Lesson #42 – Three Basic Trigonometric Functions

|  |  |
| --- | --- |
| A2.A.66 | Determine the trigonometric functions of any angle, using technology |
| A2.A.64 | Use inverse functions to find the measure of an angle, given its sine, cosine, or tangent |

|  |  |  |
| --- | --- | --- |
| **Term** | **Formula/Symbol** | **Important Information** |
| Theta  |  | A variable used in trigonometry to represent an unknown angle. It is not like π because π has a value (approximately 3.14). Think of it like using x, y, t, or any other variable. |
| Pythagorean Theorem: |  | This can only be used with right triangles!  |
| Sine of an angle |  | In a right triangle, the ratio between the side Opposite an angle and the Hypotenuse  |
| Cosine of an angle |  | In a right triangle, the ratio between the side Adjacent or next to an angle and the Hypotenuse |
| Tangent of an angle |  | In a right triangle, the ratio between the side Opposite an angle and the side Adjacent to the angle |
| Memory Device: |
| Note: There is a lot more to the definitions of sine, cosine, and tangent. We will be expanding our understanding of these trig. functions throughout the unit. |

We read sin(x) as “sine of x” and not “sine times x” because SINE IS A FUNCTION OF X. The sine notation, **sin(x) is the same as function notation f(x)**. Sine is the specific name of a function where your **input is an angle** and your **output is the ratio** between the side opposite that angle and the hypotenuse in a right triangle.

Make sure your calculator is in **degree mode**!

**Evaluate** each of the following trigonometric functions to the nearest ten thousandth.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| 1.   | cos (73°)= |

 |

|  |  |
| --- | --- |
| 2.   | sin (73°)= |

 |

|  |  |
| --- | --- |
| 3.   | tan 81°= |

 |
|

|  |  |
| --- | --- |
| 4.   | sin 59degrees= |

 |

|  |  |
| --- | --- |
| 5.   | sin 33degrees= |

 |

|  |  |
| --- | --- |
| 6.   | tan 245degrees= |

 |
|

|  |  |
| --- | --- |
| 7.   | f(x) = tan(x). Find f(38.9degrees). |

 |

|  |  |
| --- | --- |
| 8.   | f() = tan(). Find f(57degrees). |

 |

|  |  |
| --- | --- |
| 9.   | g(x) = cos(x). Find g(-100degrees). |

 |
| How are questions 6 and 9 possible when those angles would not fit in a right triangle? Obviously there is more to sine, cosine, and tangent than what you know so far.**The inverse trigonometric functions**The inverse function for y=sin(x) is y=. Similarly, the inverse function for y=cos(x) is \_\_\_\_\_\_\_\_\_\_.And the inverse function for y=tan(x) is \_\_\_\_\_\_\_\_\_\_\_.The input (x) for a trig function is the angle, and the output (y) is the trigonometric ratio. Therefore the input for an inverse trigonometric function is the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ and the output is the \_\_\_\_\_\_\_\_\_\_\_.**Evaluate** each of the following inverse trigonometric functions to the nearest degree.

|  |  |  |
| --- | --- | --- |
|  |  |  |

**Solve** for the variable to the nearest degree. |
| 1. Solve for : sin = .7224
 | 1. Solve for x: cos(x) = .5
 | 1. Solve for : tan =
 |
| 1. Solve for x: tan(x) = 5
 | 1. Solve for : cos =
 | 1. Solve for y: sin(y)=.8888
 |

**Finding Trigonometric Function Values from Triangles**

X

Y

Z

3. In right triangle XYZ, Z is the right angle. If XY=10, YZ= , and XZ=5, find the following trigonometric values.

SinX = SinY =

CosX = CosY =

TanX = TanY=

2. In right triangle DEF, E is the right angle. If DE=1 and DF=3, find tanF.

1. In right triangle ABC, C is the right angle. If AB=13 and BC=12, find sinA and sinB.

E

F

D

A

B

 C

**Finding the values of the 3 trig ratios**

Ex) If , find the values of the other two trigonometric functions.

Steps

1. **Draw a right triangle** and label the sides with the ratio

you are given. (Remember, the size of the triangle does not matter.)

