Name: ______________________________________ Date: ___________________

INTRODUCTION TO FUNCTIONS
COMMON CORE ALGEBRA II

Most higher level mathematics is built upon the concept of a function. Like most of the important concepts in mathematics, the definition of a function is simple to the point of being easily overlooked. Make sure to know the following definition:

> **DEFINITION:** A **function** is any "rule" that assigns exactly one output value (y-value) for each input value (x-value). These rules can be expressed in different ways, the most common being equations, graphs, and tables of values. We call the input variable **independent** and output variable **dependent**.

***Exercise* #1:** An internet music service offers a plan whereby users pay a flat monthly fee of $5 and can then download songs for 10 cents each.

(a) What are the independent and dependent variables in this scenario?

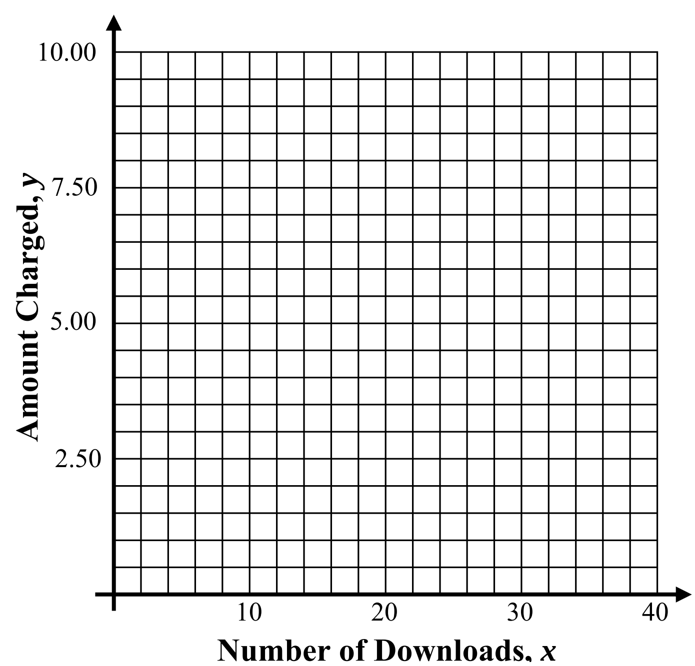
Independent: Dependent:

(b) Fill in the table below for a variety of independent values:

Number of downloads, x	0	5	10	20
Amount Charged, y				

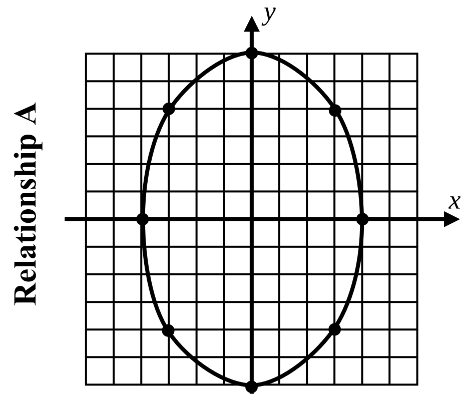
(c) Let the number of downloads be represented by the variable x and the amount charged be represented by the variable y, write an equation that models y as a function of x.

(d) Based on the equation you found in part (c), produce a graph of this function for all values of x on the interval $0 \le x \le 40$. Use a calculator **TABLE** to generate additional coordinate pairs to the ones you found in part (b).



***Exercise* #2:** One of the following graphs shows a relationship where y is a function of x and one does not.

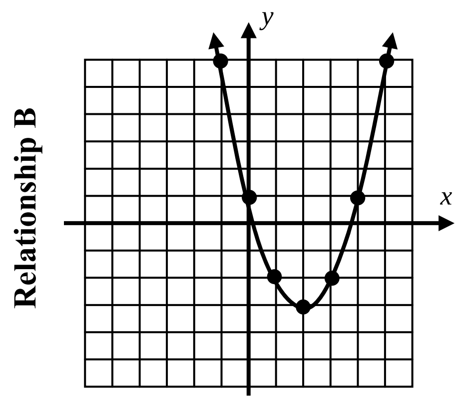
(a) Draw the vertical line whose equation is $x=3$ on both graphs.



(b) Give all output values for each graph at an input of 3.

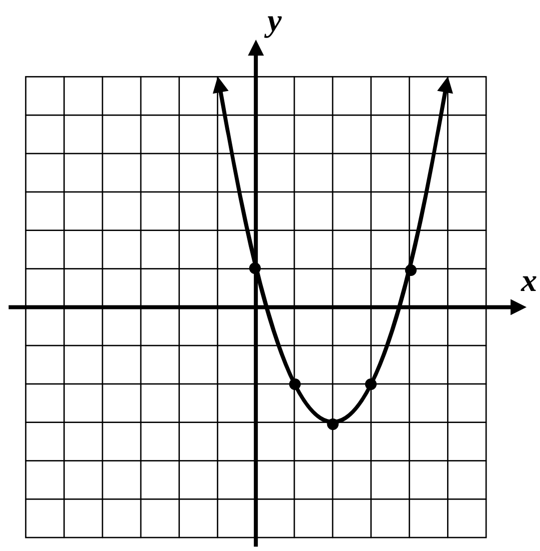
Relationship A: Relationship B:

(c) Explain which of these relationships is a function and why.



***Exercise* #3:** The graph of the function $y=x^2-4x+1$ is shown below.

(a) State this function's y-intercept.



(b) Between what two consecutive integers does the larger x-intercept lie?

(c) Draw the horizontal line $y=-2$ on this graph.

(d) Using these two graphs, find all values of x that solve the equation below:

$$x^2-4x+1=-2$$

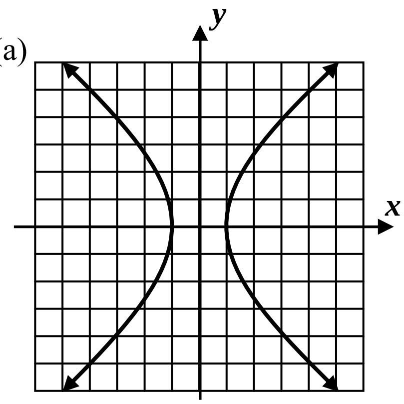
(e) Verify that these values of x are solutions by using **STORE** on your graphing calculator.

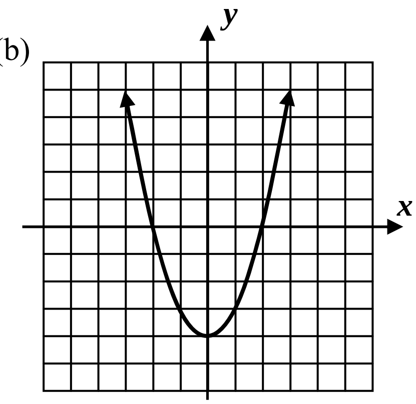
Name: ____________________________________ Date: __________________

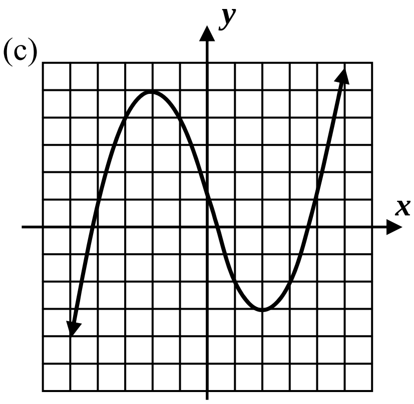
Introduction to Functions
Common Core Algebra II Homework

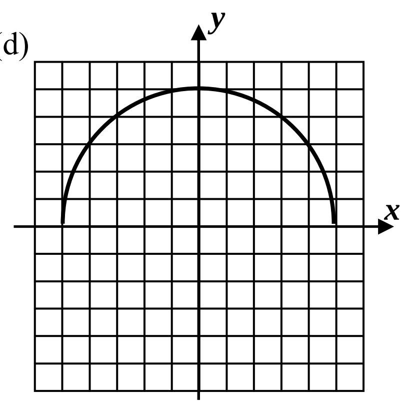
Fluency

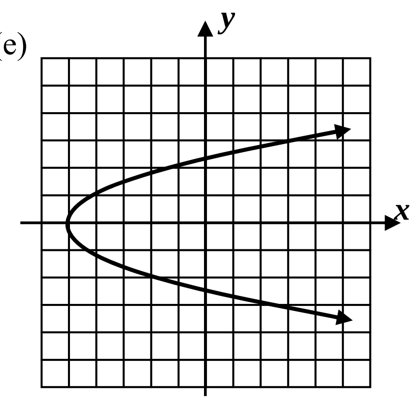
1. Determine for each of the following graphed relationships whether y is a function of x using the Vertical Line Test.

