Name: ____________________________________ Date: __________________

QUADRATIC FUNCTION REVIEW
COMMON CORE ALGEBRA II

Linear and exponential functions are used throughout mathematics and science due to their simplicity and applicability. **Quadratic functions** comprise another very important category of functions. You studied these extensively in Common Core Algebra I, but we will review many of their important characteristics in this unit.

QUADRATIC FUNCTIONS

Any function of the form $f(x)=ax^2+bx+c$ where the leading coefficient, a, is not zero.

Exercise #1: *Without* the use of your calculator, evaluate each of the following quadratic functions for the specified input values. Recall that, according to the formal Order of Operations, exponent evaluation should always come first.

(a) $f(x)=x^2$

$f(-3)=$

$f(5)=$

(b) $g(x)=2x^2-5$

$g(2)=$

$g(-1)=$

(c) $h(x)=-x^2+4x$

$h(-2)=$

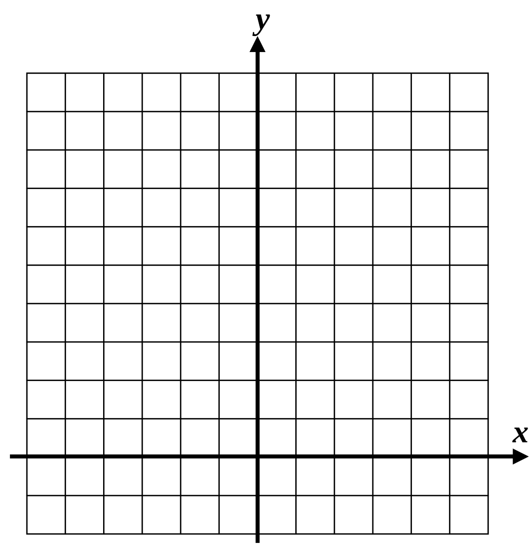
$h(3)=$

Graphs of quadratic functions form what are known as **parabolas**. The simplest quadratic function, and one that you should be very familiar with, is reviewed in the next exercise.

Exercise #2: Consider the simplest of all quadratic functions $y=x^2$.

(a) Create a table of values to plot this function over the domain interval $-3\le x\le 3$.

x	-3	-2	-1	0	1	2	3
$y=x^2$							



(b) Sketch a graph of this function on the grid to the right.

(c) State the coordinates of the **turning point** of this parabola.

(d) State the equation of this parabola's **axis of symmetry**.

(e) Over what interval is this function increasing?

All quadratic functions that have unlimited domains (domains that consist of the set of all real numbers) have turning points and an axis of symmetry. It is important to be able to sketch a parabola using your graphing calculator to generate a table of values.

***Exercise* #3:** Consider the quadratic function $f(x) = -x^2 + 6x + 5$.

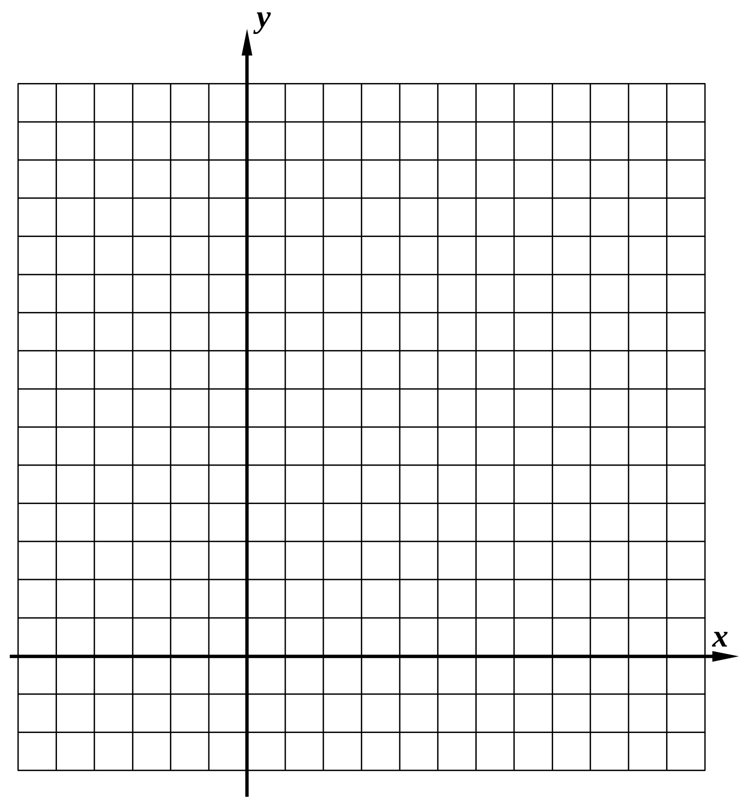
(a) Using a **TABLE** on your graphing calculator, determine the turning point of this function.

(b) What is the range of this quadratic?

(c) Graph this function of the grid to the right.

(d) Why does this parabola open downward as opposed to $y = x^2$ which opened upward?

(e) Between what two consecutive integers does the larger solution to the equation $-x^2 + 6x + 5 = 0$ lie? Show this point on your graph.



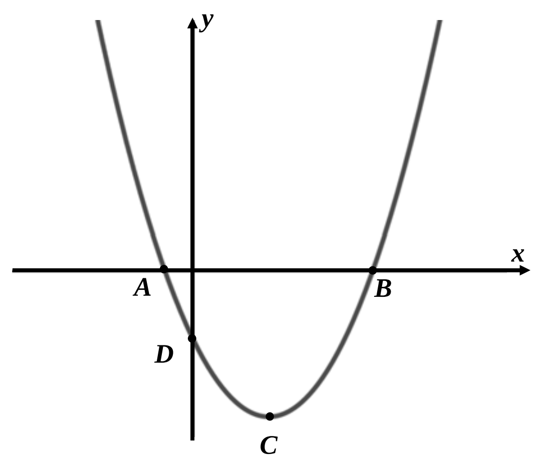
***Exercise* #4:** A sketch of the quadratic function $y = x^2 - 11x - 26$ is shown below marked with points at its intercepts and its turning point. Using tables or a graph on your calculator, determine the coordinates for each of the points.

The x-intercepts: (Zeroes) A B

The y-intercept: C

The turning point: D

Over what interval is this function positive?



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QUADRATIC FUNCTION REVIEW
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Without the use of your calculator, evaluate each of the following quadratic functions for the specified input values.

(a) $g(x)=x^2-9$ (b) $f(x)=-2x^2+8x$ (c) $h(x)=x^2-2x+6$

$g(5)=$ $f(3)=$ $h(0)=$

$g(-3)=$ $f(-1)=$ $h(-2)=$

2. Which of the following represents the *y*-intercept of the graph of the quadratic function $y=2x^2-7x+9$? (Recall, that the *y*-intercept of a graph **always** occurs when $x=0$.)

(1) 7 (3) -7

(2) 2 (4) 9

____

3. For a particular quadratic function, the leading coefficient is *negative* and the function has a turning point whose coordinates are $(-3,14)$. Which of the following must be the *range* of this quadratic?

(1) $\{y\mid y\geq -3\}$ (3) $\{y\mid y\leq 14\}$

(2) $\{y\mid y\leq -3\}$ (4) $\{y\mid y\geq 14\}$

____

4. A parabola has one *x*-intercept of $x=-2$ and an axis of symmetry of $x=4$. Which of the following represents its other *x*-intercept? (Hint, think of how far the given *x*-intercept is away from the axis.)

(1) $x=3$ (3) $x=6$

(2) $x=10$ (4) $x=8$

____

5. A quadratic function is shown in the table below Which of the following statements is *not* true about the function based on this table? Explain your choice.

(1) The function has an *x* intercept of 3.

(2) The function has a *y*-intercept of -3.

(3) The function's leading coefficient is negative.

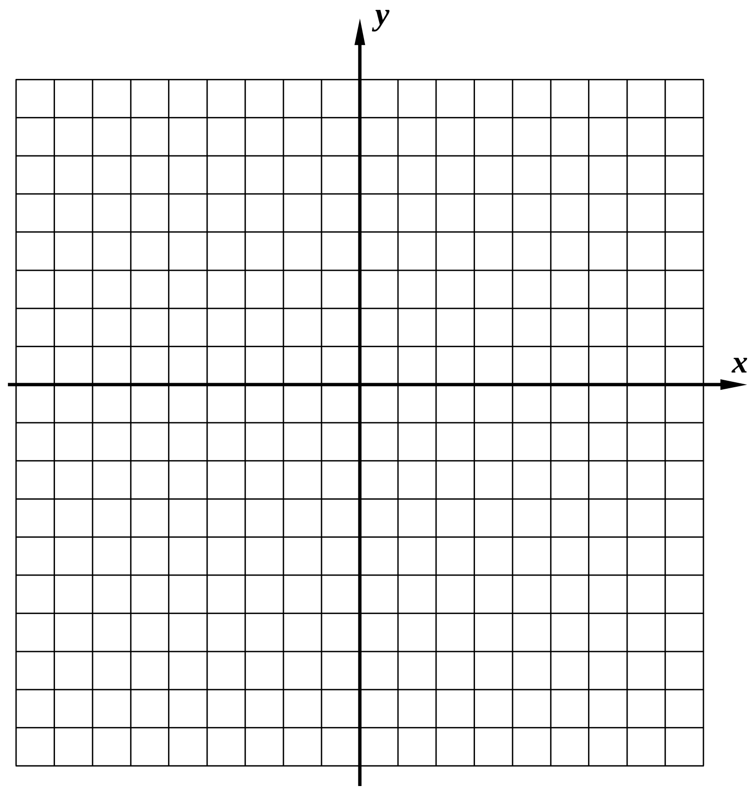
(4) The function has a turning point of $(1,-4)$

x	$f(x)$
-1	0
0	-3
1	-4
3	0
5	12

____

6. Consider the quadratic function whose equation is $f(x)=x^2+2x-8$.

(a) Sketch a graph of *f* on the grid provided.