2. Find the third side using Pythagorean theorem.

3. Find the other basic trig ratios using SOH-CAH-TOA.

1. If t is an acute angle and , find the values of the other two trigonometric functions. Express your answers in simplest radical form.
2. If is an acute angle and , find the value of tanθ. Express your answer in simplest radical form.
3. If t is an acute angle and , find the values of the other two trigonometric functions.
4. If is an acute angle and , find sinθ. Express your answer in simplest radical form.
5. If is an acute angle and , find the values of the other two trigonometric functions. Express your answers in simplest radical form.
6. If t is an acute angle and , find the values of the other two trigonometric functions.
7. If t is an acute angle and , find the values of the other two trigonometric functions. Express your answers in simplest radical form.

Lesson #43 – Trigonometric Reciprocals and Quotients

|  |  |
| --- | --- |
| A2.A.55 | Express and apply the six trigonometric functions as ratios of the sides of a right triangle |
| A2.A.58 | Know and apply the co-function and reciprocal relationships between trigonometric ratios |

Current trigonometric functions:

sin= cos= tan=

What other ways could be make ratios out of the sides of a right triangle?

These other three ratios are also trigonometric functions. They are called the **reciprocal trigonometric functions** because they are reciprocals of the three original trigonometric functions. Each of the reciprocal functions also has its own name.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Name | Abbreviation | Ratio with sides of a right triangle | Relation to sine, cosine, or tangent | Value with respect to <A |
| Cosecant |  |  |  |  |
| Secant |  |  |  |  |
| Cotangent |  |  |  |  |



Life is not perfect. Sine and cosecant are reciprocals and cosine and secant are reciprocals. In this case, opposites attract.

sinA= sinB=

cosA= cosB=

tanA= tanB=

cscA= cscB=

secA= secB=

cotA= cotB=



**Finding Reciprocal Values from a Triangle**

A

B

 C

1. In right triangle ABC, C is the right angle. If AB=25 and AC=7, find secA and secB.

E

F

D

2. In right triangle DEF, E is the right angle. If DE=2 and DF=4, find cotF.

**Finding the values of all 6 trig ratios**

Ex) If , find the values of the other five trig ratios.

We learned that

 

Therefore, if , 

If we know a basic trig ratio, we can flip that value to find the reciprocal ratio.

1. Draw a right triangle and label the sides

with the ratio you are given.

2. Find the third side using Pythagorean theorem.

3. Find the other basic trig ratios.

(You can find secant, cosecant, and cotangent quicker by

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_).

1. If *t* in an acute angle and , find the values of the other five trigonometric functions.

Z

Y

1. If $θ$ is an acute angle and , find the values of the other five trigonometric functions.

1. If *A* is an acute angle and , find the value of in simplest radical form.
2. If β is an acute angle and , find the value of sin(β) in simplest radical form.

Evaluating Reciprocal Trig Functions

To find the values of the reciprocal functions, you must enter it in reciprocal form in the calculator. For example, to the nearest hundredth, .

**Common Mistakes**

1. Flipping the angle.

Ex) ,

 not .

2. Confusing the reciprocal with inverse.

Ex) ,

 not .

3. Forgetting to check your **mode.**

1. Find  to the nearest hundredth.

1. Find  to the nearest thousandth.
2. Find the value of  to the nearest tenth.
3. f(x)=cot(x). Find the value of f(10°) to the nearest tenth.
4. f(x)=sec(x). Find the value of f(52°) to the nearest tenth.

The Quotient Relationships

Q: Why are there six trigonometric functions?

A: There are six different ratios you can make between the sides of a right triangle.

Even though there are six trigonometric functions, we can actually express them in terms of sine and cosine ONLY. We already know that  and abut how can we express tanθ and cotθ in terms of sine and cosine?

|  |  |
| --- | --- |
| Prove:  | Prove: Quotient Relationships |
|  |  |

These are the quotient identities. They will come in handy throughout the rest of the unit. Let’s verify these quotient relationships with a couple of examples.

1.  tan(46)°=
2.  cot(46)°=

Use the quotient identities to complete the following problem.

1. If sinθ= and cosθ=, find tanθ and cotθ without drawing a triangle.

Lesson #44 - Special Triangles and Trig Values

|  |  |
| --- | --- |
| A2.A.59 | Use the reciprocal and co-function relationships to find the value of the secant, cosecant, and cotangent of 0°, **30°, 45°, 60°,** 90°, 180°, and 270° |
| A2.A.56 | Know the exact and approximate values of the sine, cosine, and tangent of 0°, **30°, 45°, 60°,** 90°, 180°, and 270° |
| A2.A.58 | Know and apply the co-function and reciprocal relationships between trigonometric ratios |

The two special triangles you learned last year were 30-60-90 and 45-45-90 triangles. They are called “special” because they have specific ratios between their sides. Since all 45-45-90 triangles are similar and all 30-60-90 triangles are similar, size does not matter. The ratios are always true.