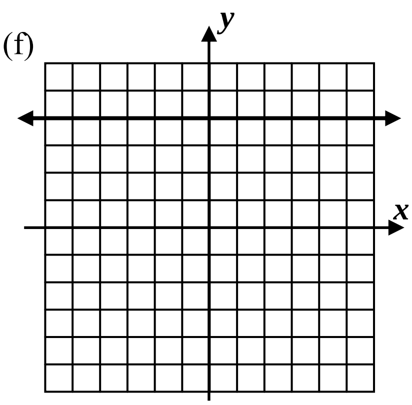
(a)


(b)


(c)


(d)


(e)


(f)


2. What are the outputs for an input of $x = 5$ given functions defined by the following formulas:

(a) $y = 3x - 4$ (b) $y = 50 - 2x^2$ (c) $y = 2^x$

APPLICATIONS

3. Evin is walking home from the museum. She starts 38 blocks from home and walks 2 blocks each minute. Evin's distance from home is a function of the number of minutes she has been walking.

(a) Which variable is independent and which variable is dependent in this scenario?

(b) Fill in the table below for a variety of time values.

Time, t, in minutes	0	1	5	10
Distance from home, D, in blocks				

(c) Determine an equation relating the distance, D, that Evin is from home as a function of the number of minute, t, that she has been walking.

(d) Determine the number of minutes, t, that it takes for Evin to reach home.

REASONING

4. In one of the following tables, the variable y is a function of the variable x. Explain which relationship is a function and why the other is not.

x	y
-2	11
0	7
2	11
4	23
6	43

Relationship #1

x	y
0	0
1	-1
1	1
4	-2
4	2

Relationship #2

Name: ___________________________________ Date: _________________

FUNCTION NOTATION
COMMON CORE ALGEBRA II

Functions are fundamental tools that convert inputs, values of the independent variable, to outputs, values of the dependent variable. There is a special notation that is commonly used to show this conversion process. The first exercise will illustrate this notation in the context of formulas.

***Exercise* #1:** Evaluate each of the following given the function definitions and input values.

(a) $f(x)=5x-2$

$f(3)=$

$f(-2)=$

(b) $g(x)=x^2+4$

$g(3)=$

$g(0)=$

(c) $h(x)=2^x$

$h(3)=$

$h(-2)=$

Although this notation could be confused with multiplication, the context will make it clear that it is not. The idea of function notation is summarized below.

FUNCTION NOTATION

$$y = f\ (x)$$

Output → y Rule → f Input → x

Recall that function rules commonly come in one of three forms: (1) equations (as in Exercise #1), (2) graphs, and (3) tables. The next few exercises will illustrate function notation with these three forms.

***Exercise* #2:** Boiling water at 212 degrees Fahrenheit is left in a room that is at 65 degrees Fahrenheit and begins to cool. Temperature readings are taken each hour and are given in the table below. In this scenario, the temperature, *T*, is a function of the number of hours, *h*.

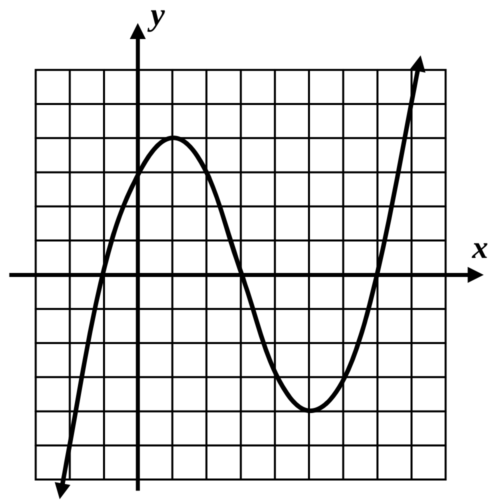
h (hours)	0	1	2	3	4	5	6	7	8
$T(h)$ $({}^\circ F)$	212	141	104	85	76	70	68	66	65

(a) Evaluate $T(2)$ and $T(6)$.

(b) For what value of *h* is $T(h)=76$?

(c) Between what two consecutive hours will $T(h)=100$?

***Exercise* #3:** The function $y = f(x)$ is defined by the graph shown below. Answer the following questions based on this graph.



(a) Evaluate $f(-1)$, $f(1)$, and $f(5)$.

(b) Evaluate $f(0)$. What special feature on a graph does $f(0)$ always correspond to?

(c) What values of x solve the equation $f(x) = 0$. What special features on a graph does the set of x-values that solve $f(x) = 0$ correspond to?

(d) Between what two consecutive integers does the largest solution to $f(x) = 3$ lie?

***Exercise* #4:** For a function $y = g(x)$ it is known that $g(-2) = 7$. Which of the following points must lie on the graph of $g(x)$?

(1) $(7, -2)$

(2) $(-2, 7)$

(3) $(0, 7)$

(4) $(-2, 0)$

____

***Exercise* #5:** Physics students drop a ball from the top of a 50 foot high building and model its height as a function of time with the equation $h(t) = 50 - 16t^2$. Using **TABLES** on your calculator, determine, to the nearest *tenth* of a second, when the ball hits the ground. Provide tabular outputs to support your answer.

Name: ___________________________________ Date: _________________

FUNCTION NOTATION
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Without using your calculator, evaluate each of the following given the function definitions and input values.

(a) $f(x)=3x+7$

$f(-4)=$

$f(2)=$

(b) $g(x)=3x^2$

$g(2)=$

$g(-3)=$

(c) $h(x)=\sqrt{x-5}$

$h(41)=$

$h(14)=$

2. Using **STORE** on your calculator, evaluate each of the following more complex functions.

(a) $f(x)=\dfrac{3x^2-5}{4x+10}$

$f(-5)=$

$f(0)=$

(b) $g(x)=\dfrac{\sqrt{25-x^2}}{x}$

$g(4)=$

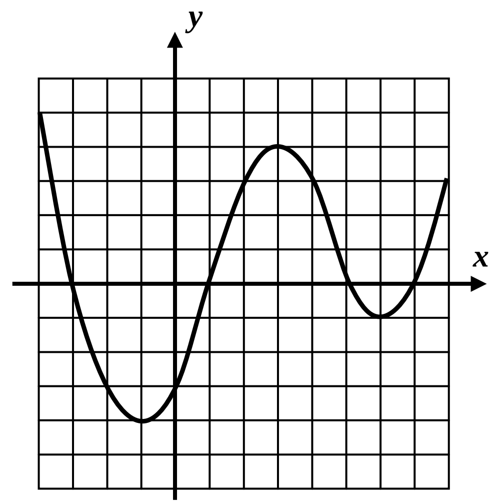
$g(-3)=$

(c) $h(x)=30(1.2)^x$

$h(3)=$

$h(0)=$

3. Based on the graph of the function $y=g(x)$ shown below, answer the following questions.



(a) Evaluate $g(-2)$, $g(0)$, $g(3)$ and $g(7)$.

(b) What values of x solve the equation $g(x)=0$

(c) Graph the horizontal line $y=2$ on the grid above and label.

(d) How many values of x solve the equation $g(x)=2$?

APPLICATIONS

4. Ian invested \$2500 in an investment vehicle that is guaranteed to earn 4% interest compounded yearly. The amount of money, *A*, in his account as a function of the number of years, *t*, since creating the account is given by the equation $A(t)=2500(1.04)^t$.

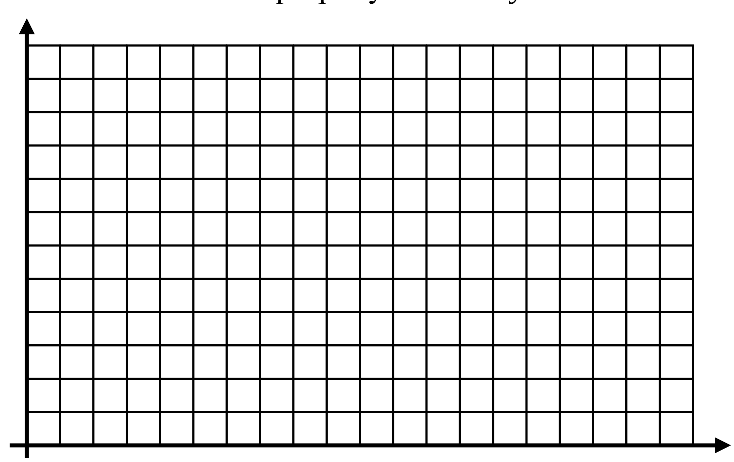
(a) Evaluate $A(0)$ and $A(10)$.