(b) Over what interval is *f* decreasing?

(c) Over what interval is $f(x)<0$?

(d) State the range of *f*.

APPLICATIONS

7. The number of meters above the ground, *h*, of a projectile fired at an initial velocity of 86 meters per second and at an initial height of 6.2 meters is given by $h(t)=-4.9t^2+86t+6.2$, where *t* represents the time, in seconds, since the projectile was fired. If the projectile hits its peak height at $t=8.775$ seconds, which of the following is closest to its greatest height?

(1) 265 meters (3) 422 meters

(2) 384 meters (4) 578 meters ________

8. Physics students were modeling the height of a ball once it was dropped from the roof of a 25 story building. The students found that the height in feet, *h*, of the ball above the ground as a function of the number of seconds, *t*, since it was dropped was given by $h(t)=300-16t^2$.

From what height was the ball dropped?

To the nearest *tenth* of a second, determine the time at which the ball hits the ground. Provide evidence from a table to support your answer or solve this algebraically if you recall how to.

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FACTORING
COMMON CORE ALGEBRA II

In the study of algebra there are certain skills that are called "gateway skills" because without them a student simply cannot enter into many more complex and interesting problems. Perhaps the most important gateway skill is that of **factoring**. The definition of factor, in two forms, is given below.

FACTOR – TWO IMPORTANT MEANINGS

(1) **Factor** (*verb*) – To rewrite a quantity as an equivalent product.
(2) **Factor** (*noun*) – Any individual component of a product.

You should be familiar with factoring integers as well as algebraic expressions from earlier courses. We will review some of the basic concepts and techniques of factoring in this lesson.

***Exercise* #1:** Factor each of the following integers completely. In other words, write them as the product of only prime numbers (called prime factorization).

(a) 12 (b) 30 (c) 16 (d) 36

Always keep in mind that when we **factor (*verb*)** a quantity, we are simply rewriting it in an different form that is completely equal to the original quantity. It might look different, but $2\cdot 3$ is still the number 6.

***Exercise* #2:** Rewrite each of the following binomials as a product of an integer with a different binomial.

(a) $5x+10$ (b) $2x-6$ (c) $6x+15$ (d) $6-14x$

The above type of factoring is often referred to as "factoring out" the greatest common factor (gcf). This greatest common factor can be comprised of numbers, variables, or both.

***Exercise* #3:** Write each of the following binomials as the product of the binomial's gcf and another binomial.

(a) $3x^2+6x$ (b) $20x-5x^2$ (c) $10x^2+25x$ (d) $30x^2-20$

***Exercise* #4:** Rewritten in factored form $20x^2-36x$ is equivalent to

(1) $2x(10x-15)$ (3) $5x(4x+7)$

(2) $4x(5x-9)$ (4) $9x(x-4)$

____

Trinomials can also sometimes be factored into the product of a gcf and another trinomial.

***Exercise* #5:** Rewrite each of the following trinomials as the product of its gcf and another trinomial.

(a) $2x^2+8x+10$ (b) $10x^2-20x+5$ (c) $8x^3-12x^2+20x$ (d) $6x^3+15x^2-21x$

Another type of factoring that you should be familiar with stems from our work in the last lesson on conjugates. Recall the conjugate multiplication pattern. This can be "reversed" in order to factor binomials that have the form of the **difference of perfect squares**.

CONJUGATE MULTIPLICATION PATTERN

$$(x-a)(x+a)=x^2-a^2$$

***Exercise* #6:** Write each of the following binomials as the product of a conjugate pair.

(a) x^2-9 (b) $4-x^2$ (c) $4x^2-25$ (d) $16-81x^2$

***Exercise* #7:** Write each of the following binomials as the product of a conjugate pair.

(a) $x^2-\frac{1}{4}$ (b) $25-\frac{x^2}{9}$ (c) $\frac{4}{81}x^2-\frac{49}{9}$ (d) $36x^2-49y^2$

Factoring an expression until it cannot be factored anymore is known as **complete factoring**. Complete factoring is an important skill to master in order to solve a variety of problems. In general, when completely factoring an expression, the **first** type of factoring always to consider is that of factoring out the gcf.

***Exercise* #8:** Using a combination of gcf and difference of perfect squares factoring, write each of the following in its completely factored form.

(a) $5x^2-20$ (b) $28x^2-7$ (c) $40-250x^2$ (d) $3x^3-48x$

Name: ___________________________________ Date: _________________

FACTORING
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Rewrite each of the following binomials as the product of an integer with a different binomial.

(a) $10x-55$ (b) $24x-40$ (c) $6x-45$ (d) $18x-9$

2. Rewrite each of the following binomials as the product of its gcf along with another binomial.

(a) $2x^2-8x$ (b) $6x+27$ (c) $30x^2-35x$ (d) $24x^3+20x^2$

3. Rewrite each of the following binomials as the product of a conjugate pair.

(a) x^2-121 (b) $64-x^2$ (c) $4x^2-1$ (d) $25x^2-\frac{1}{9}$

4. Rewrite each of the following trinomials as the product of its gcf and another trinomial.

(a) $4x^2+12x+28$ (b) $6x^2-4x+10$ (c) $14x^3+35x^2-7x$ (d) $20x^3-5x^2+15x$

5. Completely factor each of the following binomials using a combination of gcf factoring and conjugate pairs.

(a) $6x^2-150$ (b) $36-4x^2$ (c) $28x^2-7$ (d) $27x^3-12x$

(e) $80-125x^2$ (f) $2x^3-200x$ (g) $8x^2-512$ (h) $44x-99x^3$

6. When completely factored, the expression $48-3x^2$ is written as

(1) $3(16-x)(16+x)$ (3) $3(x-4)(x+4)$

(2) $3(x-16)(x+16)$ (4) $3(4-x)(4+x)$ ________

7. Which of the following represents the greatest common factor of the terms $4x^2y^6$ and $18xy^5$?

(1) $36xy$ (3) $2xy^5$

(2) $4x^2y^3$ (4) $2x^2y^2$ ________

8. Which of the following is *not* a factor of $6x^2-18x$?

(1) $x-3$ (3) 12

(2) 2 (4) x ________

9. Which of the following prime numbers is *not* a factor of the integer 330?

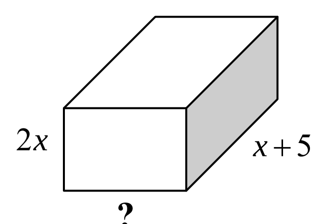
(1) 11 (3) 3

(2) 7 (4) 5 ________

APPLICATIONS

10. The area of any rectangular shape is given by the product of its width and length. If the area of a particular rectangular garden is given by $A=15x^2-35x$ and its width is given by $5x$, then find an expression for the garden's length. Justify your response.

11. The volume of a particular rectangular box is given by the equation $V=50x-2x^3$. The height and length of the box are shown on the diagram below. Find in the width of the box in terms of *x*. Recall that $V=L\cdot W\cdot H$ for a rectangular box.



12. A projectile is fired from ground level such that its height, *h*, as a function of time, *t*, is given by $h=-16t^2+80t$. Written in factored form this equation is equivalent to

(1) $h=-16t(t+4)$ (3) $h=-16t(t-5)$

(2) $h=-8t(2t-7)$ (4) $h=-8t(t-5)$ ________

Name: ____________________________________ Date: _________________

FACTORING TRINOMIALS
COMMON CORE ALGEBRA II

Factoring trinomials, expressions of the form ax^2+bx+c, is an important skill. Trinomials can be factored if they are the product of two binomials. The two main keys to factoring trinomials are: (1) the ability to quickly and accurately multiply binomials (FOIL) and (2) the ability to work with signed numbers. We practice both of these skills with four warm-up multiplication problems in *Exercise* #1.