****

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**Special Angle Trigonometric Values Summary Chart**

|  |
| --- |
| Exact Values |
|  | 30 | 45 | 60 |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

Do your calculations for the exact values first on the next page. Copy your final answers in into the table on the left. You must know the bolded part of the table by memory. The estimated values can be found using your calculator. Round to the *nearest thousandth when necessary*.

|  |
| --- |
| Estimated Values |
|  | 30 | 45 | 60 |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |



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Use the two triangles to find the **exact** (in simplest radical form when necessary) trigonometric values for sine and cosine. Use the reciprocal and quotient relationships to find the values for tangent, secant, cosecant, and cotangent. All answers should be in **simplest radical form**.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 30 | 45 | 60 | Pattern/Method |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

**HINTS FOR MEMORIZATION: The 1-2-3, 3-2-1 Method**

Look at the table on page 10. What pattern do you notice in the sine values?

What pattern do you notice in the cosine values?

|  |  |  |  |
| --- | --- | --- | --- |
|  | 30 | 45 | 60 |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

This pattern makes it much easier to memorize the trigonometric values for the special angles. If you memorize  or even type sin(30°) into your calculator to find out that it equals , you can count from there to find the other values.

In the table to the right, find the sine and cosine values using this method.

If you can use what you learned last lesson, you just need to memorize the sine AND cosine trigonometric values for the special angles. All of the others you can find using the **quotient or reciprocal** relationships. (It is still helpful to memorize the tangent values.)

**Quotient Relationships:** = =

Go back to the table above and find tangent using the quotient relationship.

**Reciprocal Relationships:**  = = =

**Arithmetic with Special Angle Trigonometric Values**

(Find the trig value before performing the operation.)

|  |  |  |
| --- | --- | --- |
| 1. sin(30°)+cos(45°)=
 | 1. $(tan(30°))^{2}$=
 | 1. sin(30°)(cos(30°)+cos(45))=
 |
| 1. sin(45°)+tan(30)°=
 | 1. cot(30°)tan(30°)=
 | 1. csc(60°)+tan(45°)+sec(30°)=
 |

Lesson #45 - Angle Measurements in Standard Position

|  |  |
| --- | --- |
| A2.A.57 | Sketch and use the reference angle for angles in standard position |

**Standard Position for an angle**: An angle with its initial (starting) side on the

 positive x-axis and its vertex on the origin. The other side of the angle

is called the terminal (ending) side.

\*\*Positive angles go \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ while negative angles go \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

How many degrees are in one rotation?

Why is that number helpful?

Draw angles in standard position with the following measures.

|  |  |  |
| --- | --- | --- |
| 120 degrees | 230 degrees | -40 degrees |
| 80 degrees | -200 degrees | -90 degrees |
| 530 degrees | -400 degrees | 360 degrees |

**Quadrantal Angle**: An angle with its terminal side on the axes.

The measurements of the first five positive

quadrantal angles would be: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_



**Coterminal angles**: Angles with the same terminal (ending) side

Look at the previous page. What angle would be coterminal

with -200°?

 Method for finding the **smallest positive coterminal angle**:

Add or subtract 360° until the angle is between 0 and 360.

We will need to find the smallest positive coterminal angle for the next section of this lesson as well as the rest of the unit.

Ex) What is the smallest positive angle that is coterminal with an angle of 450°?

**450°-360°=90°**

1. What is the smallest positive angle that is coterminal with an angle of 700°?
2. What is the smallest positive angle that is coterminal with an angle of -460°?
3. What is the smallest positive angle that is coterminal with an angle of 120°?
4. Challenge: What is the smallest positive angle that is coterminal with an angle of 10,427 degrees?

**Reference Angle**: an acute angle formed by the terminal side of the original angle and the x-axis. All angles in standard position have reference angles EXCEPT the quadrantal angles.

|  |  |  |  |
| --- | --- | --- | --- |
| reference1a | reference2 | reference3 | reference4 |

|  |  |
| --- | --- |
| Steps |  Ex) Find the reference angle for 140°. |
| 1. *If necessary*, find the smallest positive coterminal angle for the given angle.
 |  | http://www.regentsprep.org/Regents/math/algtrig/ATT3/bowtie.gif |
| 1. **Draw the angle** and decide in which quadrant the terminal side lies.
2. **Draw a reference triangle to the x-axis**
 |  |
| 1. Find the **angle measure between the terminal side and the x-axis**.