(b) What do the two values that you found in part (a) represent?

(c) Using tables on your calculator, determine, to the nearest whole year, the value of *t* that solves the equation $A(t)=5000$. Justify your answer with numerical evidence.

(d) What does the value of *t* that you found in part (b) represent about Ian's investment?

5. A ball is shot from an air-cannon at an angle of 45° with the horizon. It travels along a path given by the equation $h(d)=-\frac{1}{50}d^2+d$, where *h* represents the ball's height above the ground and *d* represents the distance the ball has traveled horizontally. Using your calculator to generate a table of values, graph this function for all values of *d* on the interval $0\le d\le 50$. Look at the table to properly scale the *y*-axis.



What is the maximum height that the ball reaches? At what value of *d* does it reach this height?

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FUNCTION COMPOSITION
COMMON CORE ALGEBRA II

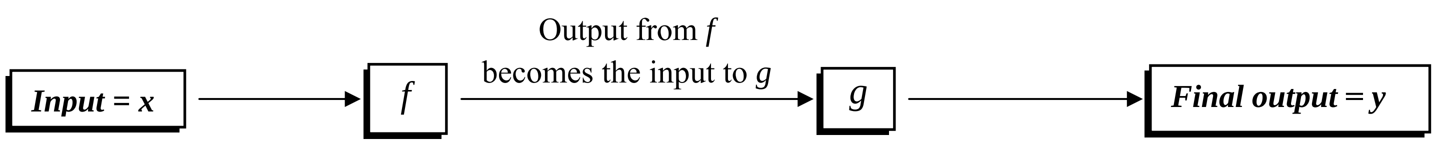
Since functions convert the value of an input variable into the value of an output variable, it stands to reason that this output could then be used as an input to a second function. This process is known as composition of functions, in other words, combining the action or rules of two functions.

***Exercise* #1:** A circular garden with a radius of 15 feet is to be covered with topsoil at a cost of \$1.25 per square foot of garden space.

(a) Determine the area of this garden to the nearest square foot.

(b) Using your answer from (a), calculate the cost of covering the garden with topsoil.

In this exercise, we see that the output of an area function is used as the input to a cost function. This idea can be generalized to generic functions, *f* and *g* as shown in the diagram below.



There are two notations that are used to indicate composition of two functions. These will be introduced in the next few exercises, both with equations and graphs.

***Exercise* #2:** Given $f(x)=x^2-5$ and $g(x)=2x+3$, find values for each of the following.

(a) $f(g(1))=$

(b) $g(f(2))=$

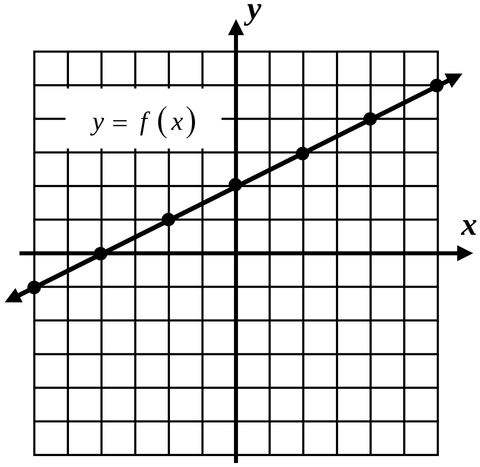
(c) $g(g(0))=$

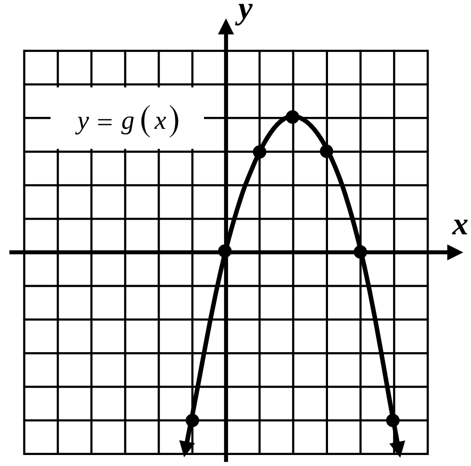
(d) $(f\circ g)(-2)=$

(e) $(g\circ f)(3)=$

(f) $(f\circ f)(-1)=$

***Exercise* #3:** The graphs below are of the functions $y = f(x)$ and $y = g(x)$. Evaluate each of the following questions based on these two graphs.





(a) $g(f(2)) =$

(b) $f(g(-1)) =$

(c) $g(g(1)) =$

(d) $(g \circ f)(-2) =$

(e) $(f \circ g)(0) =$

(f) $(f \circ f)(0) =$

On occasion, it is desirable to create a formula for the composition of two functions. We will see this facet of composition throughout the course as we study functions. The next two exercises illustrate the process of finding these equations with simple linear and quadratic functions.

***Exercise* #4:** Given the functions $f(x) = 3x - 2$ and $g(x) = 5x + 4$, determine formulas in simplest $y = ax + b$ form for:

(a) $f(g(x))$

(b) $g(f(x))$

***Exercise* #5:** If $f(x) = x^2$ and $g(x) = x - 5$ then $f(g(x)) =$

(1) $x^2 + 25$

(2) $x^2 - 25$

(3) $x^2 - 5$

(4) $x^2 - 10x + 25$

____

Name: ______________________________ Date: ______________

FUNCTION COMPOSITION
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Given $f(x) = 3x - 4$ and $g(x) = -2x + 7$ evaluate:

(a) $f(g(0))$ (b) $g(f(-2))$ (c) $f(f(3))$

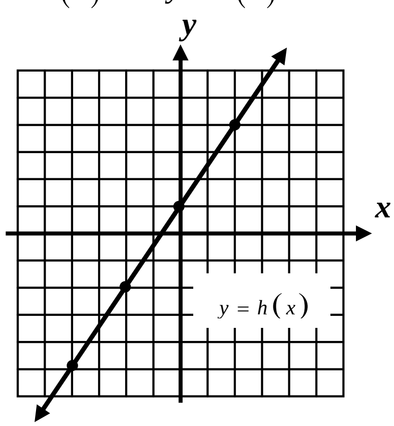
(d) $(g \circ f)(6)$ (e) $(f \circ g)(5)$ (f) $(g \circ g)(2)$

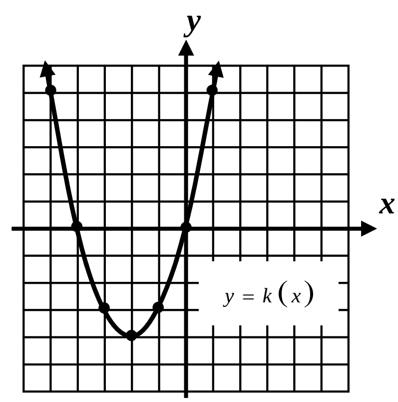
2. Given $h(x) = x^2 + 11$ and $g(x) = \sqrt{x-2}$ evaluate:

(a) $h(g(18))$ (b) $g(h(4))$ (c) $(g \circ g)(11)$

(d) $h(h(0))$ (e) $(h \circ g)(38)$ (f) $(g \circ h)(0)$

3. The graphs of $y = h(x)$ and $y = k(x)$ are shown below. Evaluate the following based on these two graphs.





(a) $h(k(-2))$ (b) $(k \circ h)(0)$ (c) $h(h(-2))$ (d) $(k \circ k)(-2)$

4. If $g(x)=3x-5$ and $h(x)=2x-4$ then $(g\circ h)(x)=?$

(1) $6x-17$ (3) $5x-9$

(2) $6x-14$ (4) $x-1$ ________

5. If $f(x)=x^2+5$ and $g(x)=x+4$ then $f(g(x))=$

(1) x^2+9 (3) $4x^2+20$

(2) $x^2+8x+21$ (4) x^2+21 ________

APPLICATIONS

6. Scientists modeled the intensity of the sun, I, as a function of the number of hours *since* 6:00 a.m., h, using the function $I(h)=\dfrac{12h-h^2}{36}$. They then model the temperature of the soil, T, as a function of the intensity using the function $T(I)=\sqrt{5000I}$. Which of the following is closest to the temperature of the soil at 2:00 p.m. ?

