Name: ___________________________________ Date: _________________

GEOMETRIC SEQUENCES
COMMON CORE ALGEBRA I

In Unit #4 we first encountered **sequences,** which just consisted of a specific **list of numbers** in a **particular order**. We extensively studied the idea of an **arithmetic sequence**, where each successive number in the list was generated by adding the same quantity to the previous number. Let's do a warm up.

***Exercise* #1:** An arithmetic sequence is defined **recursively** by the following formula:

$$a_1 = 5 \text{ and } a_n = a_{n-1} + 3$$

(a) Find the next three terms of the sequence.

(b) Find the value of a_{20} without listing out all 20 terms.

Clearly, **arithmetic sequences** share many characteristics of **linear functions.** In fact, **arithmetic sequences** are examples of **discrete linear functions**. **Exponential functions** have their own **discrete versions** and those are called **geometric sequences**. They have a very simple **recursive definition**.

> **GEOMETRIC SEQUENCES**
>
> Given the first term, a_1, then each successive term can be found by $a_i = a_{i-1} \cdot r$, where r is some constant often known as the **common ratio** of the sequence.

***Exercise* #2:** For each of the following **geometric sequences** identify the common ratio, *r*, and give the next two terms.

(a) 2, 6, 18, _______, _______

$r =$

(b) $4, -20, 100,$ _______, _______

$r =$

(c) $16, 8, 4,$ _______, _______

$r =$

As with arithmetic sequences, we should be able to predict any particular **term** in the geometric sequence by thinking about how many times we have multiplied by the **common ratio**, *r*.

***Exercise* #3:** Consider the geometric sequence given by the **recursive rule:**

$$b(1) = 3 \text{ and } b(n) = b(n-1) \cdot 2$$

(a) Find $b(2)$, $b(3)$, and $b(4)$. Write each as an extended product to see a pattern, but also find the final result.

(b) Based on (a), determine the value of $b(10)$ and $b(20)$.

One thing you might have noticed in the last exercise is how quickly a geometric sequence grows. Does this sound familiar? Let's take a look at a classic problem.

***Exercise* #4:** You have just won a very strange lottery. The lottery promises to give you money each day for a 30 day month based on one of two options:

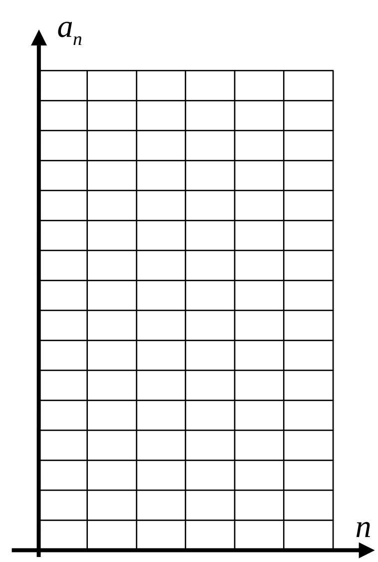
Option 1: You can receive $1000 on the first day, $2000 on the second day, $3000 on the third, in this **arithmetic sequence**.

Option 2: You can receive $0.01 (one penny) on the first day, $0.02 on the second, $0.04 on the third, $0.08 on the fourth, etcetera in this **geometric sequence**.

Of the two options, which would result in in the larger payoff on the 30th day only? Show work that supports your answer.

Graphs of geometric sequences will look familiar. Because they are a type of **discrete exponential function** they will look very similar.

***Exercise* #5:** For a geometric sequence defined by $a_1 = 16$ and $a_n = a_{n-1} \cdot \frac{1}{2}$, list and plot the first 6 terms on the grid below.



Name: ___________________________ Date: ________________

GEOMETRIC SEQUENCES
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. For each of the following geometric sequences, fill in the missing two terms and identify the common ratio, *r*. Remember, you can always find *r* by dividing two consecutive terms such as $\frac{a_2}{a_1}$.

(a) 2, 10, 50, _______ , _______

$r =$ ________

(b) 4, -8, 16, _______ , _______

$r =$ ________

(c) 40, 20, 10, _______ , _______

$r =$ ________

(d) 81, 54, 36, _______ , _______

$r =$ ________

(e) 5, -5, 5, _______ , _______

$r =$ ________

(f) 8, 20, 50, _______ , _______

$r =$ ________

2. One of the following sequences is arithmetic and one is geometric. Explain which is which.

Sequence #1: 5, 15, 45, 135, 405

Sequence #2: 5, 15, 25, 35, 45

3. In a geometric sequence the first term is 5 and the second term is 20, which of the following is the fifth term?

(1) 65
(3) 80
(2) 1,280
(4) 5,120

4. A geometric sequence is defined recursively by $a(1)=40$ and $a(n)=a(n-1)\cdot\frac{1}{2}$.

(a) Write out the first four terms of this sequence.

(b) Is the 9th term of this sequence larger or smaller than $\frac{1}{10}$? Show the calculation that you use to determine your answer.

5. Which has the larger 15th term when comparing the arithmetic and geometric sequences below? Show evidence that supports your answer.

Arithmetic Sequence: 150, 650, 1150, 1650, …

Geometric Sequence: 4, 12, 36, 108, …

APPLICATIONS

6. Maria plans to double the amount of time she spends walking per day each week. She starts, on week 1, walking 5 minutes per day. After 7 days, she then walks 10 minutes per day, etctera.

(a) How many minutes per day will Maria be walking on Week #6? Show the calculation that gives your answer.

(b) Scale the *y*-axis appropriately and graph the first six terms of this sequence. List them all if you haven't already.

(c) According to this geometric progression, how many minutes per day would Maria be walking on Week #10? Why is this not a viable answer?