 Quadrant I: Quadrant II or III: Quadrant IV: |  |

**Find the reference angle for each angle below. Draw a sketch to show your work.**

|  |  |  |
| --- | --- | --- |
| 1. **285°**

**Quadrant IV****360-285 = 75**° | 1. **78°**
 | 1. **230°**
 |
| 1. **-17°**

 | 1. **268°**
 | 1. **91°**
 |
| 1. **243°**
 | 1. **306°**
 | 1. **-299°**
 |
| 1. **-474°**

 | 1. **652°**
 | 1. **1000°**
 |
| 1. **100°**
 | 1. **-100°**
 | 1. **540°**
 |

**Expanding Our Understanding of Trigonometry**

The other two angles in a right triangle must be in the following interval: 

Sine and Cosine are functions. Are the domains sine and cosineonly?

Find sin(300°) and cos(-1000°).

How can what we know about right triangles and what we just learned about angles in standard position make it possible to make the domains of sine and cosine all real numbers?

Lesson #46 - Trigonometry on the Coordinate Plane

|  |  |
| --- | --- |
| A2.A.62RULES | Find the value of the trigonometric functions, if given a point on the terminal side of angle *q* **(and much more)** |

1. The sign of the **adjacent side** is determined by the **sign of x** in that quadrant.
2. The sign of the **opposite side** is determined by the **sign of y** in that quadrant.
3. The **hypotenuse** is **always positive**.
4. An angle in standard position is drawn in each quadrant.
5. Draw a **reference triangle** in each quadrant
6. Label each side of the triangle as positive (+) or negative (-) using the 3 rules above.
7. **Use quadrant II as an example** to complete the textboxes for quadrants I, III, and IV.

Quadrant I

sin=

cos=

tan=

Quadrant II

sin= 

cos= 

tan= 

 +y +h

 -x

sec=

csc=

cot=

sec=

csc=

cot=

Quadrant IV

sin=

cos=

tan=

Quadrant III

sin=

cos=

tan=

sec=

csc=

cot=

sec=

csc=

cot=

**Use this example to complete the problems on the next page.**

|  |
| --- |
| Problem: The terminal side of θ , an angle in standard position, passes through the point, (-4,3). Find the sin() and sec(). |
| 1. Plot the point and draw the angle in standard position that passes through the point.
 |  |
| 1. Draw the reference triangle for that angle.
 |  |
| 1. Label the side lengths based on the given point.
 |  |
| 1. Use the Pythagorean Theorem to find the hypotenuse. **(Remember it is always positive!)**
 |  |
| 1. Set up the trigonometric ratio(s) you are asked to find. Be sure to write the answer in simplest radical form if necessary.
 |  |
| 1. Check your signs.
 | (Based on page 17, in the second quadrant sine should be positive and secant should be negative. Therefore my answers have the correct signs). |

**Use the example on the previous page** to complete the four problems below.

|  |  |
| --- | --- |
| 1. The terminal side of an angle in standard position passes through the point, (-12,-35). Find cos and cot.
 |  |
| 1. The terminal side of an angle in standard position passes through the point, . Find sin and csc.
 |   |
| 1. The terminal side of , an angle in standard position, passes through the point, . Find the values of the six trigonometric functions.

  |  |
| 1. The terminal side of an angle in standard position passes through the point, (-9,-40). Find the cosecant of the angle.
 |  |

****The following chart helps you remember which of the three basic trigonometric functions are positive in each quadrant.

There is a mnemonic statement that may be helpful for remembering the positive trig values (and their reciprocals) in each quadrant**.**

**A S T C** - **A**ll **S**tudents **T**ake **C**alculus!

(We start the brainwashing early!)