***Exercise* #1:** *Without* using your calculator, write each of the following products in simplest ax^2+bx+c form.

(a) $(3x+2)(5x+7)$ (b) $(2x-3)(2x+5)$ (c) $(5x-4)(x-2)$ (d) $(4x+3)(3x-8)$

It is important that you know the fundamental rules governing the multiplication and addition of signed numbers. These rules will be key in factoring quickly and correctly. In each case, where a trinomial can be factored, it will be done using the guess-and-check method, where we **intelligently** guess binomial pairs and then check by seeing if the linear terms of the multiplication combine to form the linear term of the trinomial.

***Exercise* #2:** Consider the trinomial $6x^2-35x-6$. Below are four guesses of how this trinomial factors.

$(3x+2)(2x-3)$ $(x-3)(x+2)$ $(6x+1)(x-6)$ $(3x-2)(2x-3)$

(a) Two of these guesses are "unintelligent" – meaning that they should not even be checked. Cross them out and explain below them why they are unreasonable.

(b) Of the two that remain, check both above and determine which is the correct factorization of the trinomial.

The easiest of all trinomial factoring occurs when the leading coefficient is one $(a=1)$.

***Exercise* #3:** Using a guess-and-check technique, factor each of the following trinomials.

(a) $x^2+2x-35$ (b) $x^2+11x+24$ (c) $x^2-13x+22$ (d) $x^2-5x-50$

A step up from the last exercise occurs when the leading coefficient isn't one but is still a prime number. This is very often the case and makes at least part of the guessing much easier.

***Exercise* #3:** Using a guess-and-check technique, factor each of the following trinomials that have prime leading coefficients. Show each guess and its check.

(a) $3x^2+19x-40$

(b) $2x^2-15x+18$

Finally, the hardest trinomials to factor are those whose leading coefficients are not prime. This is due to the fact that there are so many more intelligent guesses. In future lessons we will develop ways to eliminate some of these, but for now, the key will be to just keep **guessing until you get it right**.

***Exercise* #4:** Factor each of the following trinomials. Show each guess and its check.

(a) $15x^2+13x+2$

(b) $10x^2+13x-30$

(c) $12x^2+8x-15$

(d) $36x^2-35x+6$

Name: ___________________________________ Date: _________________

FACTORING TRINOMIALS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Multiply each of the following binomial pairs and express your answer in simplest trinomial form.

(a) $(2x+5)(3x-2)$ (b) $(3x-8)(5x-1)$ (c) $(8x+3)(x+7)$ (d) $(7x-5)(5x+2)$

2. Which of the following is the correct factorization of the trinomial $12x^2-23x+10$. Hint – eliminate two of the choices because they are "unintelligent" guesses.

(1) $(6x-1)(3x-10)$ (3) $(4x-5)(3x+2)$

(2) $(6x-2)(2x-5)$ (4) $(4x-5)(3x-2)$

____

3. Written in factored form $x^2+16x-36$ is equivalent to

(1) $(x-3)(x+12)$ (3) $(x-2)(x+18)$

(2) $(x-6)(x+6)$ (4) $(x-9)(x+4)$

____

4. Write each of the following trinomials in its factored form. These are the easiest trinomials to factor because the leading coefficient is equal to one.

(a) $x^2-7x-18$ (b) $x^2+14x+24$ (c) $x^2-17x+30$ (d) x^2-5x-6

(e) x^2-5x+6 (f) $x^2-15x+44$ (g) $x^2+21x+20$ (h) $x^2-6x-16$

5. Each of the following trinomials has a leading coefficient that is prime. Using a guess-and-check technique, write each trinomial in its factored form. Show each guess and its check.

(a) $5x^2-41x+8$

(b) $3x^2+4x-20$

(c) $2x^2-29x-15$

(d) $7x^2+39x+20$

6. Each of the following trinomials has a non-prime leading coefficient. Using a guess-and-check technique, write each trinomial in its factored form. Show each guess and its check.

(a) $18x^2-25x+8$

(b) $20x^2-11x-42$

REASONING

7. Consider the trinomial $12x^2+7x-10$.

(a) Does this trinomial have a greatest common factor that could be "factored out"?

(b) Why is $(4x-2)(3x+5)$ not an intelligent guess for factoring this trinomial even though $4\cdot 3=12$ and $-2\cdot 5=-10$? Consider your answer to part (a).

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COMPLETE FACTORING
COMMON CORE ALGEBRA II

Each expression that we have factored has been the product of two quantities. But, factoring can produce many more than just two factors. In *Exercise* #1, we first warm-up by multiplying three factors together.

***Exercise* #1:** Write each of these in their simplest form. The last two should take little time to do.

(a) $2(x+4)(x+7)$ (b) $5(2x-5)(x+3)$ (c) $3(x-5)(x+5)$ (d) $4x(3x-2)(3x+2)$

To completely factor an expression means to write it as a product which includes binomials that contain no greatest common factors (gcf's).

***Exercise* #2:** Consider the trinomial $2x^2-4x-6$.

(a) Verify that both of the following products are *correct* factorizations of this trinomial.

$(2x-6)(x+1)$ $(2x+2)(x-3)$

(b) Why are neither of these completely factored?

(c) Write each of these in completely factored form by factoring out the gcf of each unfactored binomial.

(d) What is true of both complete factorizations you found in part (c)?

In practicality, it is always easiest to completely factor by looking for a gcf of the expression first. Once removed, the factoring then either consists of the difference of perfect squares or standard trinomial techniques.

***Exercise* #3:** Write each of the following in its completely factored form. These should be relatively easy.

(a) $4x^2+12x-40$ (b) $6x^2-24$ (c) $2x^2+20x+50$ (d) $75-3x^2$

***Exercise* #4:** Completely factor each of the following. These will involve final trinomials that are more difficult to guess-and-check.

(a) $10x^2+55x-105$

(b) $12x^2+57x-15$

The concept of completely factoring an expression by first removing its gcf leads to a helpful factoring tip when working with the trinomial guess-and-check method. This tip will be developed in the next exercise.

***Exercise* #5:** Consider the trinomial $4x^2+5x-6$.

(a) Does this trinomial have a gcf that can be factored out?

(b) The two products listed below are not reasonable guesses for the factorization of this trinomial. Why?

$(2x-3)(2x+2)$ $\quad$ $(4x+6)(x-1)$

(c) Could a binomial of the form $(2x-a)(2x+b)$, where *a* and *b* are divisors of 6, be a correct guess for the factorization of this trinomial? Why or why not?

(d) Factor this trinomial by intelligently guessing-and-checking.

***Exercise* #6:** Use the intelligent factoring tip developed in *Exercise* #5 to factor each of the following trinomials. Note that neither has a gcf to begin with.

(a) $6x^2-13x+6$

(b) $12x^2+29x-8$

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COMPLETE FACTORING
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Find each of the following products in their simplest ax^2+bx+c form.

(a) $5(x-6)(x-2)$ (b) $3(2x-1)(2x+1)$ (c) $2x(x+4)(x+10)$

2. Write each of the following expressions in their completely factored form. These should be moderately easy to factor.

(a) $2x^2-14x-36$ (b) $5x^2+70x+245$ (c) $3x^2-192$

(d) $6x^3+36x^2-96x$ (e) $28x-7x^3$ (f) $8x^2+12x-8$

3. Write each of the following in completely factored form. These will involve *slightly more difficult* final trinomial expressions.

(a) $15x^2-110x+120$ (b) $10x^3-26x^2-12x$

4. Use the factoring tip developed in *Exercise* #5 to write each of the following trinomials in its factored form. Note that neither has a gcf that can be first factored out.

(a) $8x^2+67x+24$

(b) $12x^2-20x+3$

5. More Practice – Write each of the following expressions in its completely factored form.

(a) $18x^2-39x-15$

(b) $45x-20x^3$

(c) $8x^2+30x+28$

(d) $90x^3-90x^2+20x$

(e) $27x^2-3$

(f) $20x^2+112x-48$

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FACTORING BY GROUPING
COMMON CORE ALGEBRA II

You now have essentially three types of factoring: (1) greatest common factor, (2) difference of perfect squares, and (3) trinomials. We can combine gcf factoring with the other two to **completely factor** quadratic expressions. Today we will introduce a new type of factoring known as **factoring by grouping**. This technique requires you to **see structure in expressions**.

***Exercise* #1:** Factor a binomial common factor out of each of the following expressions. Write your final expression as the product of two binomials.

(a) $x(2x+1)+7(2x+1)$

(b) $5x(x-2)-4(x-2)$

(c) $(x+5)(x-7)+(x-7)(x+1)$

(d) $(2x+8)(x+4)-(x-2)(x+4)$

***Exercise* #2:** Write the expression $(x+3)(x-4)+5(x+3)$ as the equivalent product of binomials. Test this equivalency with $x=2$.

Some very special polynomials can be factored by taking advantage of the structure we have seen in the last two problems. The key is to do **mindful manipulations** of expressions so that they **remain equivalent** but are written as an overall product.

***Exercise* #3:** Consider the expression $2x^3-6x^2+5x-15$. Justify each step below with one of the three major properties of real numbers, i.e. the commutative, associative, or distributive.