(1) 54 (3) 67

(2) 84 (4) 38 ________

7. Physics students are studying the effect of the temperature, T, on the speed of sound, S. They find that the speed of sound in meters per second is a function of the temperature in degrees Kelvin, K, by $S(K)=\sqrt{410K}$. The degrees Kelvin is a function of the temperature in Celsius given by $K(C)=C+273.15$. Find the speed of sound when the temperature is 30 degrees Celsius. Round to the nearest *tenth*.

REASONING

8. Consider the functions $f(x)=2x+9$ and $g(x)=\dfrac{x-9}{2}$. Calculate the following.

(a) $g(f(15))$ (b) $g(f(-3))$ (c) $g(f(x))$

(d) What appears to always be true when you compose these two functions?

Name: ___________________________________ Date: _________________

THE DOMAIN AND RANGE OF A FUNCTION
COMMON CORE ALGEBRA II

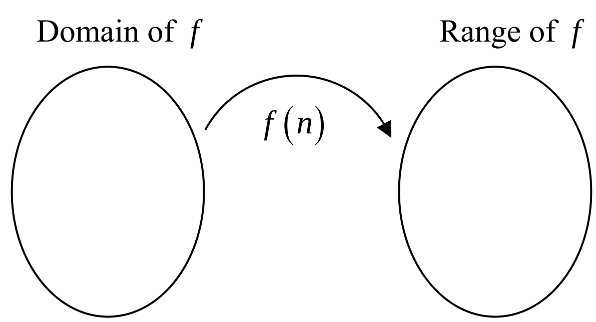
Because functions convert values of inputs into value of outputs, it is natural to talk about the sets that represent these inputs and outputs. The **set of inputs** that result in an output is called **the domain** of the function. The **set of outputs** is called **the range**.

***Exercise* #1:** Consider the function that has as its inputs the months of the year and as its outputs the number of days in each month. In this case, the number of days is a function of the month of the year. Assume this function is restricted to non-leap years.

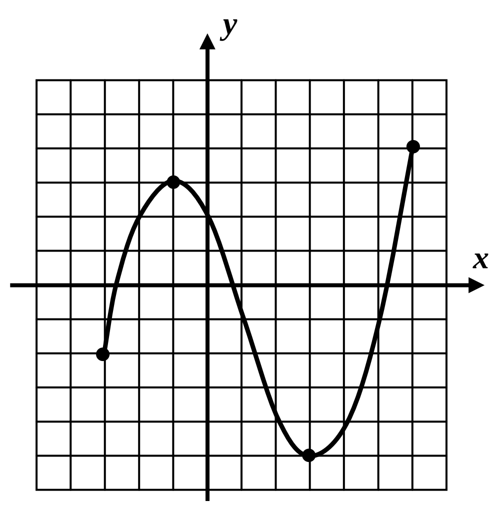
(a) Write, in roster form, the set that represents this function's domain.

(b) Write, in roster for, the set that represents this function's range.

***Exercise* #2:** State the range of the function $f(n)=2n+1$ if its domain is the set $\{1,3,5\}$. Show the domain and range in the mapping diagram below.



***Exercise* #3:** The function $y=g(x)$ is completely defined by the graph shown below. Answer the following questions based on this graph.



(a) Determine the minimum and maximum *x*-values represented on this graph.

(b) Determine the minimum and maximum *y*-values represented on this graph.

(c) State the domain and range of this function using set builder notation.

Some functions, defined with graphs or equations, have domains and ranges that stretch out to infinity. Consider the following exercise in which a standard parabola is graphed.

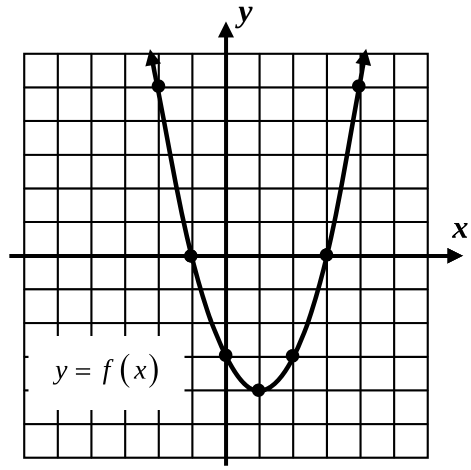
Exercise #4: The function $f(x) = x^2 - 2x - 1$ is graphed on the grid below. Which of the following represent its domain and range written in interval notation?

(1) Domain: $[-2, 4]$
Range: $[-4, 6]$

(2) Domain: $[-2, 4]$
Range: $(-4, \infty)$

(3) Domain: $(-\infty, \infty)$
Range: $[-4, \infty)$

(4) Domain: $(-2, 4)$
Range: $(-4, 6)$



For most functions defined by an algebraic formula, the domain consists of the set of all real numbers, given the concise symbol $\mathbb{R}$. Sometimes, though, there are restrictions placed on the domain of a function by the structure of its formula. Two basic restrictions will be illustrated in the next few exercises.

Exercise #5: The function $f(x) = \dfrac{2x+1}{x-4}$ has outputs given by the following calculator table.

x	$f(x)$
1	-1
2	-2.5
3	-7
4	Error
5	11
6	6.5
7	5

(a) Evaluate $f(1)$ and $f(6)$ from the table.

(b) Why does the calculator give an ERROR at $x = 4$?

(c) Are there any values except $x = 4$ that are not in the domain of f? Explain.

Exercise #6: Which of the following values of x would not be in the domain of the function $y = \sqrt{x+4}$? Explain your answer.

(1) $x = 0$

(2) $x = 5$

(3) $x = -3$

(4) $x = -8$

____

Name: ___________________________________ Date: _________________

THE DOMAIN AND RANGE OF A FUNCTION
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. A function is given by the set of ordered pairs $\{(2,5),(4,9),(6,13),(8,17)\}$. Write its domain and range in roster form.

   Domain: Range:

2. The function $h(x)=x^2+5$ maps the domain given by the set $\{-2,-1,0,1,2\}$. Which of the following sets represents the range of $h(x)$?

   (1) $\{0,6,10,12\}$ (3) $\{5,6,9\}$

   (2) $\{5,6,7\}$ (4) $\{1,4,5,6,9\}$

   ____

3. Which of the following values of x would *not* be in the domain of the function defined by $f(x)=\dfrac{x-2}{x+3}$?

   (1) $x=-3$ (3) $x=3$

   (2) $x=2$ (4) $x=-2$

   ____

4. Determine any values of x that do not lie in the domain of the function $f(x)=\dfrac{3x+2}{2x-10}$. Justify your response.

5. Which of the following values of x *does* lie in the domain of the function defined by $g(x)=\sqrt{2x-7}$?

   (1) $x=0$ (3) $x=3$

   (2) $x=2$ (4) $x=5$

   ____

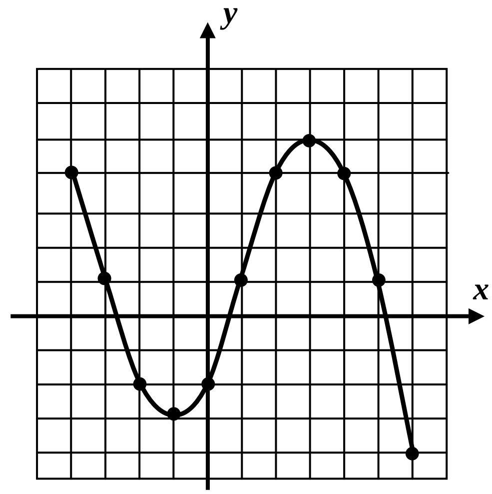
6. Which of the following would represent the domain of the function $y=\sqrt{6-2x}$?

   (1) $\{x:x>3\}$ (3) $\{x:x\le 3\}$

   (2) $\{x:x<3\}$ (4) $\{x:x\ge 3\}$

   ____

7. The function $y = f(x)$ is completely defined by the graph shown below.



(a) Evaluate $f(-4)$, $f(3)$, and $f(6)$.

(b) Draw a rectangle that circumscribes (just surrounds) the graph.

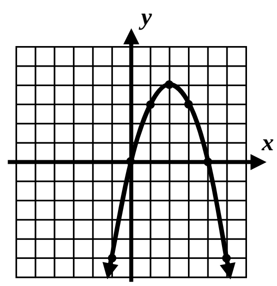
(c) State the domain and range of this function using interval notation.

Domain: Range:

8. Which of the following represents the range of the quadratic function shown in the graph below?

(1) $(4, \infty)$ (3) $(-\infty, 4)$

(2) $(-\infty, 4]$ (4) $[4, \infty)$



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APPLICATIONS

9. A child starts a piggy bank with \$2. Each day, the child receives 25 cents at the end of the day and puts it in the bank. If A represents the amount of money and d stands for the number of days then $A(d) = 2 + 0.25d$ gives the amount of money in the bank as a function of days (think about this formula).

