. We haven't done much of that yet but try your best to think about these problems in the context of solving an equation.

6. Say I knew that the solution to the equation $2x - 7 = 9$ was $x = 8$. How could I use that to help me to solve the equation $2(x + 2) - 7 = 9$?

(a) Either $x = 6$ or $x = 10$ is a solution to our new equation: $2(x + 2) - 7 = 9$. Check to see which is a solution by substituting them into the left hand expression and seeing if it is equal to 9.

(b) Explain in your own words why the correct answer was the one you found in (b). In other words, look at the **structure** of both equations. Think about what is the same and what is different.