$2x^3-6x^2+5x-15=(2x^2-6x^2)+(5x-15)$ ____

$=2x^2(x-3)+5(x-3)$ ____

$=(x-3)(2x^2+5)$ ____

So, when we **factor by grouping** we first extract common factors from pairs of binomials in the four-term polynomials. If we are **lucky** we are left with another **binomial common factor**.

***Exercise* #4:** Use the method of factoring by grouping to completely factor the following expressions..

(a) $3x^3+2x^2-27x-18$

(b) $18x^3+9x^2-2x-1$

(c) $x^5+4x^3+2x^2+8$

(d) $5x^3+10x^2+20x+40$

***Exercise* #5:** Consider the expression $x^2+ab-ax-bx$.

(a) How can you rewrite the expression so that the first two terms share a common factor (other than 1)?

(b) Write this expression as an equivalent product of binomials.

Be careful when you use factoring by grouping. Don't force the method when it does not apply. This can lead to errors.

***Exercise* #6:** Consider the expression $2x^3+10x^2+7x+21$. Explain the error made in factoring it. How can you tell that the factoring is incorrect?

$$\begin{aligned} 2x^3+10x^2+7x+21 &= 2x^2(x+5)+7(x+3) \\ &= (2x^2+7)(x+5+x+3) \\ &= (2x^2+7)(2x+8) \end{aligned}$$

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FACTORING BY GROUPING
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Rewrite each of the following as the product of binomials. Be especially careful on the manipulations that involve subtraction.

(a) $x(x+5)+7(x+5)$

(b) $4x(x-2)-3(x-2)$

(c) $(x+10)(x-3)+(x+5)(x-3)$

(d) $(2x-7)(x+4)+(x+4)(x+2)$

(e) $(4x+3)(2x-1)-(x+2)(2x-1)$

(f) $(3x+7)(x+5)-(x+5)(2x-4)$

2. Max tries to simplify the expression $(5x+2)(x+3)-(2x-3)(x+3)$ as follows:

$=(5x+2)(x+3)-(2x-3)(x+3)$
$=(x+3)(5x+2-2x-3)$
$=(x+3)(3x-1)$

Show using $x=2$ that this simplification is incorrect. Then, give the correct simplification.

3. Factor each of the following quadratic expressions completely using the method of grouping:

(a) $10x^2+6x+35x+21$

(b) $12x^2+3x-20x-5$

4. Factor each of the following cubic expressions completely.

(a) $5x^3 + 2x^2 - 20x - 8$

(b) $18x^3 - 27x^2 - 2x + 3$

(c) $x^3 + 2x^2 - 25x - 50$

(d) $8x^3 + 10x^2 + 12x + 15$

5. Factor each of the following expressions. Rearrange the expressions as needed to produce binomial pairs with common factors.

(a) $x^2 - ac - cx + ax$

(b) $xy + ab + ay + bx$

REASONING

6. Consider the expression: $x^3 - 5x^2 - 9x + 45$. Enter this expression on your calculator and find its zeroes. Provide evidence. Then, factor it completely. Do you see the relationship between the factors and the zeroes?

Name: ____________________________________ Date: _________________

THE ZERO PRODUCT LAW
COMMON CORE ALGEBRA II

One of the most important equation solving technique stems from a fact about the number zero that is **not true of any other number**:

> **THE ZERO PRODUCT LAW**
>
> If the **product** of multiple factors is **equal to zero** then at least **one of the factors must be equal to zero**.

The law can immediately be put to use in the first exercise. In this exercise, quadratic equations are given already in factored form.

***Exercise* #1:** Solve each of the following equations for all value(s) of *x*.

(a) $(x+7)(x-3)=0$

(b) $(2x-5)(x-4)=0$

(c) $4(3x+2)(4x-3)=0$

***Exercise* #2:** In *Exercise* #1(c), why does the factor of 4 have no effect on the solution set of the equation?

The Zero Product Law can be used to solve any quadratic equation that is factorable (not prime). To utilize this technique the problem solver must first set the equation equal to zero and then factor the non-zero side.

***Exercise* #3:** Solve each of the following quadratic equations using the Zero Product Law.

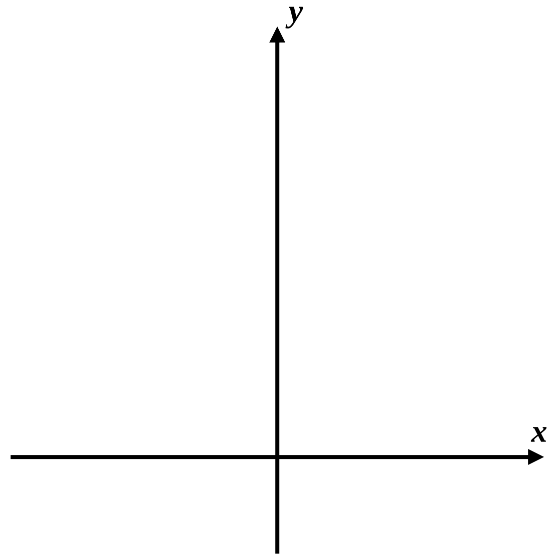
(a) $x^2+3x-14=-2x+10$

(b) $3x^2+12x-7=x^2+3x-2$

***Exercise* #4:** Consider the system of equations shown below consisting of a parabola and a line.

$y = 3x^2 - 8x + 5$ and $y = 4x + 5$

(a) Find the intersection points of these curves *algebraically*.

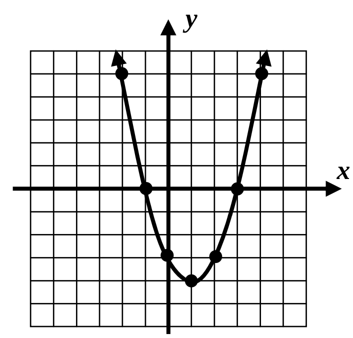


(b) Using your calculator, sketch a graph of this system on the axes to the right. **Be sure to label the curves with equations, the intersection points, and the window.**

(c) Verify your answers to part (a) by using the **INTERSECT** command on your calculator.

The Zero Product Law is extremely important in finding the **zero's** or ***x*-intercepts** of a parabola.

***Exercise* #5:** The parabola shown at the right has the equation $y = x^2 - 2x - 3$.



(a) Write the coordinates of the two x-intercepts of the graph.

(b) Find the x-intercepts of this parabola *algebraically*.

***Exercise* #6:** *Algebraically* find the set of x-intercepts for each parabola given below.

(a) $y = 4x^2 - 1$

(b) $y = 3x^2 + 13x - 10$

(c) $y = 5x^2 - 10x$

Name: ___________________________________ Date: _________________

THE ZERO PRODUCT LAW
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Solve each of the following equations for all value(s) of *x*.

   (a) $(x-2)(x+5)=0$ (b) $(7x-1)(2x+5)=0$ (c) $(3x-1)(3x+1)=0$

2. Solve each of the following quadratic equations which have already been set equal to zero.

   (a) $x^2+10x+16=0$ (b) $3x^2+11x-4=0$ (c) $12x^2+8x=0$

3. Solve each of the following quadratic equations by first manipulating them so that one side of the equation is set equal to zero.

   (a) $x^2+4x-40=10x+15$ (b) $4x^2+3x-11=3x-2$

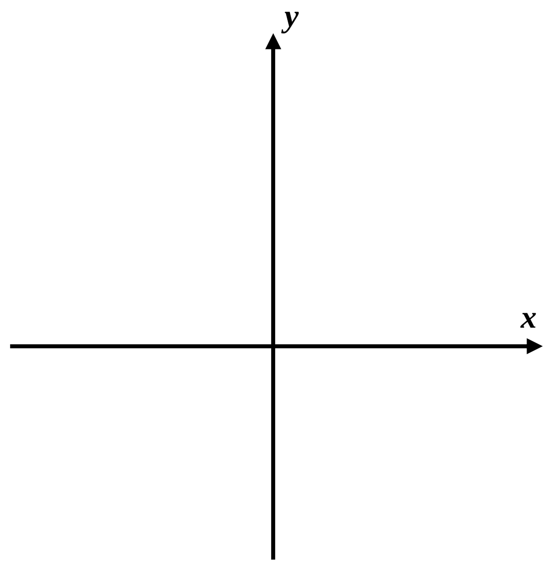
   (c) $6x^2-15x+2=2x^2+10x-4$ (d) $-16t^2+76t+5=12t+5$

APPLICATIONS

4. Consider the system of equations shown below consisting of one linear and one quadratic equation.

$y = 4x - 5$ and $y = 2x^2 - 5x - 10$

(a) Find the intersection points of this system *algebraically*.



(b) Using your calculator, sketch a graph of this system to the right. **Be sure to label the curves with equations, the intersection points, and the window.**

(c) Use the **INTERSECT** command on your calculator to verify the results you found in part (a).

