**Study Guide Examples**

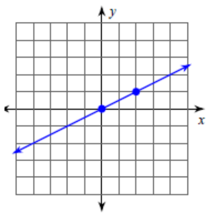
# IS IT A FUNCTION?

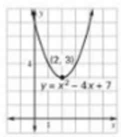
How do you recognize a Function?

## From a graph

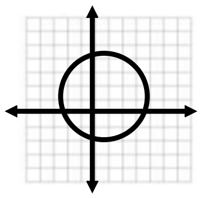
Every x is mapped to one y.
Each input has exactly one output.

"x" never repeats!





## Non-Examples



"x" is repeated
not a function.

## From a table

| x | y |
|---|---|
| -2 | 13 |
| -1 | 12 |
| 1 | 10 |
| 2 | 9 |

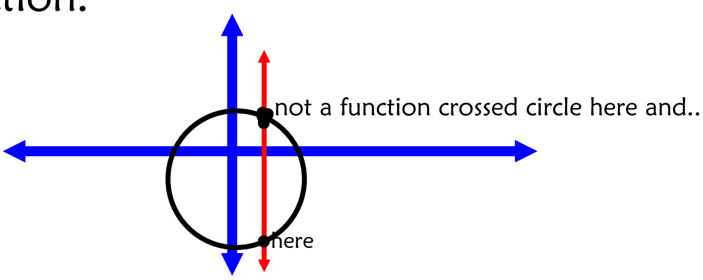
"x" does not repeat

## Non-Examples

| x | y |
|---|---|
| 1 | 2 |
| 2 | 3 |
| 2 | 5 |
| 3 | 7 |

"x" is repeated
not a function.

Do the vertical line test on graphs. If the vertical line crosses your graph in more than one spot it is not a function.



**Study Guide Examples**

Traditionally, functions are referred to by the notation $f(x)$, but $f$ need not be the only letter used in function names. Remember that $f(x)$ is telling you that the result will be "a function of $x$", or is dependent upon $x$.

The statements $y = x^2$ and $f(x) = x^2$ are basically the same.

*Example:*

A function is represented by $f(x) = 2x + 5$. Find $f(3)$. Find the value of the function when x = 3.

To find $f(3)$, replace the $x$-value with 3.

$f(\mathbf{3}) = 2(\mathbf{3}) + 5$
$\quad = 6 + 5$
$\quad = 11$

x = 3

If we reverse this and say f(x) = 11, you are now finding the finding the "x" value, so we replace f(x) in the equation with 11.

$f(x) = 2x + 5$
$11 = 2x + 5$
$6 = 2x$
$3 = x$ So you can see when x= 3 then f(x) = 11

*Note:* The $f(x)$ notation can be thought of as another way of representing the $y$-value, especially when graphing. The $y$-axis may even be labeled as the $f(x)$ axis.

Examples:

When you are given "x":

1. $f(x) = 4x^2 - 5$; find f(3)
$f(3) = 4(3)^2 - 5$
$\quad = 4(9) - 5$
$\quad = 36 - 5$
$\quad = 31$

b) $g(x) = 2x + 3$ find f(-2)

When you are given f(x)

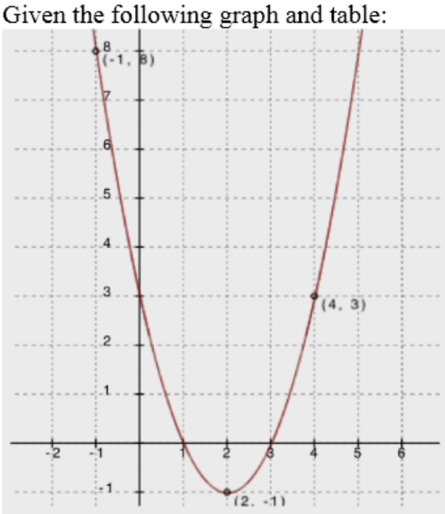
1. $f(x) = 4x^2 - 5$; find x if f(x) = 31
$31 = 4x^2 - 5$
$+5 \qquad +5$
$\frac{36}{4} = \frac{4x^2}{4}$
$9 = x^2$
$3 = x$ or $x = -3$
fvf

b) $g(x) = 2x + 3$ find x if f(x)= -1

**Study Guide Examples**

# Vocabulary - Quadratic Equations

<table>
<tr>
<td>Increasing and decreasing intervals</td>
<td>On what intervals of the domain is the function depicted by the graph above increasing?<br>(-3, +∞)<br><br>On what intervals of the domain is the function depicted by the graph decreasing.<br><br>(- ∞, -3)</td>
</tr>
<tr>
<td>Average Rate of Change on an interval</td>
<td>Given the following graph and table:<br>




