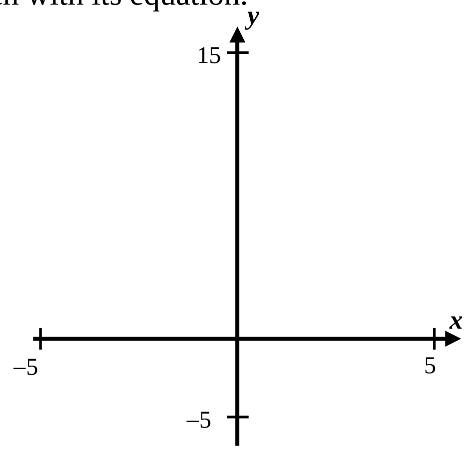
Name: ____________________________________ Date: __________________

STRETCHING PARABOLAS AND MORE COMPLETING THE SQUARE
COMMON CORE ALGEBRA I

We want to do one additional lesson to make sure we fully appreciate what the leading coefficient in a parabola tells us about it. Plus, we will get a bit more work with Completing the Square; although, it will be more challenging.

***Exercise* #1:** Let's understand what a really does in $y = ax^2 + bx + c$. So far we know that if a is positive, the parabola opens upwards and if a is negative, it opens downward. Let's see if we can deepen our understanding.

(a) Using your calculator, sketch a graph of $y = x^2$, $y = 2x^2$, and $y = 4x^2$ on the axes below. Use the window indicated on the axes. Label each with its equation.



(b) Explain what is happening when we multiply x^2 by a.

So, a stretches and compresses a parabola depending on whether it is greater than 1 or between 0 and 1. Sound familiar? This is similar to the base of an exponential function.

***Exercise* #2:** The graph below shows the curves that are listed. Write the number of the equation of each beside its curve.

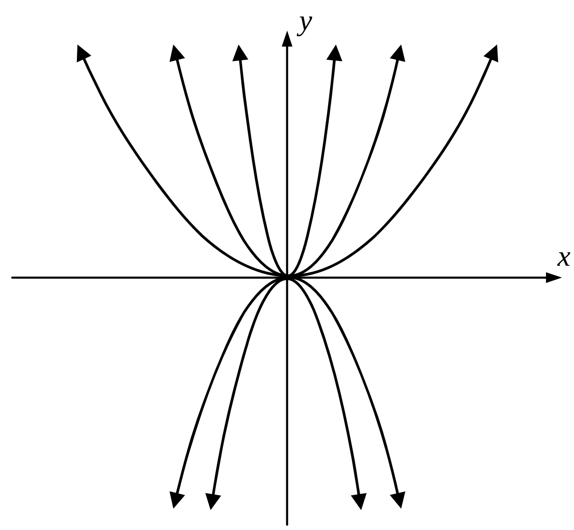
(1) $y = 3x^2$

(2) $y = \frac{1}{2}x^2$

(3) $y = -2x^2$

(4) $y = x^2$

(5) $y = -x^2$



Since **all** quadratics of the form $y = ax^2$ have their **turning points** at the **origin** we can also identify turning points if we can place a quadratic in vertex form even if *a* doesn't equal 1. This is harder, mechanically.

***Exercise* #3:** Consider the quadratic $y = 2x^2 - 12x + 11$.

(a) Use the Method of Completing the Square to write this equation in the vertex form $y = a(x-h)^2 + k$.

(b) What are the coordinates of the turning point of this quadratic based on (a)? Provide evidence from a calculator table that supports this answer.

Completing the Square, when the leading coefficient doesn't equal 1, is much more difficult to master and to understand. Always remember that you are writing an **equivalent expression** by essentially **adding zero** in one way or another.

***Exercise* #4:** Use the Method of Completing the Square to write each of the following quadratic functions in the vertex form $y = a(x-h)^2 + k$. Identify the turning point of the quadratic from this form. State whether it is a maximum or minimum.

(a) $y = 5x^2 + 20x + 23$

(b) $y = -2x^2 + 4x + 7$

(c) $y = 6x^2 - 24x + 14$

(d) $y = -x^2 - 12x - 33$

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STRETCHING PARABOLAS AND MORE COMPLETING THE SQUARE
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. The graph to the right contains the following functions. Match their letters to the correct curves.