1. If  is negative and  is negative, in which quadrant does the terminal side of  lie?

|  |  |  |  |
| --- | --- | --- | --- |
| 1) | I | 3) | III |
| 2) | II | 4) | IV |

1. If  and , then angle *x* terminates in Quadrant

|  |  |  |  |
| --- | --- | --- | --- |
| 1) | I | 3) | III |
| 2) | II | 4) | IV |

1. If  and , angle *A* terminates in Quadrant

|  |  |  |  |
| --- | --- | --- | --- |
| 1) | I | 3) | III |
| 2) | II | 4) | IV |

1. If  and , in which quadrant does the terminal side of angle  lie?

|  |  |  |  |
| --- | --- | --- | --- |
| 1) | I | 3) | III |
| 2) | II | 4) | IV |

1. If , in which quadrants could  terminate?

|  |  |  |  |
| --- | --- | --- | --- |
| 1) | I and IV | 3) | II and IV |
| 2) | I and III | 4) | II and III |

1. If , in which quadrants could angle *x* terminate?

|  |  |  |  |
| --- | --- | --- | --- |
| 1) | I and III | 3) | II and IV |
| 2) | II and III | 4) | III and IV |

1. If the tangent of an angle is negative and its secant is positive, in which quadrant does the angle terminate?

|  |  |  |  |
| --- | --- | --- | --- |
| 1) | I | 3) | III |
| 2) | II | 4) | IV |

1. If  and , in which quadrant does the terminal side of angle *x* lie?

|  |  |  |  |
| --- | --- | --- | --- |
| 1) | I | 3) | III |
| 2) | II | 4) | IV |

**Relating trigonometric functions of larger angles to their reference angles**

Steps

* Draw the angle and a reference triangle.
* Identify the quadrant.
* Find the reference angle.
	+ (If necessary, use coterminal angles first.)
* **The trigonometric values for the given angle will be the SAME as the reference angle EXCEPT the sign might be different**. Use the quadrant to decide.

The directions to this type of question can be the most confusing part. It will often say, **“Write the given function as a function of an acute angle.”**

|  |  |
| --- | --- |
| 1.
 | 1.
 |
| 1.
 | 1.
 |
| 1.
 | 1.
 |
| 1.
 | 1.

  |

**Summary**

|  |  |
| --- | --- |
| 135° sin(135°) is the same as sin(45°). cos(135°) is the same as -cos(45°). tan(135°) is the same as –tan(45°).**Check these answers on your calculator.** The angle is in the 2nd quadrant so sine will be positive, but cosine and tangent will be negative. | 260°.sin(260°) is the same as \_\_\_\_\_\_\_.cos(260°) is the same as \_\_\_\_\_\_\_.tan(260°) is the same as \_\_\_\_\_\_\_.Check these answers on your calculator.  |
| -++   |   |

**The steps for every problem can be summarized with some combination of the following steps**

1. Draw a picture (angle in standard position and reference triangle).
2. Use coterminal angles until the angle is smaller than 360°.
3. Find the reference angle.
4. Set up the trigonometric ratio(s).
5. Decide if the trigonometric function(s) are positive or negative in that quadrant.

Lesson #47 - Radian Measure

|  |  |
| --- | --- |
| A2.M.1 | Define radian measure |
| A2.A.61 | Determine the length of an arc of a circle, given its radius and the measure of its central angle |
| A2.M.2 | Convert between radian and degree measures |

Why do we use degees to measure angles?

Are there other ways to measure angles?

|  |  |  |
| --- | --- | --- |
| Image | Angle Measurement | Number in a full rotation |
| degrees-360 | Degrees |  |
| http://www-math.cudenver.edu/~wcherowi/clock.gif | “Clockians” |  |
| http://dclips.fundraw.com/zobo500dir/clipart_by_nicu_buculei_01.jpg | “Petaloids” |  |
| http://rml3.com/a20p/images/trig_radians.gif | Radians |  |

What is another way that you know 2π radii (plural for radius) fit perfectly around the outside of a circle?

**Arc length**: (symbol:***s***) The length of an arc which is a section or part of a circle's perimeter (a.k.a. circumference)

**Central Angle**: An angle positioned with its vertex at the center of a

circle and its endpoints on the circle

**Subtend**: to intersect or be opposite

**One Radian**: (abbreviation: *rad*) The measure of a central angle that subtends an arc length that is one radius long.

<http://id.mind.net/~zona/mmts/trigonometryRealms/radianDemo1/RadianDemo1.html>

How many radii fit around a circle?



The size of the circle does not affect the number of radians in the circle because the **radius and the circumference will grow or shrink in proportion to each other.**

THE MEASURE OF THE CENTRAL ANGLE IS **ONE RADIAN** WHEN THE **ARCLENGTH IS EQUAL TO THE RADIUS**.

