Name: ____________________________________ Date: __________________

INTRODUCTION TO SEQUENCES
COMMON CORE ALGEBRA I

A **sequence** is a very special type of function. When students first encounter sequences, they often think of them as just a list of numbers in some particular order (and then they have to find the pattern). We will start with the technical definition of a sequence in terms of a function.

SEQUENCE DEFINITION

A **sequence** is a function whose set of inputs, the **domain**, is a subset of the natural numbers, i.e. $\{1, 2, 3, 4, ...\}$. A sequence is often shown as an ordered list of numbers, called the **terms** or **elements** of the sequence. Sequence function notation can be tricky.

***Exercise* #1:** Consider the sequence below. If we represent this sequence with the letter *a* then do the following.

$$4, 8, 16, 32, 64, 128, 256$$

(a) Find $a(3)$

(b) Find $a(1)+a(7)$

(c) Find a_2.

(d) Find $(a_1)^2$

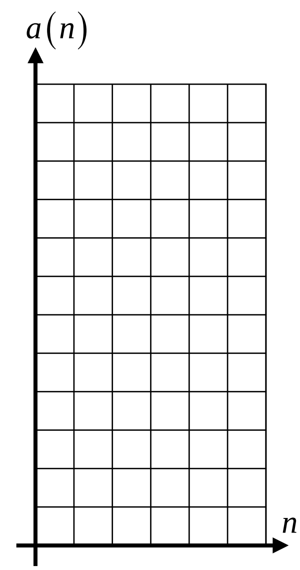
(e) Find $a_5 - a_4$

(f) Solve for *n*: $a(n)=128$.

Sequences are functions. The key here is that the input is simply the **number's place in line** so to speak and the output is the actual **number in the list**.

***Exercise* #2:** Consider the sequence defined by the formula $a(n)=2n+1$.

(a) Write out the first 5 elements of this sequence.



(b) Graph the sequence on the grid shown below for $1 \le n \le 5$.

(c) Why shouldn't we connect the points plotted with a continuous straight line?

(d) What is the 21st term of this sequence? Show how you arrived at your answer.

Sequences can be defined by a classic function formula, like what we saw in Exercise #2, and they also can be defined **recursively**. A **recursive formula** is one where each term in the sequence **depends on a term or terms** that came **before it**.

***Exercise* #3:** Consider a sequence of numbers given by the following definition:

$$b_1 = 7 \text{ and } b_i = b_{i-1} + 4$$

(a) Give a common sense interpretation for this **recursive** function rule.

(b) Write out the rule for the first 4 terms and evaluate each one of them (except b_1 which is given).

One of the most famous of all **recursively defined** sequences is known as the **Fibonacci Sequence**. Let's play around with it in the next exercise.

***Exercise* #4:** The Fibonacci Sequence is defined recursively as follows:

$$a(1) = 1,\ a(2) = 1 \text{ and } a(n) = a(n-1) + a(n-2)$$

(a) How do you interpret this recursive rule? Write it down in your own words.

(b) Write down the rule for $a(3)$, $a(4)$, and $a(5)$ and determine their values.

Sequences often show up in the real world, where they are sometimes defined in terms of a recursive process.

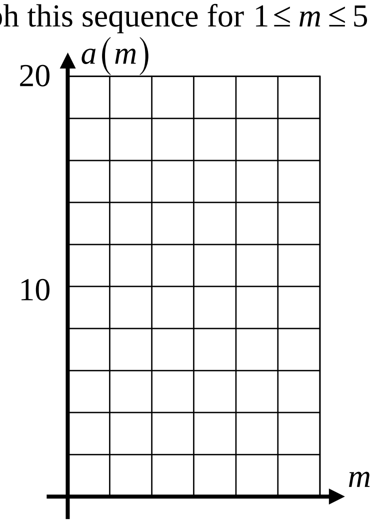
***Exercise* #5:** Kirk is trying to train for the marathon. His first month, he runs 5 miles per workout. He adds an additional 3 miles to his workout for each month that he trains.

(a) Fill out the table below for the amount of miles he runs as a function of how many months he has been running.

m	1	2	3	4	5
$a(m)$	5				

(b) Give a **recursive definition** for the sequence $a(m)$. Don't forget to give an initial value.

(c) Graph this sequence for $1 \le m \le 5$.



Name: ____________________________________ Date: __________________

INTRODUCTION TO SEQUENCES
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Consider the sequence below. If we represent this sequence with the letter a then do the following.

$$1, 7, 13, 19, 25, 31, 37, 43$$

(a) Find $a(5)$

(b) Find $a_2 + a_6$

(c) Find $a(4) + 2a(6)$

(d) Find $\sqrt{a(5)}$

(e) Find $\dfrac{a(5) - a(3)}{2}$

(f) Find a recursive definition for the sequence $a(n)$.

2. Consider the sequence defined in the table below.

n	1	2	3	4	5
$b(n)$	2	12	22	32	42

(a) Find $b(4)$

(b) Find $\dfrac{2b(2) - b(3)}{4}$

(c) Find a recursive definition for the sequence $b(n)$.

3. Consider a sequence of numbers given by the definition $c_1 = 2$ and $c_i = c_{i-1} \cdot 3$

(a) Write out the first 4 terms of this sequence.

(b) Find the value of $c_4 - c_2$. Show your calculation.

APPLICATIONS

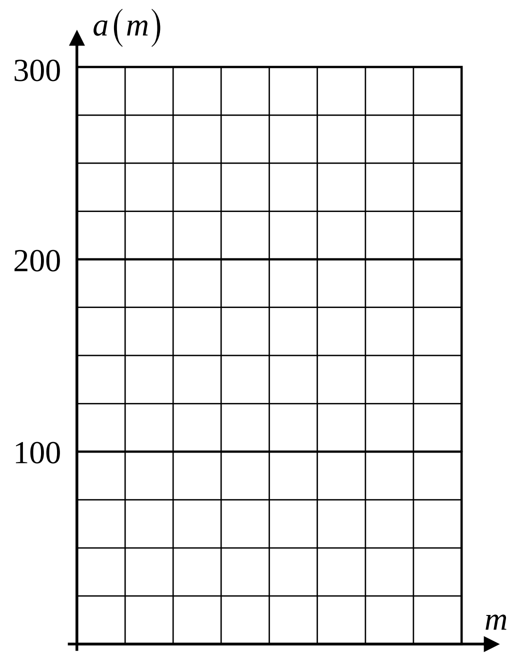
4. Erin is traveling abroad this summer and would like to have a bit of spending cash while she's overseas. She has 100 dollars already saved and she plans on saving 40 dollars a month.

(a) Fill out the table below for the amount of money she saves as a function of how many months she has been saving.

m	1	2	3	4	5
$a(m)$	140				

(b) Give a **recursive definition** for the sequence $a(m)$. Don't forget to give an initial value.

(c) Graph this sequence for $1 \le m \le 5$.



REASONING

5. A sequence is defined recursively as follows: $a(1) = 1$, and $a(n) = a(n-1) + n$

(a) How do you interpret this recursive rule? Write it down in your own words.

(b) Write down the rule for $a(2)$, $a(3)$, and $a(4)$ and determine their values.