<table>
<tr><th>x</th><th>f(x)</th></tr>
<tr><td>−1</td><td>8</td></tr>
<tr><td>0</td><td>3</td></tr>
<tr><td>1</td><td>0</td></tr>
<tr><td>2</td><td>−1</td></tr>
<tr><td>3</td><td>0</td></tr>
<tr><td>4</td><td>3</td></tr>
<tr><td>5</td><td>8</td></tr>
</table>

What is the average rate of change for the following intervals:<br><br>[-1,0]: [0,3]<br><br>[0,1]: [1,3]</td>
</tr>
</table>

**Study Guide Examples**

slope

## Slope

### From equation

y =2x y =mx

slope = 2

y = 3x + 6

slope = 3

### From two points

(5, -9)

(3, 4)

$\frac{-9 - 4}{5 - 3} = \frac{-13}{2}$

Simplify if possible

### From table

| x | y |
|---|---|
| 0 | 8 |
| 2 | 9 |
| 4 | 10 |
| 6 | 11 |
| 8 | 12 |

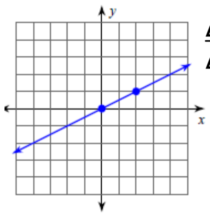
Choose two points

(2,9)

(4,10)

$\frac{9 - 10}{2 - 4} = \frac{-1}{-2} = \frac{1}{2}$

### From Graph




$\frac{\Delta y}{\Delta x} = \frac{1}{2}$

Slope-
A constant rate of change

Another word for slope is unit rate

**Study Guide Examples**

## Forms of Linear Equations

STANDARD FORM

$2x + 3y = 6$

YOU CAN SOLVE FOR

Y and put in y=mx +b form

SLOPE -INTERCEPT FORM

$y = mx + b \quad 2x + 3y = 6$

sovle for y:

$2x + 3y = 6$

$-2x \qquad -2x$

$$\frac{3y}{3} = \frac{-2x}{3} + \frac{6}{3}$$

$$y = \frac{-2x}{3} + 2$$

slope = $\frac{-2}{3}$

y-intercept = 2

Point- slope form

$y - y_1 = m(x - x_1)$

$y - 2 = 3(x + 6)$

slope = 3

goes through

ordered pair (-6,2)

Name the form

1. $2x + 6y = 12$

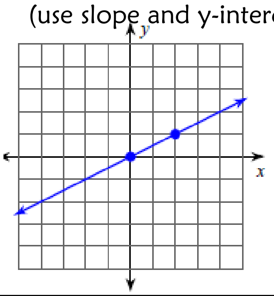
2. $y - 2 = 5(x + 7)$

3. $y = 3x + 8$

**Study Guide Examples**

# From a graph

(use slope and y-intercept)



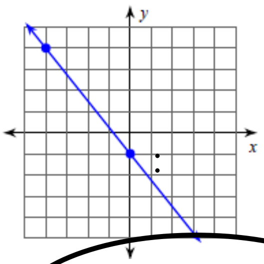

$m = \frac{1}{2}$

b = 0

y = mx +b

$y = \frac{1x}{2} + 0$

$y = \frac{1x}{2}$




$m = \frac{-5}{4}$

b = -1

y = mx +b

$y = \frac{-5x}{4} + 1$

How do you write LINEAR Equations to Describe Functions?

## Given m = 2 and b = -5

y = mx +b

y = 2x -5 is the equation.