Note: From now on, if an angle does not have a degree symbol, you must assume it is measured in radians.

An angle is 1 radian when the \_\_\_\_\_\_\_\_\_ is equal to the \_\_\_\_\_\_\_\_.

If the arclength (s) is 5 inches and the radius (r) is 5 inches the radian measure of the angle would be \_\_\_\_\_\_\_\_.

In other words, the radian measure of an angle is equal to the ratio of the arclength (s) to the radius (r).

*s* = arclength

ø = angle in radians

*r* = radius

This gives us the formula, .

If we use as the variable for the angle, this gives us the formula, .

NOTE:  **must be in radians to use this formula, not degrees**.

Draw the following conditions: If the radius of the circle to the right is 5 and the arc length subtended by the central angle is 10, would be \_\_\_ radians. This makes sense because \_\_\_ radii fit around the arclength.

* 1. Find the arclength if radius is 6 inches and the measure of the central angle is 3 radians.

**New Formula**



 = central angle in radians

s = arc length

r = radius

* 1. Find the exact value of the central angle if the radius of the circle is 10 inches and the arclength is 5π inches.

* 1. Find the radius of a circle if the arclength is 12 in and the central angle is  to the nearest tenth.
	2. Find the arclength of a circle if the radius is 2 cm and the measure of the central angle is 3.2 radians.

Converting between radian and degree measure

A full rotation has \_\_\_ degrees or \_\_\_ radians.

Therefore the ratio of degrees to radians is \_\_\_\_:\_\_\_\_\_\_. Divide both sides by 2.

\_\_\_\_:\_\_\_\_\_\_.

We will use this fact to convert between radians and degrees.

|  |  |
| --- | --- |
| **Converting from degrees to radians** Multiply by **To radians**: “put in π take out 180°Ex) Convert 120° to radians.Exact answer:  Nearest Hundredth: You can enter 120/180 in your calculator, change it to a fraction, and put π after it. | **Converting from radians to degrees****To °**: “put in 180° take out π” Multiply by Ex) Convert  radians to degrees.  |

1. Convert to radians. Write as an exact answer **and** rounded to the nearest tenth.

|  |  |
| --- | --- |
| * 1. 30º
 | * 1. 260º
 |
| * 1. 45º
 | * 1. 180º
 |

1. Convert each radian measure to degrees. Round to the nearest degree when necessary.

|  |  |
| --- | --- |
| * 1.
 | * 1.
 |
| * 1. 2.45
 | * 1.
 |

Lesson #48 – The Unit Circle

|  |  |
| --- | --- |
| A2.A.60 | Sketch the unit circle and represent angles in standard position |
| A2.A.59 | Use the reciprocal and co-function relationships to find the value of the secant, cosecant, and cotangent of **0°**, 30°, 45°, 60°, **90°, 180°, and 270°** |
| A2.A.56 | Know the exact and approximate values of the sine, cosine, and tangent of **0°**, 30°, 45°, 60°, **90°, 180°, and 270°** |

You learned in Lesson #42 that the size of the triangle does not matter when finding the trigonometric functions of an angle.

You learned in Lesson #46 that when given a point on an angle in standard postion and  Draw a picture of this concept in the axes provided.

Since the size of the triangle does not matter, how could

we make cos=x and sin=y?

A way to make sure our triangle always has a **hypotenuse of 1 is to contain the triangle in a circle with a radius of 1, centered at (0,0).**

This is the parent relation for circles we learned about in unit 4. Its equation is and we call it the **unit circle** because it has a radius of one unit.

* On the axes draw an angle with its terminal side in the first quadrant.
* Draw the **unit circle**.
* Draw a reference triangle using the point

where the angle intersects the circle

as the end of your hypotenuse.

* Label the sides of the triangle.

This is only true when the point is on the unit circle!

 

How is the unit circle helpful?

1. It allows us to **work with trigonometric functions without using a triangle** because the hypotenuse is always one. All we need is a point on the unit circle, (x,y).
2. The **x-value** of the point on the unit circle is the same as the **cosine of the angle**.
3. The **y-value** of the point on the unit circle is the same as the **sine of the angle**.
4. It allows us to find the **trigonometric values of the** **quadrantal angles** (90°, 180°, etc.).
5. It is the basis for creating the **function graphs of sine and cosine** (next lesson).
6. It is the easiest way to find many **trigonometric identities** that we will learn about at the end of the year.