(a) Evaluate $A(1)$, $A(7)$, and $A(30)$.

(b) For what value of d will $A(d) = \$10.50$.

(c) Explain why the domain does not contain the value $d = 2.5$.

(d) Explain why the range does not include the value $A = \$3.10$.

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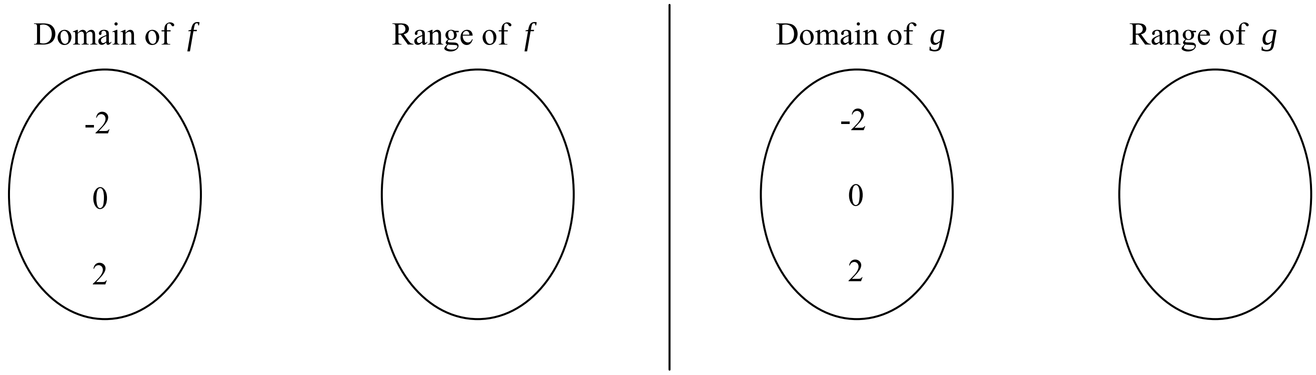
ONE-TO-ONE FUNCTIONS
COMMON CORE ALGEBRA II

Functions as rules can be divided into various categories based on shared characteristics. One category is comprised of functions known as one-to-one. The following exercise will be illustrate the difference between a function that is one-to-one and one that is not.

***Exercise* #1:** Consider the two simple functions given by the equations $f(x) = 2x$ and $g(x) = x^2$.

(a) Map the domain $\{-2, 0, 2\}$ using each function. Fill in the range and show the mapping arrows.



(b) What is fundamentally different between these two functions in terms of how the elements of this domain get mapped to the elements of the range.

ONE-TO-ONE FUNCTIONS

A function $f(x)$ is called one-to-one if $x_1 \neq x_2$ implies that $f(x_1) \neq f(x_2)$.

(In other words, different inputs give different outputs.)

***Exercise* #2:** Of the four tables below, one represents a relationship where *y* is a one-to-one function of *x*. Determine which it is and explain why the others are not.

(1)

x	*y*
4	2
4	-2
9	3
9	-3

(2)

x	*y*
-2	1
-1	0
0	1
1	2

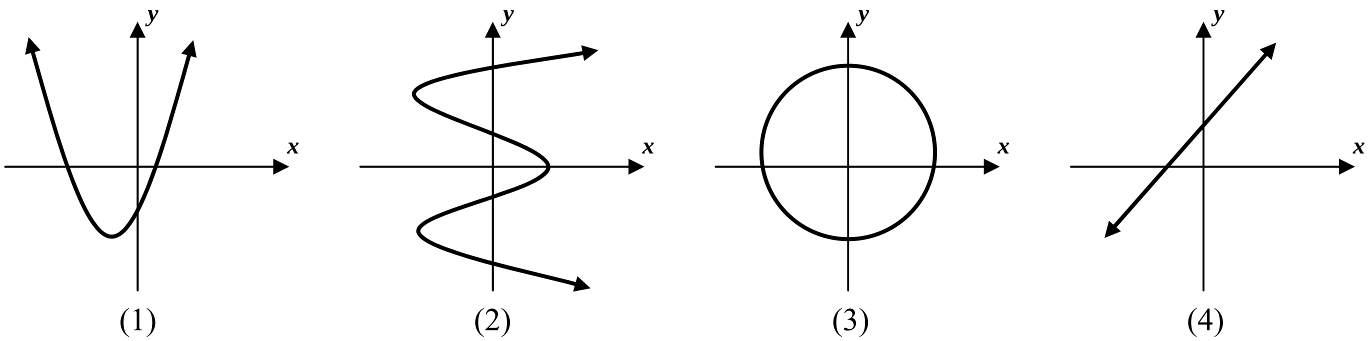
(3)

x	*y*
1	2
2	4
3	8
4	16

(4)

x	*y*
-3	10
-2	9
-1	7
-2	10

***Exercise* #3:** Consider the following four graphs which show a relationship between the variables y and x.



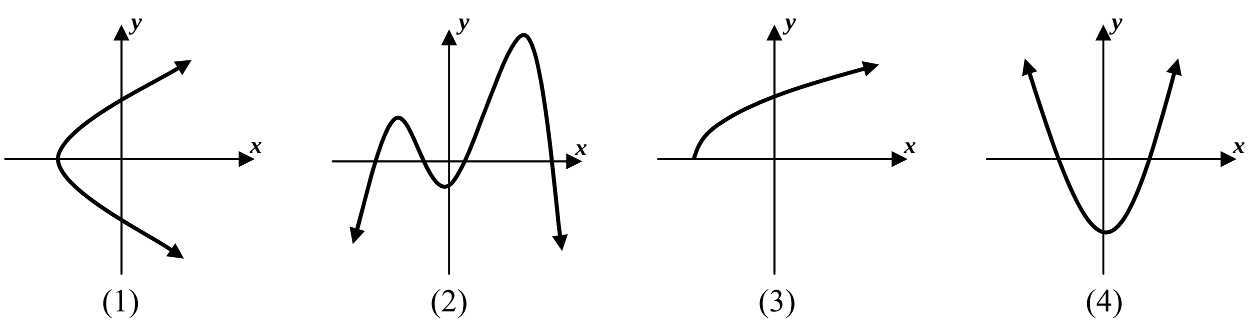
(a) Circle the two graphs above that are functions. Explain how you know they are functions.

(b) Of the two graphs you circled, which is one-to-one? Explain how you can tell from its graph.

THE HORIZONTAL LINE TEST

If any given horizontal line passes through the graph of a function at most one time, then that function is one-to-one. This test works because horizontal lines represent constant y-values; hence, if a horizontal line intersects a graph more than once, an output has been repeated.

***Exercise* #4:** Which of the following represents the graph of a one-to-one function?



______

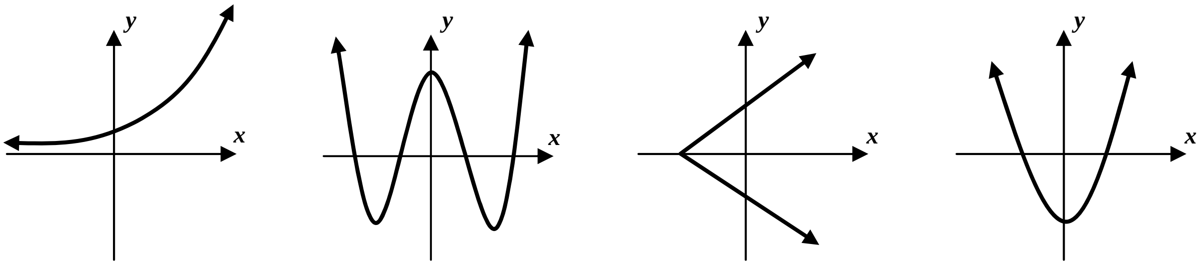
***Exercise* #5:** The distance that a number, x, lies from the number 5 on a one-dimensional number line is given by the function $D(x)=|x-5|$. Show by example that $D(x)$ is *not* a one-to-one function.