## From a table

| x | y |
|---|---|
| -2 | 13 |
| -1 | 12 |
| 1 | 10 |
| 2 | 9 |

Choose two ordered pairs
(-1,12)
(1,10)
Find Slope

$\frac{12 - 10}{-1 - 1} = \frac{2}{-2} = -1$

Now use the equation y = mx +b and any point from table. Substitute values for y, m and x into the equation and solve for b.

(2,9) x = 2  y = 9 m = -1

y = mx +b
9 = -1(2) + b
9 = -2 + b
+2  +2
11 =b  so y = -x +11

## From a Description

Kevin has a paper route. He is paid $10 each week and $0.20 for each paper he delivers. Write an equation that shows the relationship between the number of papers he delivers and his total income for one week.

Total cost = rate for delivering paper(#paper delivered) + weekly pay
total cost = y
rate for delivering papers is $0.20
weekly pay is $10

y= .20x + 10 represents the function.

# Rules of Exponents

When you multiply with the same base you add the exponents.

$$x^4 * x^7 = x^{4+7} = x^{11}$$

$$2^4 * 2^7 = 2^{4+7} = 2^{11} = 2048$$

When you divide with the same base you SUBTRACT the exponents.

$$\frac{X^{10}}{X^7} = X^{10-7} = X^3$$

$$\frac{2^{10}}{2^7} = 2^{10-7} = 2^3 = 8$$

When you raise to a power you multiply the exponents.

$$(x^5)^4 = x^{5(4)} = x^{20}$$

Rewrite negative exponents as a positive exponent.

$$2^{-4} = \frac{1}{2^4} = \frac{1}{16}$$

ANYTHING TO THE ZERO POWER IS ALWAYS ONE!

$x^0 = 1$

$2^0 = 1$

# Solving equations

<u>One step</u>

| | | |
|---|---|---|
| 1. $x + 5 = 12$<br><br>$x + 5 = 12$<br>$\underline{\quad -5 \qquad -5}$<br>$x = 7$ | 2. $2x = 10$<br><br>$\frac{2x}{2} = \frac{10}{2}$<br>$x = 5$ | 3. $\frac{x}{2} = 6$ multiply both sides by 2.<br>3. $2*\frac{x}{2} = 6*2$<br>$x = 12$ |

<u>Two step</u>

4. $2x + 5 = 11$

$$\begin{aligned} 2x + 5 &= 11 \\ \underline{\quad -5} &\quad \underline{-5} \\ \frac{2x}{2} &= \frac{6}{2} \\ x &= 3 \end{aligned}$$

<u>CHECK</u>

$2(3) + 5 = 11$

$6 + 5 = 11$

$11 = 11$

<u>Two step</u>

5. $5x = 3x + 20$

$$\begin{aligned} 5x &= 3x + 20 \\ \underline{-3x} &\quad \underline{-3x} \\ \frac{2x}{2} &= \frac{20}{2} \\ x &= 10 \end{aligned}$$

<u>CHECK</u>

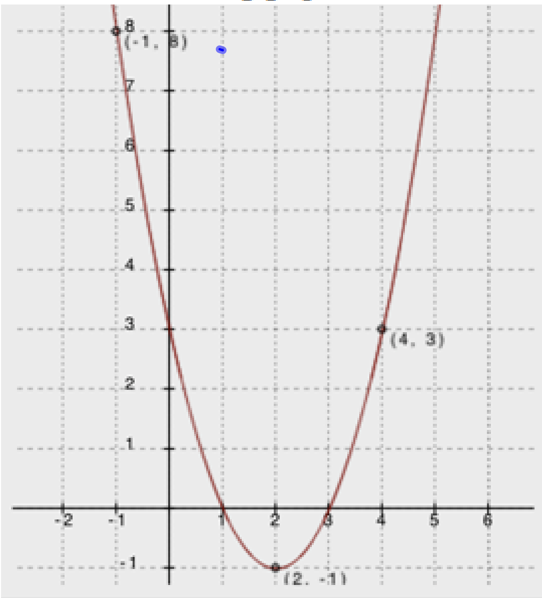
$5(10) = 3(10) + 20$

$50 = 30 + 20$

$50 = 50$

**Study Guide Examples**

Given the following graph and table:




| $x$ | $f(x)$ |
|---|---|
| −1 | 8 |
| 0 | 3 |
| 1 | 0 |
| 2 | −1 |
| 3 | 0 |
| 4 | 3 |
| 5 | 8 |

average

What is the average rate of change for the following intervals:

[-1,0]: Write your ordered pairs [0,3]

(-1,8)

(0,3)

Find the average rate of change

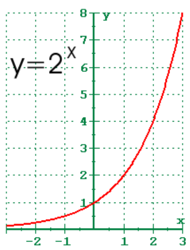
$$\frac{8-3}{-1-0} = \frac{5}{-1} = -5$$

**Study Guide Examples**

## $y$ = initial amount (growth factor)$^x$

### From a graph

a = y-intercept b = growth factor




Step 1: look for y-intercept (where x=0).

Step 2: Choose two points and find growth factor
(0,1)
(1,2) growth factor = 2 Each y was multiplied by 2 as "x" increased by 1.

Equation $y = ab^x$

$y = 1(2)^x$

How do you write exponential Equations? $y = ab^x$

$y = 25(4^x)$

25 is the initial amount (also the y-intercept)

4 is the growth factor. How much your original amount is growing by.

### From a table

| x | f(x) |
|---|---|
| 0 | 2 |
| 1 | 6 |
| 2 | 18 |
| 4 | 54 |
| 5 | 162 |

Step 1: look for y-intercept (where x=0). In the table when x = 0 y= 2

$\frac{\text{Value}}{\text{Previous Value}}$ $\frac{6}{2} = 3$ $\frac{18}{6} = 3$

check each ordered pair to see if ratio is 3

Equation $y = ab^x$

$y = 2(3)^x$

### From a Description

A newly discovered microbe has a growth factor of 5 every hour. If you start with 4 microbes in a petri dish, Write an equation to model this scenario.

$y = ab^x$

$y = 4(5)^x$

a = initial amount
so a = 4
b= growth factor
so b = 5

These are all examples of exponential growth. In the equation $y = ab^x$, when your "b" is greater than 1. It is exponential growth.

**Study Guide Examples**

## Solvingequationswithvariablesonbothsides

1.AtSilverGym,membershipis$25permonth,andpersonaltrainingsessionsare $30each.AtFitFactor,membershipis$65permonth,andpersonaltraining sessionsare$20each.Inonemonth,howmanypersonaltrainingsessionswould Sarahhavetobuytomakethetotalcostatthetwogymsequal?

***A.Writeanexpression representingthetotalmonthcostatSilverGym.***

Monthlymembership+Costfor trainingsessions

25 + 30 X

***B.Writeanexpression representingthetotalmonthlycostatFitFactor.***

Monthlymembership+Costfor trainingsessions

65 + 20x

***C.Howcanyouindthenumberofpersonaltrainingsessionsinonemonththat wouldmakethetotalcostsofthegymsequal?***

Set the expressions equal to each other and solve for x.

***D.Writeanequation thatcanbesolvedtoindthenumberoftrainingsessions inonemonththatmakesthetotalcostsequal.***

TotalcostofSilverGym=TotalcostofFit Factor

To solve the equation
(Rememberyouwantallvariablesononesidetosolve!)

25 + 30x = 65 +20x
-20x -20x

25 + 10x = 65
-25 -25

10x = 40
x = 4

Alwayswriteyouranswerinsentenceform:

**Sarahwouldhavetobuy___4___personaltrainingsessionstomake thetotalcostatthetwogymsequal.**

# Solve the system algebraically

$2x - y = 6$
$3x + 4y = 31$

---

STEP 1: Determine which equation to solve for a variable.

$2x - y = 6$ Solve for y! Good equation. I do not end up with fractions!

$3x + 4y = 31$ Not a good equation to solve for a variable DO NOT TRY. (You end up with fractions.)

---

Step 2: Solve one equation for x or for y.

$2x - y = 6$
$-2x \qquad -2x$

negative y, so divide by -1 $\frac{-y}{-1} = \frac{-2x}{-1} + \frac{6}{-1}$

$y = 2x - 6$ Now substitute 2x- 6 for y into other equation

---

Step 3: Replace your variable with your subsitution.