Information from a Point on the Unit Circle

If you know a point on the unit circle intersected by , you know sin and cos.

1. The terminal side of intersects the unit circle at the point . What is Sin? What is Cos?
2. The terminal side of intersects the unit circle at the point . What is Sin? What is Cos?
3. The terminal side of intersects the unit circle at the point . What is Sin? What is Cos?
4. The terminal side of intersects the unit circle at the point . What is Sin? What is Cos?

**You can also find the values of other trigonometric functions from the point on the unit circle.**

1. The terminal side of intersects the unit circle at the point . What is Tan?
2. The terminal side of intersects the unit circle at the point . What is sec?
3. The terminal side of intersects the unit circle at the point . What is csc?
4. The terminal side of intersects the unit circle at the approximate point, . What is cot?

Quadrantal Angles

Draw a unit circle on the axes below to find the trigonometric values for the quadrantal angles.

This is really the only way to conceptualize sin90°, cos180°, etc. because you cannot draw a reference triangle for a quadrantal angle.

All of the values in the chart can be checked on the calculator.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Point on Unit Circle | sin | cos | tan | sec | csc | cot |
| 0°, 360° | (1,0) | 0 | 1 |  |  |  |  |
| 90° |  |  |  |  |  |  |  |
| 180° |  |  |  |  |  |  |  |
| 270° |  |  |  |  |  |  |  |

To find the values for the other four trigonometric functions use the reciprocal and quotient relationships!

Use the unit circle to find the following trigonometric values. Show your work.

|  |  |
| --- | --- |
| 1.
 | 1.
 |
| 1.
 | 1.
 |
| 1.
 | 1.
 |

1. What point on the unit circle is intersected by an angle of 0°?
2. What point on the unit circle is intersected by an angle of 270°?

Exact Values and Special Angles

If is a **special angle**, we do not need to round the coordinates of the point on the unit circle because we know the **exact value** of sin and cos. Remember to first find the reference angle and consider the quadrant for the sign of the answer.

1. What point on the unit circle is intersected by an angle of 45°?
2. What point on the unit circle is intersected by an angle of 30°?
3. What point on the unit circle is intersected by an angle of 60°?

**The Unit Circle Diagram**

The unit circle diagram helps you start to think about special angles in the other quadrants as well as the radian measures of the special angles.

Patterns:

1. What do you notice about the **points** on the angles with a reference angle of 30°?
2. What do you notice about the **points** on the angles with a reference angle of 45°?
3. What do you notice about the **points** on the angles with a reference angle of 60°?
4. What do you notice about the **radian measure** of the angles with a reference angle of 30°?
5. What do you notice about the **radian measure** of the angles with a reference angle of 45°?
6. What do you notice about the **radian measure** of the angles with a reference angle of 60°?

**FOR EACH OF THE FOLLOWING PROBLEMS, DRAW A SKETCH OF THE UNIT CIRCLE.**

Let’s look at how the unit circle diagram summarizes information about the special angles.

We will use 210° as an example.

\*We could convert the angle to radians: 

\*We could find the sine and cosine values of 210° using reference angles and the special angle ratios.

 

**All of this information is displayed concisely on the unit circle diagram.**

1. What point on the unit circle is intersected by an angle of 150°?
2. What point on the unit circle is intersected by an angle of 315°?
3. What point on the unit circle is intersected by an angle of -120°?
4. What point on the unit circle is intersected by an angle of 180°?
5. What point on the unit circle is intersected by an angle of 225°?

You can also start with a point on the unit circle intersected by a special angle or a quadrantal angle and work backwards to find the angle measure in either degrees or radians.

1. Find the measure of the angle in both degrees and radians that intersects the point on the unit circle?
* The angle is in the **fourth quadrant** (+,-).
* when **θ=60°**. (when θ=60° also).
* Therefore θ=360-60=300°
* In radians: 
1. Find the measure of the angle in both degrees and radians that intersects the point (0,-1) on the unit circle.
2. Find the measure of the angle that intersects the point on the unit circle.
3. Find the measure of the angle in both degrees and radians that intersects the point  on the unit circle.
4. Find the measure of the angle in both degrees and radians that intersects the point (1,0) on the unit circle.
5. Find the measure of the angle in both degrees and radians that intersects the point  on the unit circle.
6. Find the measure of the angle in both degrees and radians that intersects the point on the unit circle.