Name: ___________________________________ Date: ___________________

ONE-TO-ONE FUNCTIONS
COMMON CORE ALGEBRA II HOMEWORK

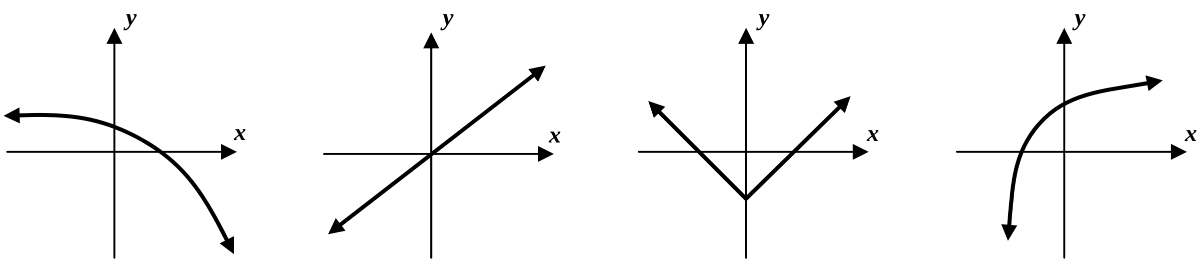
FLUENCY

1. Which of the following graphs illustrates a one-to-one relationship?



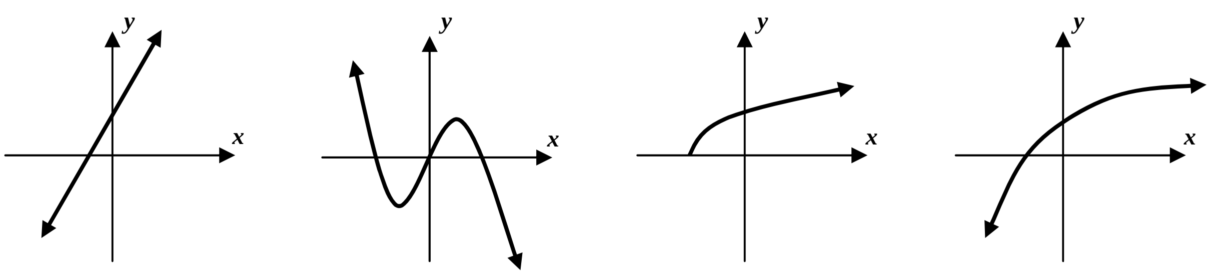
(1) (2) (3) (4) __________

2. Which of the following graphs does *not* represent that of a one-to-one function?



(1) (2) (3) (4) __________

3. In which of the following graphs is each input *not* paired with a *unique* output?



(1) (2) (3) (4) __________

4. In which of the following formulas is the variable y a one-to-one function of the variable x? (Hint – try generating some values either in your head or using TABLES on your calculator.)

(1) $y = x^2$ (3) $y = 2x$

(2) $y = |x|$ (4) $y = 5$ __________

5. Which of the following tables illustrates a relationship in which y is a one-to-one function of x?

(1)

x	y
-2	-1
0	-3
2	-1
4	1
6	3

(2)

x	y
-2	-8
-1	-1
0	0
1	1
2	8

(3)

x	y
-2	-5
-1	-4
0	-1
-1	7
-2	5

(4)

x	y
-2	11
-1	-4
0	-5
1	-4
2	11

______

APPLICATIONS

6. Physics students drop a basketball from 5 feet above the ground and its height is measured each tenth of a second until it stops bouncing. The height of the basketball, h, is clearly a function of the time, t, since it was dropped.

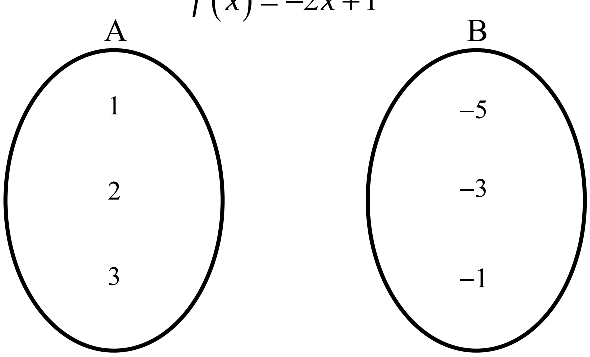
(a) Sketch the general graph of what you believe this function would look like.

(b) Is the height of the ball a one-to-one function of time? Explain your answer.

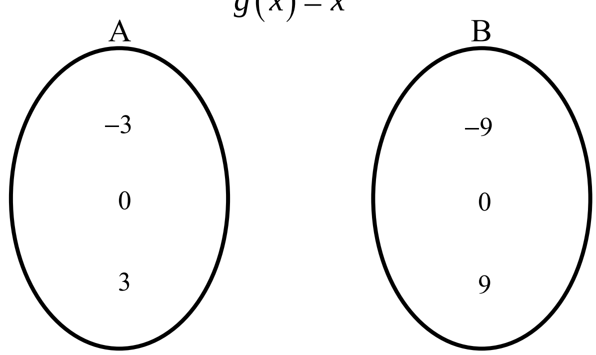
REASONING – ONTO FUNCTIONS (OR MAPPINGS) – Another important type of function is known as **onto**. An **onto function or onto mapping** occurs when a function maps the elements from set A to set B and all elements in set *B* get mapped to. Every member of the output set must be hit for a function to be **onto**.

7. In each case below, show how elements in set A get mapped to elements in set B. Then, state which mapping is **onto** and which is not **onto**.

(a) $f(x) = -2x + 1$



(b) $g(x) = x^2$



Name: ___________________________________ Date: _________________

INVERSE FUNCTIONS
COMMON CORE ALGEBRA II

The idea of inverses, or opposites, is very important in mathematics. So important, in fact, that the word is used in many different contexts, including the additive and multiplicative inverses of a number. The actions of certain functions can be reversed as well. The rules governing the reversal themselves can be functions.

***Exercise* #1:** Consider the two linear functions given by the formulas $f(x)=\frac{3x+7}{2}$ and $g(x)=\frac{2x-7}{3}$.

(a) Calculate $f(5)$ and $g(11)$.

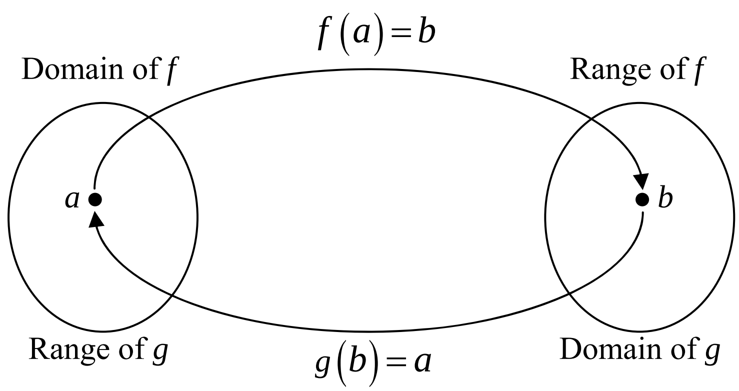
(b) Calculate $f(0)$ and $g\left(\frac{7}{2}\right)$.

(c) Calculate $f(g(-1))$.

(d) Calculate $f(g(5))$.

(e) Without calculation, determine the value of $f(g(\pi))$.

The two functions seen in Exercise #1 are inverses because they literally "undo" one another. The general idea of inverses, $f(x)$ and $g(x)$, is shown below in the mapping diagram.



***Exercise* #2:** If the point $(-3, 5)$ lies on the graph of $y = f(x)$, then which of the following points must lie on the graph of its inverse?

(1) $(3, -5)$

(3) $(5, -3)$

(2) $(-5, 3)$

(4) $\left(-\frac{1}{3}, \frac{1}{5}\right)$

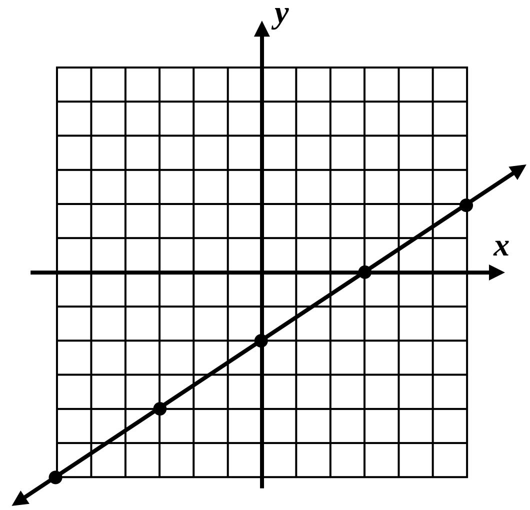
____

Inverse functions have their own special notation. It is shown in the box below.

INVERSE FUNCTION NOTATION

If a function $y = f(x)$ has an inverse that is also a function we represent it as $y = f^{-1}(x)$.