5. *Algebraically*, find the *x*-intercepts of each quadratic function given below.

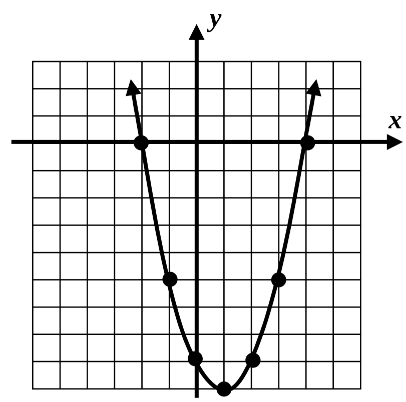
(a) $y = x^2 - 81$

(b) $y = 12x^2 - 18x$

(c) $y = 2x^2 - 6x - 8$

REASONING

6. A quadratic function of the form $y = x^2 + bx + c$.



(a) What are the *x*-intercepts of this parabola?

(b) Based on your answer to part (a), write the equation of this quadratic function first in factored form and then in trinomial form.

Name: ___________________________________ Date: _________________

QUADRATIC INEQUALITIES IN ONE VARIABLE
COMMON CORE ALGEBRA II

At the heart of solving any inequality is finding all values of the variable (or variables) that make the inequality true. This basic notion of inequalities is critical to understand before proceeding.

***Exercise* #1:** Determine if each of the following is a solution to the inequality given. Show work to justify your response.

(a) $x^2 - 3x - 10 > 0$ for $x = 4$

(b) $2x^2 + 13x - 7 \geq 0$ for $x = 2$

(c) $x^2 - x - 12 < 0$ for $x = -3$

Most of the time, there are an infinite number of solutions to an inequality. The **solution set** of inequalities like these cannot be written in **roster form** (where one lists the solutions). In *Exercise* #2, we will explore how to determine this solution set by using tables on your calculator.

***Exercise* #3:** Consider the quadratic inequality $x^2 + 2x - 3 < 0$.

(a) Solve the corresponding equation $x^2 + 2x - 3 = 0$ algebraically for all values of *x*.

(b) Using your calculator and the equation $y = x^2 + 2x - 3$ fill in the table below.

x	-5	-4	-3	-2	-1	0	1	2	3
y									

(c) Explain why the zeroes you found in part (a) are **not** part of the solution set of the inequality.

(d) Write the solution set of the inequality and represent it on a number line.

The key to algebraically solving a quadratic inequality is to first find the zeroes and then test points between the zeroes and outside the zeroes.

***Exercise* #3:** Which of the following is the solution set of the inequality $x^2 - 4 > 0$?

(1) $\{x \mid x > 2\}$

(2) $\{x \mid -2 < x < 2\}$

(3) $\{x \mid x > 2 \text{ or } x < -2\}$

(4) $\{x \mid x > -2\}$

____

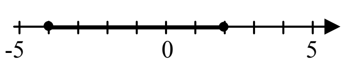
***Exercise* #4:** Solve each of the following quadratic inequalities. Write you final answers in set-builder notation and represent the solution set on a number line.

(a) $x^2 - 5x - 36 \le 0$

(b) $5x^2 + 28x - 12 > 0$

(c) $2x^2 - 4x - 8 \ge 10x - 8$

(d) $x^2 + 14x - 6 < 14x + 19$

***Exercise* #5:** The number line graph 

is the solution to which of the following inequalities?

(1) $x^2 - 2x - 8 > 0$

(2) $x^2 + 2x - 8 < 0$

(3) $x^2 - 2x - 8 \ge 0$

(4) $x^2 + 2x - 8 \le 0$

***Exercise* #6:** Which of the following represents the solution set of the inequality $-2x^2 + 7x - 3 > 0$?

(1) $\left\{x \mid \frac{1}{2} < x < 3\right\}$

(2) $\left\{x \mid -\frac{1}{2} < x < 3\right\}$

(3) $\left\{x \mid x < -3 \text{ or } x > \frac{1}{2}\right\}$

(4) $\left\{x \mid x < \frac{1}{2} \text{ or } x > 3\right\}$

Name: ______________________________ Date: ______________

QUADRATIC INEQUALITIES IN ONE VARIABLE
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Which of the following values of x is in the solution set of the inequality $x^2+x-2>0$? Hint – to make this problem easier, generate a table on your calculator using $y=x^2+x-2$.

(1) 1 (3) 0

(2) -2 (4) -4 ________

2. Which of the following values of x is *not* in the solution set of the inequality $5x^2+35x\leq 0$?

(1) -1 (3) 0

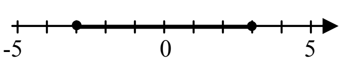
(2) 2 (3) -7 ________

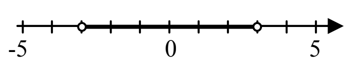
3. The solution set of the inequality $x^2>25$ is which of the following?

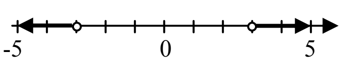
(1) $(5,\infty)$ (3) $(-\infty,-5)\cup(5,\infty)$

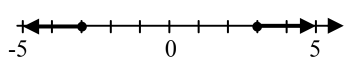
(2) $[-5,5]$ (4) $(-\infty,5]$ ________

4. The solution to the inequality $x^2-9<0$ can be expressed graphically as

(1) 

(3) 

(2) 

(4) 

____

5. Which of the following is the solution set of $(x+5)(x-3)<0$?

(1) $\{x\,|-5<x<3\}$ (3) $\{x\,|\,x<-5 \text{ or } x>3\}$

(2) $\{x\,|-5\leq x\leq 3\}$ (4) $\{x\,|-3<x<5\}$ ________

6. Which inequality below represents all solutions to $x^2\geq 5x+24$?

(1) $\{x\,|-6\leq x\leq 4\}$ (3) $\{x\,|\,x\leq -8 \text{ or } x\geq 3\}$

(2) $\{x\,|-2\leq x\leq 12\}$ (4) $\{x\,|\,x\leq -3 \text{ or } x\geq 8\}$ ________

7. Find the solution set to each of the quadratic inequalities shown below. Represent your solution set using any acceptable notation and graphically on a number line.

(a) $2x^2 + 9x - 35 < 0$

(b) $x^2 \geq 5x + 6$

(c) $8x^2 + 50x - 5 < 10x - 5$

(d) $4x^2 + 23x - 6 \geq 0$

(e) $x^2 \leq 10x + 24$

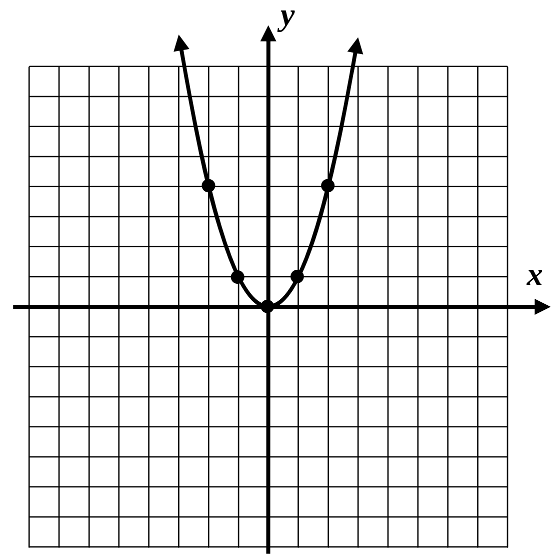
(f) $7x^2 + 4x + 3 > 3x^2 + 4x + 4$

Name: ___________________________________ Date: _________________

SHIFTING PARABOLAS AND THEIR TURNING POINTS
COMMON CORE ALGEBRA II

Parabolas, and graphs more generally, can be moved horizontally and vertically by simple manipulations of their equations. This is known as **shifting** or **translating** a graph. You worked with this extensively in Common Core Algebra I. The first exercise will review how to use a method known as **completing the square** to identify shifts and the turning point of a parabola.

***Exercise* #1:** The function $y = x^2$ is shown already graphed on the grid below. Consider the quadratic whose equation is $y = x^2 - 8x + 18$.

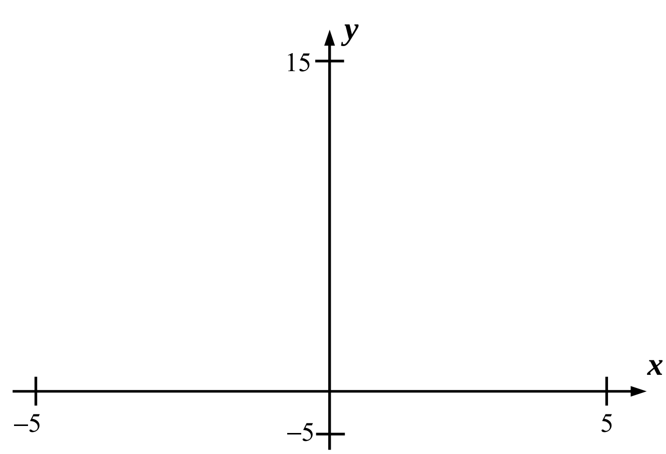


(a) Using the method of completing the square, write this equation in the form $y = (x-h)^2 + k$.

