

THE SHIFTED FORM OF A PARABOLA COMMON CORE ALGEBRA I



Although the standard form of a parabola has advantages for certain applications, it is not helpful locating the most important point on the parabola, the **turning point**. In this lesson, we will learn about a form of a parabola where the turning point is fairly obvious. First, though, a review of simple parabolas.

Exercise #1: Without using your calculator, sketch each of the parabolas shown below on your own set of axes. State the coordinates of the turning point of both.

(a) $y = 2x^2$

(b) $y = -3x^2$

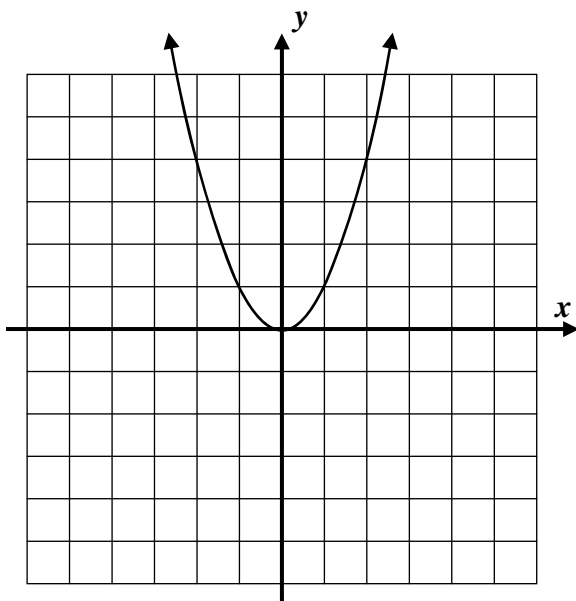
(c) If we have a parabola in the form $y = ax^2$ then it has a turning point at _____.

Now we would like to try to develop a pattern to see how a function can have its graph **shifted**.

Exercise #2: Consider the basic quadratic function $f(x) = x^2$ and the more complex quadratic function $g(x) = (x-2)^2 - 4$. The graph of $f(x) = x^2$ is shown on the grid already.

(a) Using your calculator to generate a table, sketch a graph of g .

(b) How would you need to shift the graph of $f(x)$ to get the graph of $g(x)$?



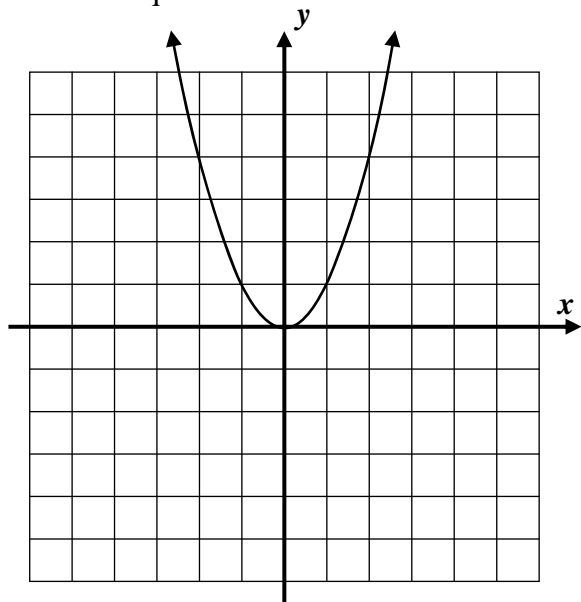
(c) What is the turning point of $g(x)$? Where do you “see” the turning point in the function’s equation?



Let's keep looking at this pattern but more simply.

Exercise #3: The parabola $y = x^2$ is again shown on the grid below. Consider the quadratic functions $y = x^2 + 2$ and $y = x^2 - 4$.

- (a) Using your calculator to generate tables, sketch these two quadratics and label.



- (b) What was the effect of adding a constant to the overall function?

- (c) State the coordinates of the turning points of each of the parabola you drew in (a).

$$y = x^2 + 2$$

$$y = x^2 - 4$$

- (d) What would the coordinates of the turning point of the parabola $y = x^2 - 150$ be?

Now let's see about that number added and subtracted from the input variable, x , before it is even squared.

Exercise #4: Yet (again), the parabola $y = x^2$ is graphed below. Now consider $y = (x+3)^2$ and $y = (x-1)^2$.

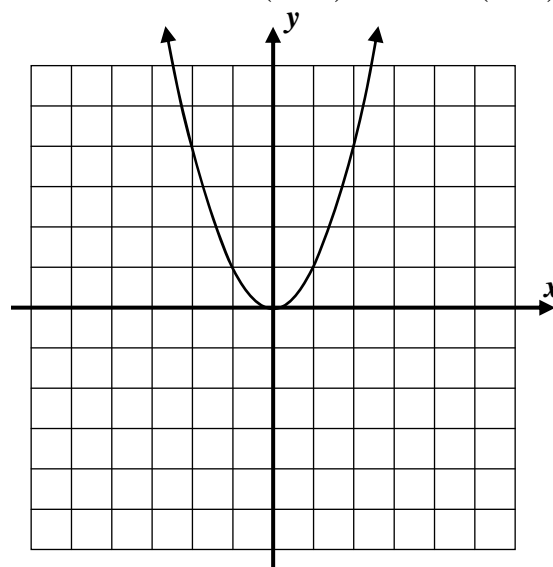
- (a) Using your calculator to generate tables, sketch these two quadratics and label.