$3x + 4y = 31$

distribute $3x + 4(2x - 6) = 31$

(Com LikeTerms) $3x + 8x - 24 = 31$

$11x - 24 = 31$
$+24 \quad + 24$

$11x = 55$
$x = 5$

You found x now find y

Now find the other variable.

$2x - y = 6$
$2(5) - y = 6$
$10 - y = 6$
$-10 \qquad -10$
$\frac{-y}{-1} = \frac{-4}{-1}$
$y = 4$

---

## The solution is (5,4).

CHECK YOUR SOLUTION!

BY substituting in x = 5 and y = 4 into both equations.

$3x + 4y = 31$
$3(5) + 4(4) = 31$
$15 + 16 = 31$
$31 = 31$ yes!

$2x - y = 6$
$2(5) - 4 = 6$
$10 - 4 = 6$
$6 = 6$ yes!

# Name the solution

Infinite solutions One Solution No Solution

1. $2x + 3 = 2x - 6$

   No solutions
   They have the same slope;
   they are parallel.

2. $3x - 1 = 3x - 2$

   No solutions
   They have the same slope;
   they are parallel.

3. $5x + 6 = 2x - 10$

   One solution.
   Solve the equation

4. $2x + 6 = 2(x+3)$

   Same line (the slope and y-intercept are the same.)
   Infinite solutions

**Study Guide Examples**

Solve the system by graphing.

$3x + 2y = 9$

$-4x + y = -1$

$3x + 2y = 9$

$-3x \qquad -3x$

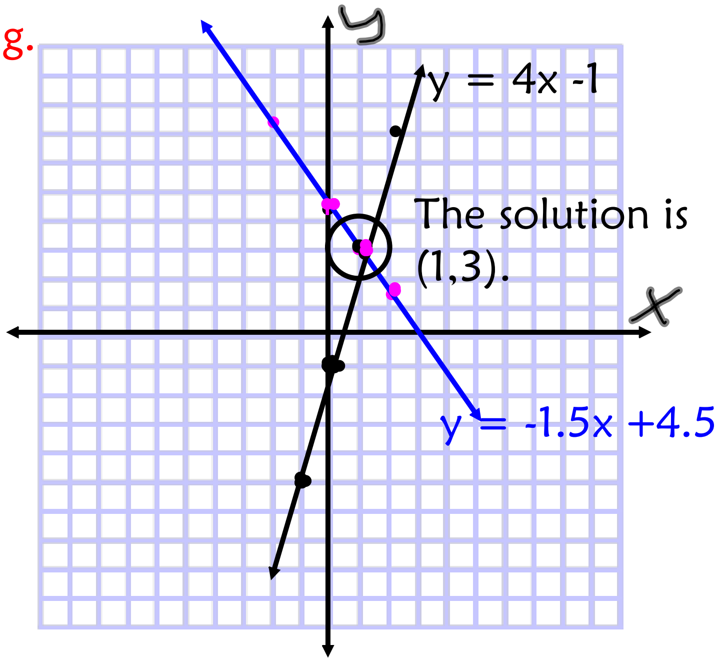
$\frac{2y}{2} = \frac{-3x}{2} + \frac{9}{2}$

$y = \frac{-3x}{2} + 4.5$

$-4x + y = -1$

$+4x \qquad +4x$

$y = 4x - 1$




Solve the system algebraically.

$3x + 2y = 9$

$-4x + y = -1$

$y = 4x - 1$

$3x + 2(4x - 1) = 9$

$3x + 8x - 2 = 9$ distributive property

$11x - 2 = 9$ CLT

$11x = 11$

$x = 1$

Finding y:

$-4x + y = -1$

$-4(1) + y = -1$

$-4 + y = -1$

$+4 \qquad +4$

$y = 3$

Check your solution

Verify this is a solution to the system of equations.

| | |
|---|---|
| $3x + 2y = 9$ | $-4x + y = -1$ |
| $3(1) + 2(3) = 9$ | $-4(1) + 3 = -1$ |
| $3 + 6 = 9$ | $-4 + 3 = -1$ |
| $9 = 9$ | $-1 = -1$ |
| Yes, it is a solution. | Yes, it is a solution. |