(b) Describe how the graph of $y = x^2$ would be shifted to produce the graph of $y = x^2 - 8x + 18$.

(c) Sketch the graph of $y = x^2 - 8x + 18$ by using its **vertex form** in (a). What are the coordinates of its turning point (vertex).

***Exercise* #2:** Using your calculator and a window show below, sketch the graph of $y = x^2$, $y = 3x^2$, and $y = \frac{1}{2}x^2$.



Every quadratic of the form $y = ax^2$ has a turning point at:

The **algorithm** of completing the square works best when $a=1$ and b is even in the form $y=ax^2+bx+c$. But, it does work in every case, even the messy ones.

***Exercise* #3:** Place each of the following quadratic functions in vertex form and identify the turning point.

(a) $y=3x^2+12x-2$

(b) $y=2x^2+6x+1$

***Exercise* #4:** The method of completing the square can be performed on the standard quadratic equation $y=ax^2+bx+c$ and after much manipulation can be placed in the form:

$$y=a\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a}+c$$

(a) Based on this formula, what is the *x*-coordinate of the turning point of any parabola? Be careful.

(b) Use this formula to find the turning point of the parabola $y=x^2+10x-2$.

(c) Verify your answer from part (a) by placing the quadratic $y=x^2+10x-2$ into vertex form.

(d) Verify both answers by examining a table on your calculator using the original equation.

***Exercise* #5:** Use the formula $x=-\frac{b}{2a}$ to find the turning points for each of the following quadratic functions.

(a) $f(x)=2x^2-12x+7$

(b) $g(x)=-\frac{1}{4}x^2+5x-20$

Name: ___________________________________ Date: _________________

SHIFTING PARABOLAS AND THEIR TURNING POINTS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Which of the following equations would result from shifting $y = x^2$ five units right and four units up?

(1) $y = (x-5)^2 + 4$ (3) $y = (x-4)^2 - 5$

(2) $y = (x+5)^2 + 4$ (4) $y = (x+4)^2 - 5$

____

2. Which of the following represents the turning point of the parabola whose equation is $y = (x+3)^2 - 7$?

(1) $(3, -7)$ (3) $(-7, -3)$

(2) $(-3, 7)$ (4) $(-3, -7)$

____

3. Which of the following quadratic functions would have a turning point at $(6, -2)$?

(1) $y = (x+6)^2 - 2$ (3) $y = 5(x-6)^2 - 2$

(2) $y = 3(x+2)^2 - 2$ (4) $y = 2(x-1)^2 + 6$

____

4. Which of the following is turning point of $y = x^2 + 12x - 4$?

(1) $(12, -4)$ (3) $(6, 104)$

(2) $(-6, -40)$ (4) $(-4, 12)$

____

5. In vertex form, the parabola $y = x^2 - 10x + 8$ would be written as

(1) $y = (x-5)^2 - 33$ (3) $y = (x-10)^2 - 92$

(2) $y = (x-5)^2 - 17$ (4) $y = (x-10)^2 - 108$

____

6. The turning point of the parabola $y = x^2 + 5x - 2$ is

(1) $(2.5, 12.75)$ (3) $(-2.5, -8.25)$

(2) $(-5, -10.5)$ (4) $(-2.5, -17.5)$

____

7. Write each of the following quadratic functions in its vertex form by completing the square. Then, identify its turning point.

(a) $y = x^2 + 12x + 50$

(b) $y = -3x^2 + 30x + 7$

8. Use the formula $x = -\frac{b}{2a}$ to find the turning points of each of the following quadratic functions. Then, place the function in vertex form to verify the turning points.

(a) $y = 5x^2 - 30x + 55$

(b) $y = -2x^2 - 24x - 67$

9. Consider the quadratic function whose equation is $y = x^2 + 6x - 40$.

(a) Determine the y-intercept of this function algebraically.

(b) Write the function in its vertex form. State the coordinates of its turning point.

(c) Algebraically find the zeroes of the function using the zero product law.

(d) Sketch a graph of the parabola, showing all relevant features found in parts (a) through (c).

Name: ___________________________________ Date: _________________

MODELING WITH QUADRATIC FUNCTIONS
COMMON CORE ALGEBRA II

We have already seen quadratic functions used to model a wide variety of real-world applications. In this lesson we will focus on using quadratic models to predict and optimize. Almost always our solution will depend on either the intercepts or the turning point of the quadratic model.

***Exercise* #1:** An object is fired upwards with an initial velocity of 112 feet per second. Its height, in feet above the ground, as a function of time, in seconds since it was fired, is given by the equation $h(t) = -16t^2 + 112t$.

(a) At what height was the object fired?

(b) Sketch a general curve of this equation below.

(c) Algebraically, find the time that the rocket reaches its greatest height and the maximum height. Label these on the graph that you drew in part (b).

(d) Algebraically, determine the time when the rocket reaches the ground. Label this on your graph in (b).

***Exercise* #2:** A skateboard half-pipe ramp has a shape in the form of a parabola whose equation is $y = 0.06x^2 - 1.2x + 7$ where x represents the horizontal distance across the 20-foot wide half-pipe and y represents the ramp's height above the ground in feet. With the help of your calculator, sketch a graph of the half-pipe below Label its height at its endpoints and its minimum point.

***Exercise* #3:** The Crazy Carmel Corn company has determined that the percentage of kernels that pop rises and then falls as the temperature of the oil the kernels are cooked in increases. It modeled this trend using the equation

$$P=-\frac{1}{250}T^2+2.8T-394$$

Where P represents the percent of the kernels that pop and T represents the temperature of the oil in degrees Fahrenheit.

(a) Algebraically determine the temperature at which the highest percentage of kernels pop. Also, determine the percent of kernels that pop at this temperature.

(b) Using your calculator, sketch a curve below for $P\geq 0$. Label your window.

(c) Using the **ZERO** command on your calculator, determine, to the nearest degree, the two temperatures at which $P=0$. Label them on your graph drawn in part (b).

(d) If a typical batch of popcorn consists of 800 kernels, how many does the Crazy Carmel Corn company expect to pop at the optimal temperature?

(e) For a batch of popcorn to be successful, the company wants at least 85% of its kernels to pop. Write an inequality whose solution represents all temperatures that would ensure a successful batch. Solve this inequality graphically, to the nearest degree, and show your graph to below, labeling all relevant points.

Name: ___________________________________ Date: _________________

MODELING WITH QUADRATIC FUNCTIONS
COMMON CORE ALGEBRA II HOMEWORK

APPLICATIONS

1. The height of a missile t seconds after it has been fired is given by $h=-4.9t^2+44.1t$. Which of the following represents the number of seconds it will take for the rocket to reach its greatest height?

(1) 108 (3) 99

(2) 4.5 (4) 7.5

____

2. The daily cost per car manufactured at a certain automotive plant decreases as the number of cars increase and then increases again due to overtime production costs. The cost C, per car, is given by $C(n)=0.3n^2-90n+12,450$ where n represents the number of cars produced. Which of the following is the lowest per car cost?

(1) \$5,700 (3) \$12,450

(2) \$150 (4) \$2,150

____

3. A decathlete at the Olympics throws a javelin such that its height, h, above the ground can be modeled as a quadratic function of the horizontal distance, d, that it has traveled. Which of the following is a realistic quadratic function for this scenario?

(1) $h=\frac{1}{100}d^2+75d+3$ (3) $h=-\frac{1}{100}d^2+75d+3$

(2) $h=\frac{1}{100}d^2+75d-3$ (4) $h=-\frac{1}{100}d^2+75d-3$

____

4. A ball thrown vertically in the air reaches its peak height after 3.5 seconds. If its height, as a function of time, is given by $h=-16t^2+bt+4$, then which of the following is the value of b?

(1) 56 (3) -112

(2) -56 (4) 112

____

5. A tour company has a ticket price that goes down \$2 for every additional person who signs up for a group trip. They charge, per person, $52-2n$ where n is the number of people that go on the trip. Their total revenue, R, as a function of the number of people who go on the trip is $R=52n-2n^2$. How many people maximize the revenue for the tour company?

(1) 13 (3) 26

(2) 39 (4) 22

____

6. Bacteria tend to grow very fast in a Petri dish at first because of unlimited food and then begin to die out due to competition. In a certain culture, the number of bacteria is given by $N(t) = -2t^2 + 92t + 625$, where t represents the hours since 625 bacteria were introduced to the Petri dish. Determine the maximum number of bacteria that occur in the dish.

7. A tennis ball is thrown upwards from the top of a 30-foot high building. Its height, in feet above the ground, t-seconds after it is thrown is given by $h = -16t^2 + 80t + 30$.

(a) Algebraically determine the time when the tennis ball reaches its greatest height? What is that height?