- (b) Why is the horizontal shift counterintuitive?

- (c) State the coordinates of the turning points of each of the parabola you drew in (a).

$$y = (x+3)^2$$

$$y = (x-1)^2$$



- (d) Determine the coordinate of the turning points of each of the following quadratics. Note that the value of a is irrelevant.

$$y = (x-8)^2 + 5$$

$$y = 5(x+1)^2 - 4$$

$$y = -2(x-3)^2 - 10$$



THE SHIFTED FORM OF A PARABOLA
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

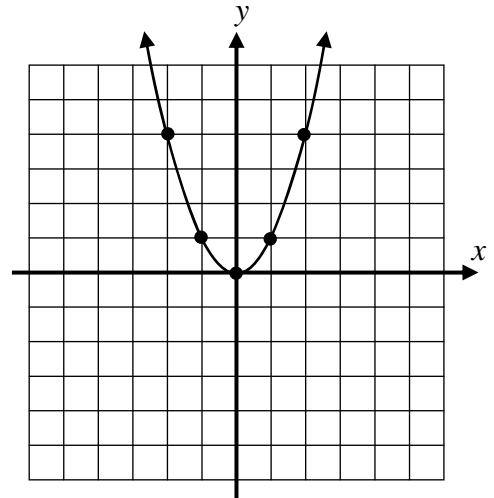
1. The grid below shows the graph of $y = x^2$ with particular points emphasized. On the same grid, draw the following **quadratic functions**. Try to do these as best as possible without using your calculator and then check your answers. Label each with its letter or equation.

(a) $y = x^2 - 6$

(b) $y = x^2 + 1$

(c) $y = (x + 3)^2$

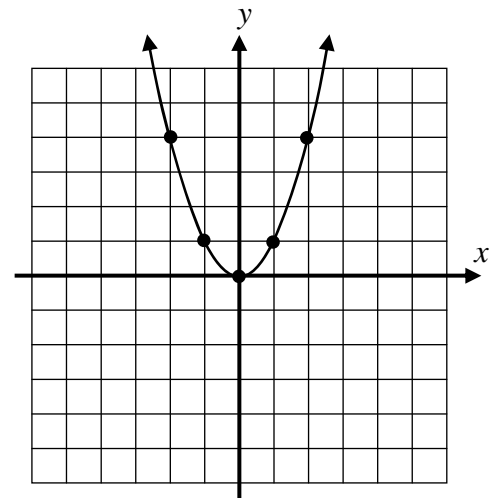
(d) $y = (x - 4)^2$



2. Again, the function $y = x^2$ is shown below. Graph each of the following more complicated quadratics without the use of your calculator. Then, use it to check that you have shifted the correct amounts. Label each with its letter or equation.

(a) $y = (x - 1)^2 - 4$

(b) $y = (x + 3)^2 - 1$



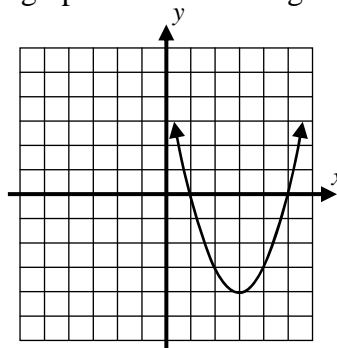
3. Which of the following equations represents the graph shown below given that it is a shift of the function $y = x^2$. Explain your choice.

(1) $y = (x - 3)^2 - 4$

(2) $y = (x + 3)^2 - 4$

(3) $y = (x - 3)^2 + 4$

(4) $y = (x + 3)^2 + 4$



4. State the turning points for each of the following quadratic functions and state whether the parabola opens upwards or downwards. Remember, the direction it opens only depends on the leading coefficient.

(a) $y = 4(x-2)^2 + 7$

(b) $y = -3(x+6)^2 + 4$

(c) $y = -(x+4)^2 - 3$

(d) $y = \frac{1}{2}(x+1)^2 - 7$

(e) $y = 9 - x^2$

(f) $y = -16(x-5)^2 + 11$

APPLICATIONS

5. An object traveling under an acceleration due to gravity alone will have a height, h , in meters above the ground t -seconds after it was fired given by

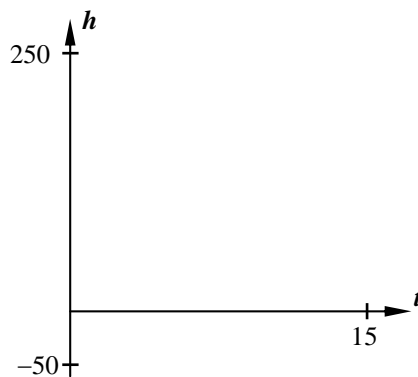
$$h = -4.9(t-6)^2 + 210$$

- (a) At what height does the object begin at $t = 0$?
Show work that supports your answer.

- (b) What is the peak height this object reaches in meters? When does it reach this height, in seconds?

- (c) Although you should be able to answer part (b) without your calculator, provide evidence in a table form that supports your answer from (b).

- (d) Using your calculator, sketch a graph of the height over the interval shown. Label your answers from (a) and (b).



REASONING

6. Why does the turning point of the quadratic $y = a(x-h)^2 + k$ **not** depend on the value of a . In other words why do both $y = 5(x-2)^2 + 3$ and $y = -8(x-2)^2 + 3$ have turning points of $(2, 3)$?