**Study Guide Examples**

Solve the system by graphing

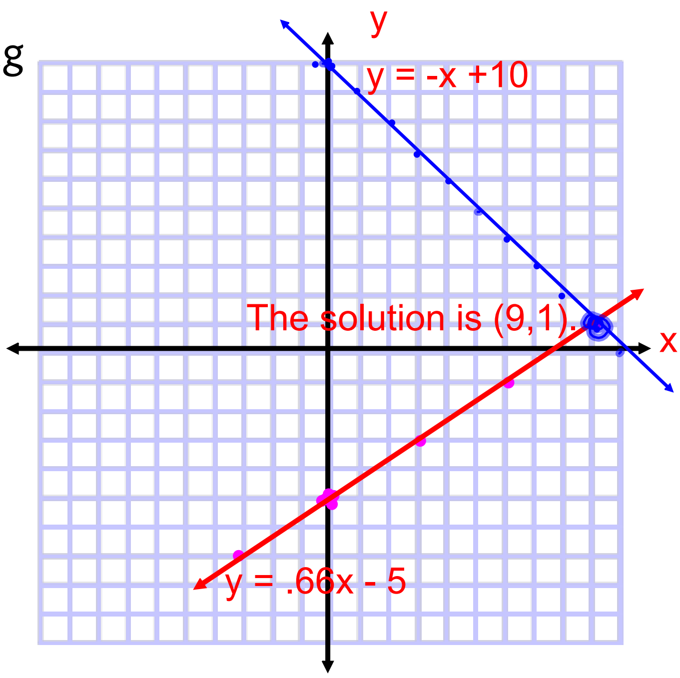
$y = \frac{2x}{3} - 5$

$y + x = 10$

$y + x = 10$

$\quad -x \qquad -x$

$y = -x + 10$




Solve the system algebraically.

$y = \frac{2x}{3} - 5$

$y + x = 10$ $\qquad y = -x + 10$

$-x + 10 = \frac{2x}{3} - 5$

Check your solution

**Study Guide Examples**

Forms of Linear Equations

STANDARD FORM

2x + 3y = 6

YOU CAN SOLVE FOR

Y and put in y=mx +b form

SLOPE -INTERCEPT FORM

y= mx + b    2x + 3y = 6

sovle for y:

2x + 3y = 6

-2x          -2x

$\frac{3y}{3} = \frac{-2x}{3} + \frac{6}{3}$

$y = \frac{-2x}{3} + 2$

slope = $\frac{-2}{3}$

y-intercept = 2

Point- slope form

$y - y_1 = m (x - x_1)$

y - 2 = 3(x + 6)

slope = 3

goes through

ordered pair (-6,2)

Name the form

1. 2x + 6y = 12

2. y - 2 =5(x +7)

3. y = 3x + 8

## Study Guide Examples

### STANDARD FORM

$ax^2 + bx + c = f(x)$

To find the roots or zeroes Of an quadratic equation — set equation = 0 to find roots

$x^2 + 8x + 15 = 0$

$(x + 5)(x + 3) = 0$ factor the quadratic equation

$x + 5 = 0$ or $x + 3 = 0$ set each factor = 0

$-5 \quad -5$ $\quad -3 \quad -3$ solve for x

$x = -5$ or $x = -3$ These are the roots.

$\{-5, -3\}$ This is in solution set form.

### VERTEX FORM - YOU WOULD SOLVE BY SQUARE ROOTS.

$h(t) = -16(t - 2)^2 + 144$ replace h(t) with 0

$0 = -16(t - 2)^2 + 144$ now solve for t by -144

$-144 = -16(t - 2)^2$ now divide by -16

$9 = (t - 2)^2$ Now take square root of both sides

$\pm\sqrt{9} = \sqrt{(t-2)^2}$ remember to take the positive and negative answer of square root

$\pm 3 = t - 2$

$t - 2 = 3$ or $t - 2 = -3$

$t = 5$ or $t = -1$ time can not be negative so in 5 sec. it would hit ground.