(b) Using your calculator, sketch a general graph showing the ball's height for all times where $t \geq 0$ and $h \geq 0$. Label the information you found in part (a).

(c) Using the **ZERO** command on your calculator, determine the amount of time the ball stays in the air. Round your answer to the nearest tenth of a second and label this on your graph drawn in part (a).

(d) The ball can be seen from the ground whenever it is at a height of at least 100 feet. Graphically determine the interval of time that the ball can be seen. Show the work on your graph from in part (b).

8. The area of a rectangle whose perimeter is a fixed 80 feet is given by $A = 40w - w^2$, where w is the width of the rectangle. Determine the width of the rectangle that gives the maximum area. What type of special rectangle is necessary to produce this maximum area? Justify.

Name: ___________________________________ Date: _________________

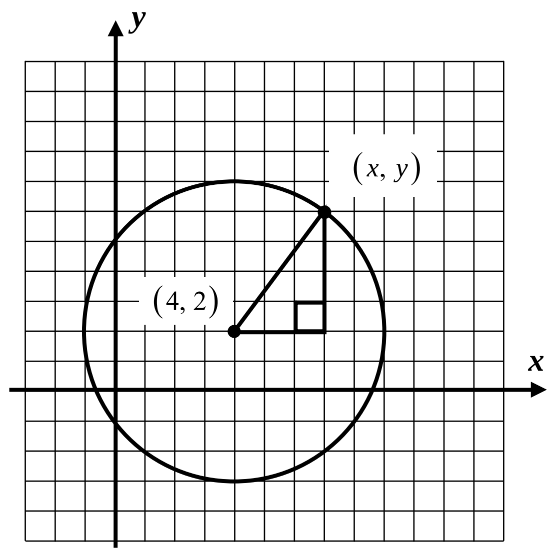
EQUATIONS OF CIRCLES
COMMON CORE ALGEBRA II

Various quadratic relationships can be placed into equations by knowing the **locus definition** of the relationship. We will explore this for parabolas in a future lesson. In this one, we will develop the **equation** of a **circle** by using the **distance formula** that you learned from Common Core Geometry.

THE DISTANCE FORMULA

The distance between two points (x_1, y_1) and (x_2, y_2) is given by: $D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

***Exercise* #1:** A circle is the collection of all points that are a set distance (the radius) away from a point (its center). The circle shown below has a radius of 5 and a center at the point $(4, 2)$. An arbitrary point on the circle, (x, y), is shown marked.



(a) Using the distance formula show that the point $(7,-2)$ must lie on this circle (verify graphically).

(b) Letting $(x_2, y_2)=(x, y)$ and $(x_1, y_1)=(4, 2)$, write the distance formula for all points on this circle.

(c) Square both sides of the equation from (b) to create the standard form of a circle.

(d) Show algebraically that the point $(1,-2)$ must also lie on the circle.

THE EQUATION OF A CIRCLE

A circle whose center is at (h, k) and whose radius is r is given by: $(x-h)^2+(y-k)^2=r^2$

***Exercise* #2:** Which of the following equations would have a center of $(-3, 6)$ and a radius of 3?

(1) $(x-3)^2+(y+6)^2=9$ (3) $(x-3)^2+(y-6)^2=3$

(2) $(x+3)^2+(y-6)^2=9$ (4) $(x+3)^2+(y+6)^2=3$ __________

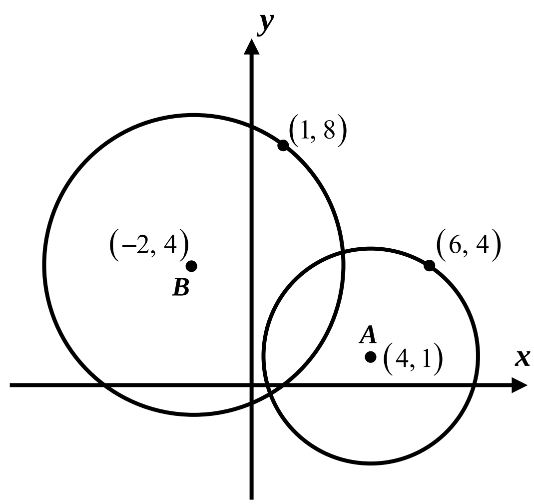
***Exercise* #3:** For each of the following equations of circles, determine both the circle's center and its radius. If its radius is not an integer, express it in decimal form rounded to the nearest *tenth*.

(a) $(x-2)^2+(y-7)^2=100$ (b) $(x-5)^2+(y+8)^2=4$ (c) $x^2+y^2=121$

(d) $(x+1)^2+(y+2)^2=1$ (e) $x^2+(y-3)^2=49$ (f) $(x+6)^2+(y-5)^2=18$

(g) $x^2+y^2=64$ (h) $(x-4)^2+(y-2)^2=20$ (i) $x^2+y^2=57$

***Exercise* #4:** Write equations for circles *A* and *B* shown below. Show how you arrive at your answers.



***Exercise* #5:** By completing the square on both quadratic expressions in *x* and *y* determine the center and radius of a circle whose equation is

$$x^2+10x+y^2-2y=10$$

Name: ___________________________________ Date: _________________

EQUATIONS OF CIRCLES
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Each of the following is an equation of a circle. State the circle's center and radius. In the cases where the radius is not an integer, give its value rounded to the nearest tenth.

(a) $x^2+y^2=144$ (b) $(x-3)^2+(x+7)^2=36$ (c) $(x+5)^2+(y+1)^2=64$

(d) $(x-2)^2+(y-9)^2=100$ (e) $x^2+y^2=1$ (f) $x^2+(y+5)^2=25$

(g) $x^2+y^2=50$ (h) $(x-3)^2+y^2=200$ (i) $(x-6)^2+(y+6)^2=20$

2. Which of the following is true about a circles whose equation is $(x+5)^2+(y-3)^2=36$?

(1) It has a center of $(5,-3)$ and an area of 12π.
(2) It has a center of $(-5,3)$ and a diameter of 6.
(3) It has a center of $(-5,3)$ and an area of 36π.
(4) It has a center of $(5,-3)$ and a circumference of 12π. __________

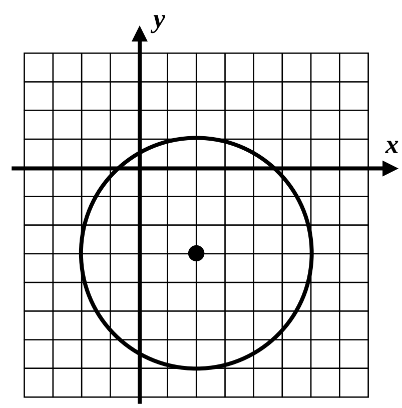
3. Which of the following represents the equation of the circle shown graphed below?

(1) $(x-2)^2+(y+3)^2=16$

(2) $(x+2)^2+(y-3)^2=4$

(3) $(x-2)^2+(y+3)^2=4$

(4) $(x+2)^2+(y-3)^2=16$ __________



4. By completing the square on each of the quadratic expressions, determine the center and radius of a circle whose equation is shown below.

$x^2-6x+y^2+10y=66$

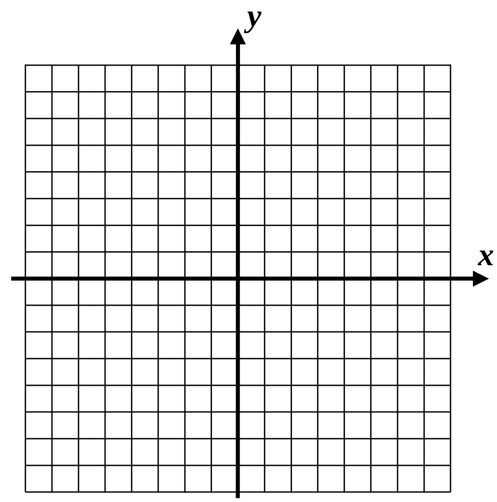
5. Circles are described below by the coordinates of their centers, *C*, and one point on their circumference, *A*. Determine an equation for each circle in center-radius form.

(a) $C(5, 2)$ and $A(11, 10)$ (b) $C(-2, -5)$ and $A(3, -17)$ (c) $C(5, -1)$ and $A(-2, -5)$

6. Solve the following system of equations *graphically*.

$x^2 + y^2 = 25$

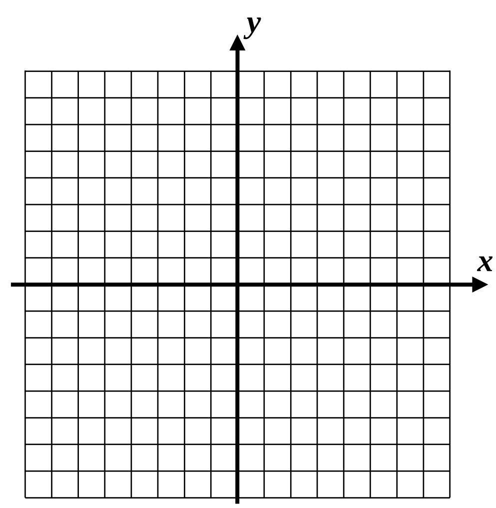
$y = 5 - x^2$



7. Find the intersection of the circle $x^2 + y^2 = 29$ and $y = x - 3$ algebraically.

APPLICATIONS

7. Jonas is designing a circular garden whose equation is $x^2 + y^2 = 49$. He wishes to place a walkway within the garden at all points within the circle that satisfy the inequality $-2 \le y \le 2$. Graph the circle on the grid to the right and shade in all points that represent the walkway.