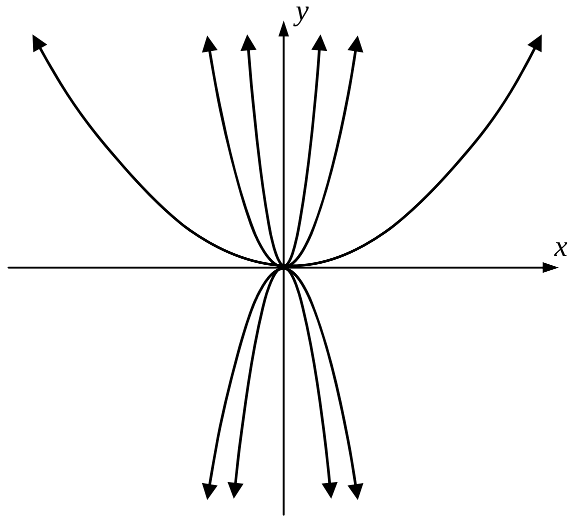
(A) $y = x^2$

(B) $y = -2x^2$

(C) $y = \frac{1}{4}x^2$

(D) $y = -x^2$

(E) $y = 3x^2$



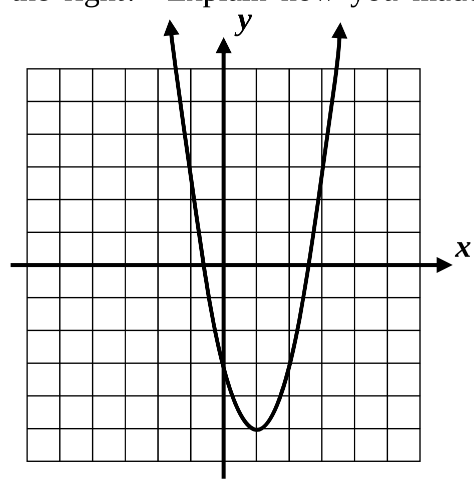
2. Which of the following equations models the graph shown at the right? Explain how you made your choice?

(1) $y = (x-1)^2 - 5$

(2) $y = -3(x+1)^2 - 5$

(3) $y = (x+1)^2 - 5$

(4) $y = 2(x-1)^2 - 5$



3. Use the method completing the square to write each of the following quadratic functions in the form $y = a(x-h)^2 + k$. Then, identify the turning point and whether it is a maximum or minimum.

(a) $y = 3x^2 - 12x + 17$

(b) $y = -5x^2 + 40x - 70$

APPLICATIONS

4. The vertical height of projectiles above level ground can be modeled by equations in the form:

$$h(t) = -16(t - t_{max})^2 + h_{max}$$

where h_{max} is the maximum height in feet and t_{max} is the time, in seconds, when it occurs.

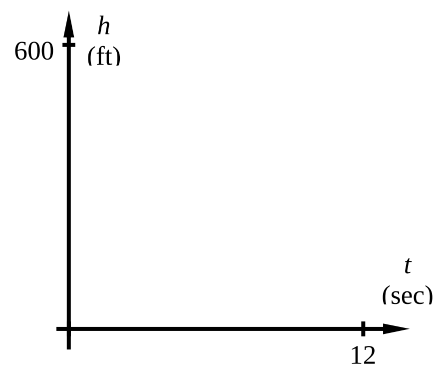
(a) A given projectile has a height function given by $h(t) = -16(t-8)^2 + 156$. What is its maximum height and at what time, *t*, does it occur?

(b) A projectile has a height function given by $h(t) = -16t^2 + 160t + 120$. Write this in the form shown above (vertex form).

(c) What is the maximum height and at what time does it occur for the projectile from (b)?

(d) At what height does the projectile in (b) start above the ground? Show the work that leads to your answer.

(e) Using your calculator, sketch a graph of the height on the axes below for the projectile from (b). Mark your answers from (c) and (d) on the graph.



REASONING

5. Every quadratic function can be placed in a vertex form: $y = a(x-h)^2 + k$. If we know the turning point of the parabola and **one other point** we can uniquely find this equation. Let's say we want to find the equation of a parabola that has a turning point at $(3, 9)$ and passes through the point $(5, 29)$.

(a) Write the equation of this parabola in the form $y = a(x-h)^2 + k$, leaving *a* as an unknown **constant** or **parameter**.

(b) Substitute the point $(5, 29)$ into the equation from part (a) and find the value of *a*.

(c) State the final equation of this parabola in vertex form and verify that it has the correct turning point and passes through $(5, 29)$ by examining a table on your calculator.