***Exercise* #3:** The linear function $f(x) = \frac{2}{3}x - 2$ is shown graphed below. Use its graph to answer the following questions.



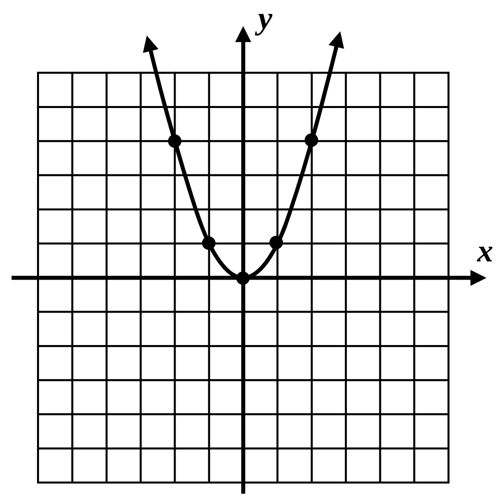
(a) Evaluate $f^{-1}(2)$ and $f^{-1}(-4)$.

(b) Determine the y-intercept of $f^{-1}(x)$.

(c) On the same set of axes, draw a graph of $y = f^{-1}(x)$.

***Exercise* #4:** A table of values for the simple quadratic function $f(x) = x^2$ is given below along with its graph.

x	-2	-1	0	1	2
$f(x)$	4	1	0	1	4



(a) Graph the inverse by switching the ordered pairs.

x					
$f^{-1}(x)$					

(b) What do you notice about the graph of this function's inverse?

EXISTENCE OF INVERSE FUNCTIONS

A function will have an inverse that is also a function if and only if it is one-to-one. Hence, a quick way to know if a function has an inverse that is also a function is to apply the Horizontal Line Test.

Name: ___________________________________ Date: _________________

INVERSE FUNCTIONS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. If the point $(-7, 5)$ lies on the graph of $y = f(x)$, which of the following points must lie on the graph of its inverse?

(1) $(5, -7)$ (3) $(7, -5)$

(2) $\left(-\frac{1}{7}, \frac{1}{5}\right)$ (4) $\left(\frac{1}{7}, -\frac{1}{5}\right)$ __________

2. The function $y = f(x)$ has an inverse function $y = f^{-1}(x)$. If $f(a) = -b$ then which of the following must be true?

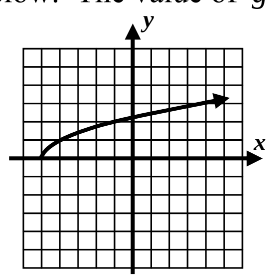
(1) $f^{-1}(-b) = -a$ (3) $f^{-1}(-b) = a$

(2) $f^{-1}\left(\frac{1}{a}\right) = -\frac{1}{b}$ (4) $f^{-1}(b) = -a$ __________

3. The graph of the function $y = g(x)$ is shown below. The value of $g^{-1}(2)$ is

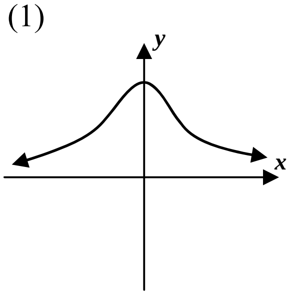
(1) 2.5 (3) 0.4

(2) −4 (4) −1 __________

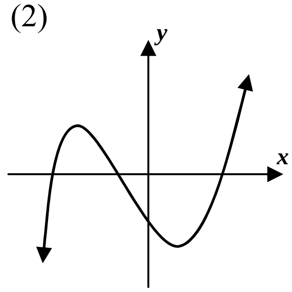


4. Which of the following functions would have an inverse that is also a function? __________

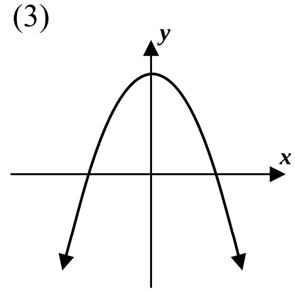
(1)



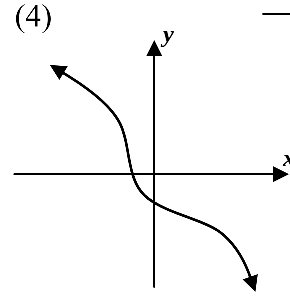
(2)



(3)



(4)



5. For a one-to-one function it is known that $f(0) = 6$ and $f(8) = 0$. Which of the following must be true about the graph of this function's inverse?

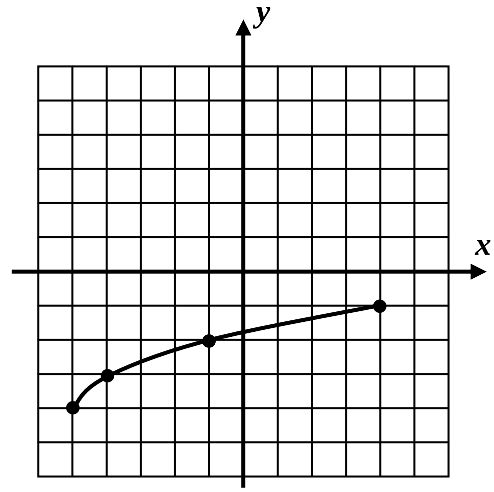
(1) its y-intercept = 6 (3) its x-intercept = −6

(2) its y-intercept = 8 (4) its x-intercept = −8 __________

6. The function $y = h(x)$ is entirely defined by the graph shown below.

(a) Sketch a graph of $y = h^{-1}(x)$. Create a table of values if needed.

(b) Write the domain and range of $y = h(x)$ and $y = h^{-1}(x)$ using interval notation.



$y = h(x)$

Domain:

Range:

$y = h^{-1}(x)$

Domain:

Range:

Applications

7. The function $y = A(r) = \pi r^2$ is a one-to-one function that uses a circle's radius as an input and gives the circle's area as its output. Selected values of this function are shown in the table below.

r	1	2	3	4	5	6
$A(r)$	π	4π	9π	16π	25π	36π

(a) Determine the values of $A^{-1}(9\pi)$ and $A^{-1}(36\pi)$ from using the table.

(b) Determine the values of $A^{-1}(100\pi)$ and $A^{-1}(225\pi)$

(c) The original function $y = A(r)$ converted an input, the circle's radius, to an output, the circle's area. What are the inputs and outputs of the inverse function?

Input:

Output:

Reasoning

8. The domain and range of a one-to-one function, $y = f(x)$, are given below in set-builder notation. Give the domain and range of this function's inverse also in set-builder notation.

$y = f(x)$

Domain: $\{x \mid -3 \le x < 5\}$

Range: $\{y \mid y > -2\}$

$y = f^{-1}(x)$

Domain:

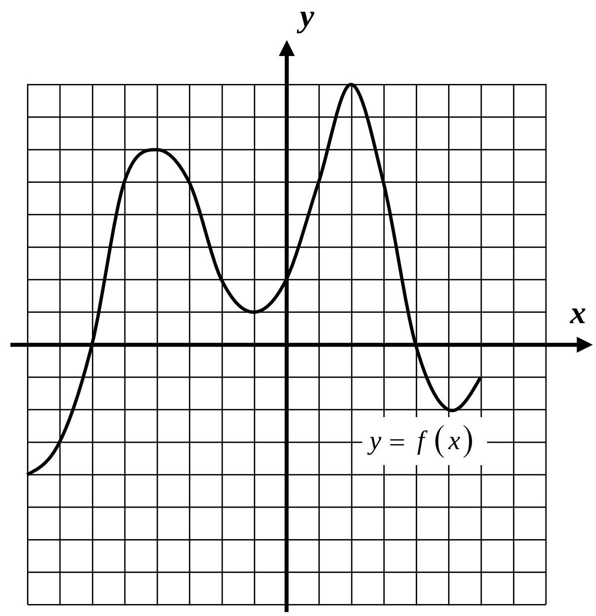
Range:

Name: ____________________________________ Date: __________________

KEY FEATURES OF FUNCTIONS
COMMON CORE ALGEBRA II

The graphs of functions have many key features whose terminology we will be using all year. It is important to master this terminology, most of which you learned in Common Core Algebra I.

***Exercise* #1:** The function $y = f(x)$ is shown graphed to the right. Answer the following questions based on this graph.



(a) State the y-intercept of the function.

(b) State the x-intercepts of the function. What is the alternative name that we give the x-intercepts?

(c) Over the interval $-1 < x < 2$ is $f(x)$ increasing or decreasing? How can you tell?

(d) Give the interval over which $f(x) > 0$. What is a quick way of seeing this visually?