Name: ______________________________________ Date: ___________________

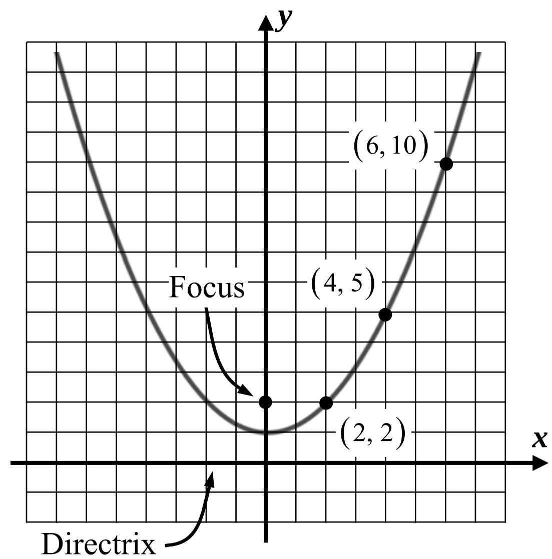
THE LOCUS DEFINITION OF A PARABOLA
COMMON CORE ALGEBRA II

The circle had a relatively easy locus definition, i.e. the collection of all points **equidistant** from a given point. Parabolas have a slightly more complex definition that we will explore in this lesson.

THE LOCUS DEFINITION OF A PARABOLA

A parabola is the collection of all points **equidistant** from a fixed point (known as its **focus**) and a fixed line (known as its **directrix**).

***Exercise* #1:** The parabola $y=\frac{1}{4}x^2+1$ is shown graphed below with selected points shown. For this parabola, its focus is the point $(0,2)$ and its directrix is the x-axis.



(a) How far is the turning point $(0,1)$ from both the focus and directrix? How far is the point $(2,2)$ from both?

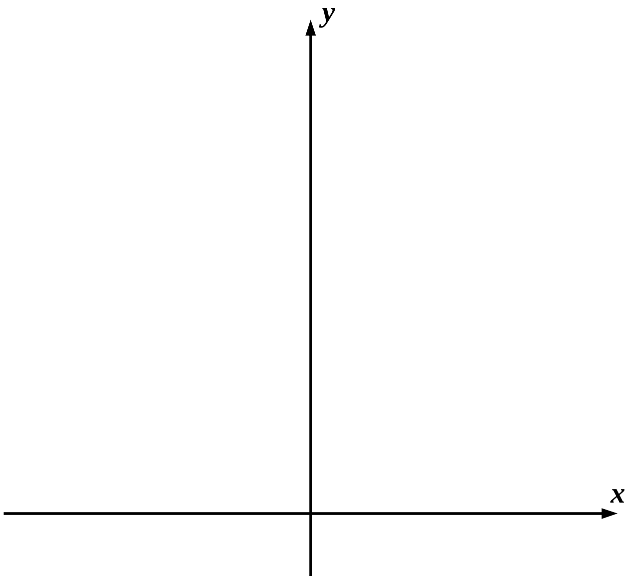
(b) Use the distance formula to verify that the point $(4,5)$ is the same distance away from the focus and directrix. Draw line segments from the focus and directrix to this point to visualize the distance. Repeat for the point $(6,10)$

(c) Use the distance formula to show that the equation of this parabola is $y=\frac{1}{4}x^2+1$ based on the locus definition of a parabola.

The algebra for finding the equation of a parabola based on its focus and directrix can be challenging, but you know all of it from previous work you have done. Just be careful with each step.

***Exercise* #2:** Consider a parabola whose focus is the point $(0, 7)$ and whose directrix is the line $y = 3$.

(a) Sketch a diagram of the parabola below and identify its turning point.



(b) Determine the equation of the parabola using the locus definition.

Any line and any point not on the line when used as the focus and directrix define a parabola. The most challenging type of problem we will tackle in this course will be finding the equation of a parabola whose focus point is not on one of the two axes. We will, however, stick with horizontal lines as our directrices.

***Exercise* #3:** Determine the equation of the parabola whose focus is the point $(4, 1)$ and whose directrix is the horizontal line $y = -2$. First, draw a diagram that shows the parabola, then carefully use the distance formula to derive its equation.

Name: ____________________________________ Date: __________________

The Locus Definition of a Parabola
Common Core Algebra I Homework

Fluency

1. Fill in the following locus definition of a parabola with one of the words shown listed below. Words may be used more than once.

point, line, equidistant, directrix, collection, focus

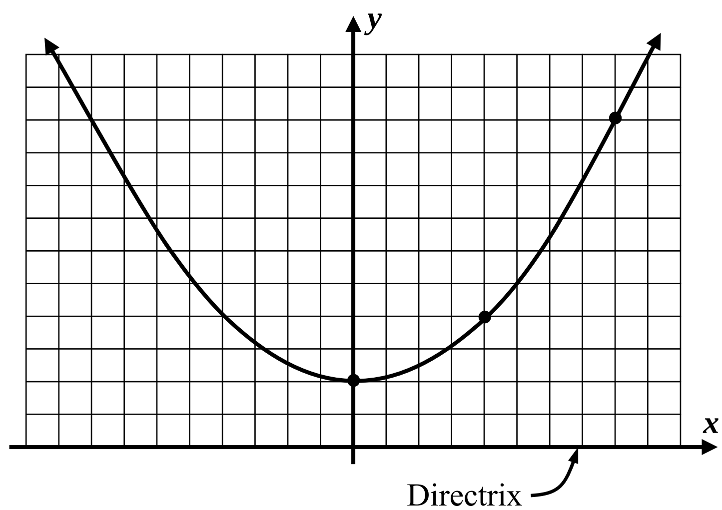
A parabola is the ____________________ of all points ______________________ from a fixed ____________________ and a fixed ______________________.

The fixed _____________________ is known as the parabola's _______________________.

The fixed _____________________ is known as the parabola's _______________________.

2. The parabola whose equation is $y = \frac{1}{8}x^2 + 2$ is shown graphed on the grid below. Its directrix is the x-axis.

(a) Explain why the focus must be the point $(0, 4)$. Label this point on the diagram.



(b) How far is the point $(4, 4)$ from both the focus and the directrix?

(c) Show that the point $(8, 10)$ is equidistant from the focus and directrix.

(d) Using the locus definition of a parabola, show that the equation is $y = \frac{1}{8}x^2 + 2$.

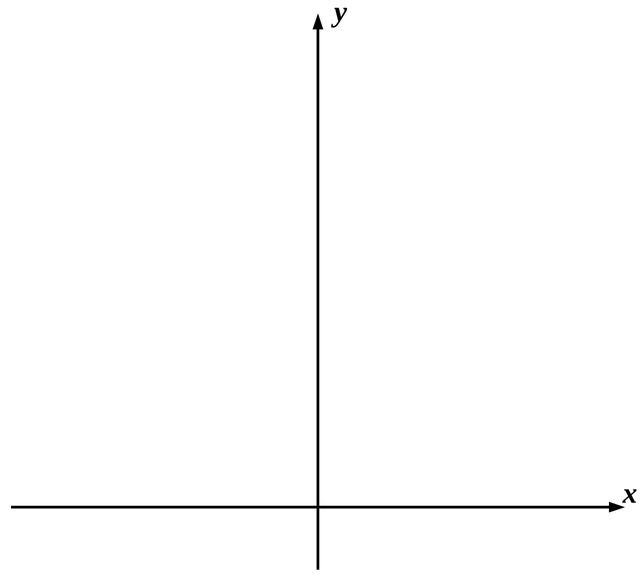
2. Consider parabola that is the collection of all points equidistant from the point $(0,8)$ and the line $y=2$.

(a) Give each of the following:

Directrix: ___________________

Focus: _____________________

(b) Draw a diagram of this parabola and label its turning point on the diagram below.



(c) Find the equation of this parabola using the locus definition.

3. Parabolas can be constructed using the classic geometric tools of a compass and a straightedge. The circles below represent all the points equidistant from the focus $(0,4)$. Given this focus point and a directrix of the *x*-axis, do the following.

(a) Draw in the horizontal lines $y=2, y=3, y=4, y=5, y=6$, $y=7$, and $y=8$. These lines represent points that are a given distance away from the *x*-axis (the directrix).

(b) Draw the intersections of the lines your drew in (a) with the circles and connect with a smooth parabolic curve.