Name: ________________________________ Date: ________________

EXPONENTS AS REPEATED MULTIPLICATION
COMMON CORE ALGEBRA I

We've used exponents a little so far, but they will become much more important to us as our studies in algebra progress. So, in the next few lessons we are going to work with some basic exponents. Recall that an exponent is a way to indicate **repeated multiplication by the same number**.

EXPONENTS AS REPEATED MULTIPLICATION

By definition, if n is a **positive integer**, i.e. $\{1, 2, 3, \ldots\}$, then $x^n = \underbrace{x \cdot x \cdot x \cdots x}_{\text{multiplied } n\text{-times}}$

***Exercise* #1:** Write out what each of the following exponents means as an extended product and find its value.

(a) 2^4 (b) 3^2 (c) 5^3

Of course, just as with numbers, variables can also be raised to exponents (other than 1).

***Exercise* #2:** Write out what each of the following terms involving exponents means as an extended product. Consider carefully your order of operations and remember that exponents come before multiplication.

(a) x^3 (b) x^2y^4 (c) $(2x)^2$

(d) $4x^4y^3$ (e) $(9x^2)^3$ (f) $(-4x^3)^2$

One of the nice aspects of exponents is that they follow very predictable patterns, often known as **exponent rules** (and they DO RULE!). Let's figure out the simplest one in the next exercise.

***Exercise* #3:** Write out each of the following products and then express them in the form x^n.

(a) x^2x^3 (b) x^5x^2 (c) x^4x^4

***Exercise* #4:** So, what's the pattern? Can you give a generic rule for what happens when we multiply two terms that have the same **base**?

EXPONENT RULE #1: $x^a \cdot x^b =$

This exponent rule allows us to multiply larger powers of variables without actually having to write out the products. Make sure to **internalize this rule**. In other words, think about it until you are absolutely sure you understand why it works. Eventually, we will extend exponent rules to weird situations with all sorts of exponents.

***Exercise* #5:** Quickly write each of the following products as a variable raised to a single power.

(a) $x^4 x^9$

(b) $x^2 x^3 x^4$

(c) $y^2 y^6$

Often you will need to be able to multiply more complicated terms and write them in as convenient (**simplest**) form as possible. The next exercise walks us how to do this and the laws and properties needed.

***Exercise* #6:** The steps to simplifying the product: $5x^3 \cdot 2x^7$ to simplest terms are shown below. Write in what justifies each step.

Step 1: $5x^3 \cdot 2x^7 = 5 \cdot 2 \cdot x^3 \cdot x^7$ Justification: ______________________________

Step 2: $5 \cdot 2 \cdot x^3 \cdot x^7 = (5 \cdot 2) \cdot (x^3 \cdot x^7)$ Justification: ______________________________

Step 3: $(5 \cdot 2) \cdot (x^3 \cdot x^7) = 10x^{10}$ Justification: ______________________________

Each step in an algebraic manipulation can ultimately be justified using a property that was established about numbers (like the commutative) or a pattern, like Exponent Rule #1 above. But, we also need to become **fluent** in these manipulations. The next exercise gives you some opportunity to do so.

***Exercise* #7:** Rewrite each of the following as equivalent expressions in simplest exponential form.

(a) $2x^7 \cdot 8x^5$

(b) $(-4x^3)(2x^2)$

(c) $(-6x^3)^2$

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Exponents as Repeated Multiplication
Common Core Algebra I Homework

Fluency

1. Rewrite each of the following terms as an extended product. Consider carefully your order of operations and remember that exponents come before multiplication. You do not need to simplify the products.

(a) 4^3 (b) $3^2 \cdot 3^3$ (c) $\left(2^3\right)^4$

(d) x^3y^4 (e) $8x^2y^5$ (f) $\left(9x^2\right)^2$

2. Write out each of the following products and then express them in simplest exponential form.

(a) x^4x^7 (b) y^3y^6 (c) $x^3y^2x^5y^2$

3. Rewrite each of the following as equivalent expressions in simplest exponential form. There is one that cannot be simplified. Identify it.

(a) $4x^3 \cdot 7x^6$ (b) $x^5y^3x^2$ (c) $\left(-x^2\right)\left(3x^{10}\right)$

(d) $x^2y^3z^3$ (e) $\left(4x\right)^3$ (f) $\left(-3x^2\right)^2$

APPLICATIONS

4. One of the most common uses of exponents is when dealing with **scientific notation**. Recall that 3.2×10^4 is written in scientific notation where 10 is being raised to the 4th power. If 3.2×10^4 is the length of a park in meters and 2.5×10^6 is the width in meters, what is the area of the park if it is in the shape of a rectangle? It may help to write the terms out as an extended product and then regroup them.

$$\text{Area} = \text{Length}\times\text{Width} = \left(3.2\times10^4\right)\left(2.5\times10^6\right) =$$

REASONING

5. The steps to simplifying the product $\left(2x^3\right)^3$ to simplest terms are shown below. Write in what justifies each step.

Step 1: $(2x^3)^3 = 2x^3 \bullet 2x^3 \bullet 2x^3$ Justification: ________________________________

Step 2: $2x^3 \bullet 2x^3 \bullet 2x^3 = 2\bullet2\bullet2\bullet x^3 \bullet x^3 \bullet x^3$ Justification: ________________________________

Step 4: $2\bullet2\bullet2\bullet x^3 \bullet x^3 \bullet x^3 = (2\bullet2\bullet2)\bullet(x^3 \bullet x^3 \bullet x^3)$ Justification: ________________________________

Step 3: $(2\bullet2\bullet2)\bullet(x^3 \bullet x^3 \bullet x^3) = 8x^9$ Justification: ________________________________

6. So far we have come up with an exponent rule for the multiplying two monomials with like bases. We saw this to be $x^a \cdot x^b = x^{a+b}$. We can also find a rule for simplifying the expression $\left(x^a\right)^b$. Try the following questions and see if you can find the pattern that helps simplify this type of expression.

(a) Rewrite the following terms as extended products and then express them in the form 2^n or x^n.

(i) $\left(2^2\right)^4$

(ii) $\left(x^3\right)^4$

(b) Looking back at part (a) see if you can see a connection between your answer and the question. Make a general rule for all terms in the form of $(x^a)^b$

$(x^a)^b =$

KNOW THIS RULE!!!