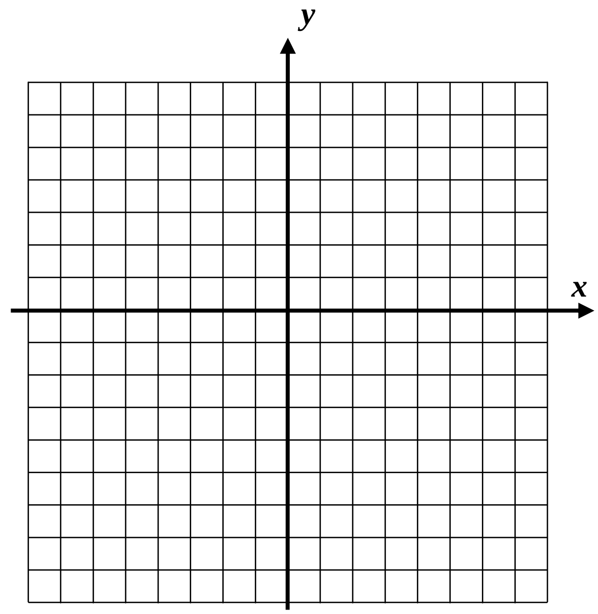
(e) State all the x-coordinates of the relative maximums and relative minimums. Label each.

(f) What are the absolute maximum and minimum values of the function? Where do they occur?

(g) State the domain and range of $f(x)$ using interval notation.

(h) If a second function $g(x)$ is defined by the formula $g(x) = \frac{1}{2} f(x+2)$, then what is the y-intercept of g?

***Exercise* #2:** Consider the function $g(x)=2|x-1|-8$ defined over the domain $-4\le x\le 7$.



(a) Sketch a graph of the function to the right.

(b) State the domain interval over which this function is decreasing.

(c) State zeroes of the function on this interval.

(d) State the interval over which $g(x)\le 0$

(e) Evaluate $g(0)$ by using the algebraic definition of the function. What point does this correspond to on the graph?

(f) Are there any relative maximums or minimums on the graph? If so, which and what are their coordinates?

You need to be able to think about functions in all of their forms, including equations, graphs, and tables. Tables can be quick to use, but sometimes hard to understand.

***Exercise* #3:** A **continuous** function $f(x)$ has a domain of $-6\le x\le 13$ with selected values shown below. The function has exactly two zeroes and has exactly two turning points, one at $(3,-4)$ and one at $(9,3)$.

x	-6	-1	0	3	5	8	9	13
$f(x)$	5	0	-2	-4	-1	0	3	1

(a) State the interval over which $f(x)<0$.

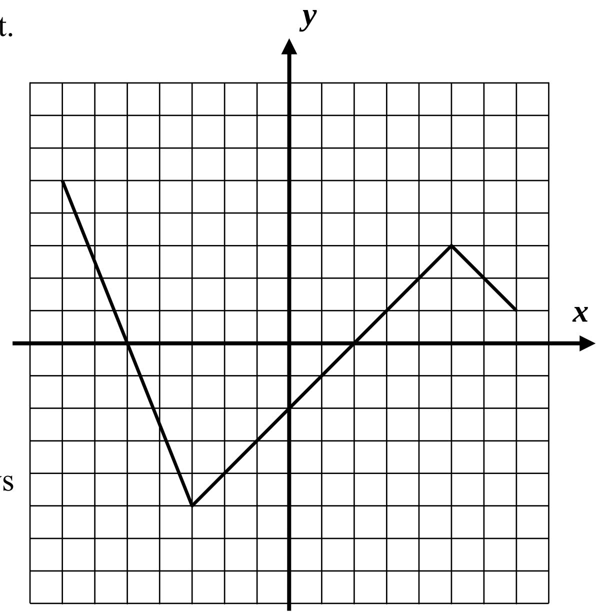
(b) State the interval over which $f(x)$ is increasing.

Name: ____________________________________ Date: __________________

KEY FEATURES OF FUNCTIONS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. The piecewise linear function $f(x)$ is shown to the right. Answer the following questions based on its graph.



(a) Evaluate each of the following based on the graph:

(i) $f(4)=$ (ii) $f(-3)=$

(b) State the zeroes of $f(x)$.

(c) Over which of the following intervals is $f(x)$ always increasing?

(1) $-7<x<-3$ (3) $-5<x<5$

(2) $-3<x<5$ (4) $-5<x<3$

(d) State the coordinates of the relative maximum and the relative minimum of this function.

Relative Maximum:________________

Relative Minimum:________________

(e) Over which of the following intervals is $f(x)<0$?

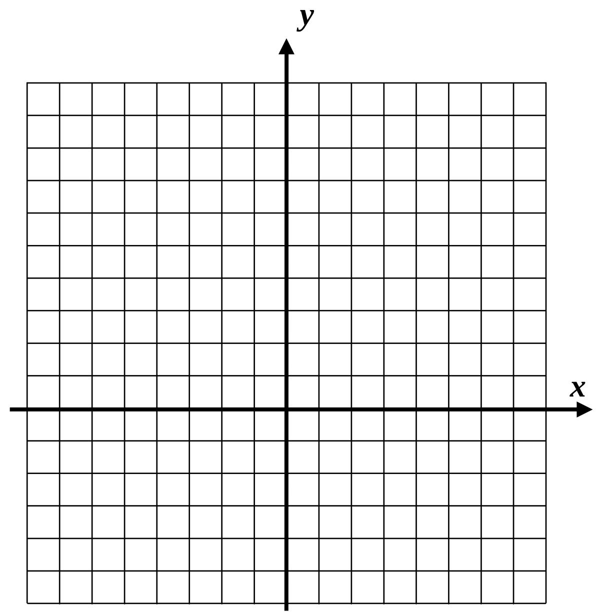
(1) $-7<x<-3$ (3) $-5<x<2$

(2) $2\le x\le 7$ (4) $-5\le x\le 2$

(f) A second function $g(x)$ is defined using the rule $g(x)=2f(x)+5$. Evaluate $g(0)$ using this rule. What does this correspond to on the graph of g?

(g) A third function $h(x)$ is defined by the formula $h(x)=x^3-3$. What is the value of $g(h(2))$? Show how you arrived at your answer.

2. For the function $g(x)=9-(x+1)^2$ do the following.



(a) Sketch the graph of g on the axes provided.

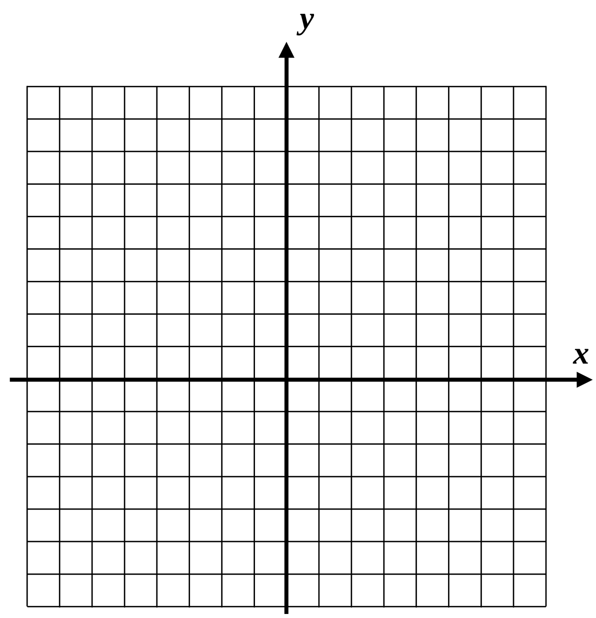
(b) State the zeroes of g.

(c) Over what interval is $g(x)$ decreasing?

(d) Over what interval is $g(x)\geq 0$?

(e) State the range of g.

3. Draw a graph of $y=f(x)$ that matches the following characteristics.



Increasing on: $-8<x<-4$ and $-1<x<5$

Decreasing on: $-4<x<-1$

$f(-8)=-5$ and zeroes at $x=-6,-2,$ and 3

Absolute maximum of 7 and absolute minimum of -5

4. A continuous function has a domain of $-7\leq x\leq 10$ and has selected values shown in the table below. The function has exactly two zeroes and a relative maximum at $(-4,12)$ and a relative minimum at $(5,-6)$.

x	-7	-4	-1	0	2	5	7	10
$f(x)$	8	12	0	-2	-5	-6	0	4

(a) State the interval on which $f(x)$ is decreasing.

(b) State the interval over which $f(x)<0$.