To find the roots, zeroes, solve for x or solution set

### Quadratic Equations

In standard form and it is not factorable. You would use quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve $2x^2 + 5x - 12 = 0$ $a = 2$ $b = 5$ $c = -12$

$X = \frac{-5 \pm \sqrt{5^2 - 4(2)(-12)}}{2(2)}$

$X = \frac{-5 \pm \sqrt{25 + 96}}{4}$

$X = \frac{-5 \pm \sqrt{121)}}{4}$

$X = \frac{-5 \pm 11}{4}$

$x = \frac{-5 + 11}{4}$ or $\frac{-5 - 11}{4}$

$x = \frac{6}{4}$ or $\frac{-16}{4}$

$x = 3/2$ or $-4$ This means the parabola crosses the x-axis at $(3/2, 0)$ and $(-4, 0)$.

Your purpose is to rewrite the equations in the form of a perfect square example $y = (x+3)^2 + 5$ This is called the **VERTEX** form of a quadratic equation. The vertex is (-3,5), which indicates that your lowest point (the minimum) is 5. (Because "a" is positive)

$y = x^2 + 6x - 2$

$y = x^2 + 6x \quad - 2$ Take half of the "b" so half of 6 is 3

$y = x^2 + 6x + 9 \quad -2 - 9$ then square it. Add 9 and subtract 9

$y = (x+3)^2 - 11$ Rewrite as a perfect square.

Thus your vertex is (-3,-11). And the lowest point would be -11, the parabola faces up, so it is the minimum.

Remember solve by completing the square is putting in vertex form then solving by square root method.

**Study Guide Examples**

## Different forms of the Quadratic Equations

**STANDARD FORM** $ax^2 + bx + c = y$

To find the vertex

Use the formula $x = \frac{-b}{2a}$ to find the x- coordinate of the vertex

Find the vertex of $y = -2x^2 - 8x - 3$ $a = -2$ $b = -8$ $c = -3$

$x = \frac{-(-8)}{2(-2)} = -2$ $x = -2$ is line of symmetry

Now take your "x" and substitute it back into equation to find y

$y = -2x^2 - 8x - 3$

$y = -2(-2)^2 - 8(-2) - 3$ substitute "x" into the equation

find y $-2(4) + 16 - 3 = 5$ so vertex is $(-2,5)$

THINK ABOUT FACTORING IN THIS FORM

**VERTEX FORM** $f(x) = -16(x - 2)^2 + 144$

Vertex = (2, 144)

This means the maximum height is 144 feet and it was reached in two seconds. (If the problem was about a projectile.)

If this was about profits in business. It would mean you maximize your profit.

**FACTORED FORM**

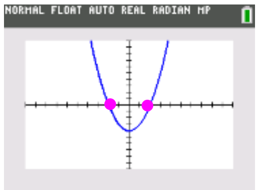
$f(x) = (x + 3)(x - 2)$

This form gives you the roots or zeroes of the function. This is where the projectile would hit the ground. Or in profits you are not making any money.

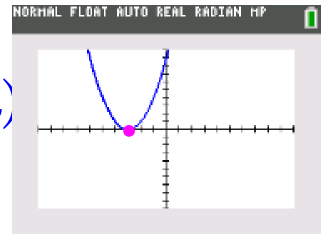
The roots here are {-3,2}.

# <u>Discriminant</u> tells you how many roots you have and if they are real.

$b^2 - 4ac > 0$ if greater than zero two real solutions. (two x-intercepts)

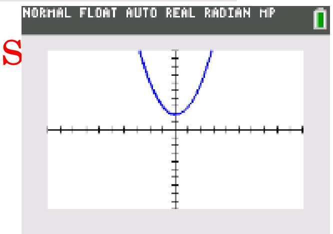



$b^2 - 4ac = 0$ one real solution (one x-intercept)




$b^2 - 4ac < 0$ No Real solutions

(no x-intercepts



# Study Guide Examples

Scientific Notation    Standard Form

$1.25 \times 10^5$    125,000

1. Showtheprocesstoexplainwhynegativeexponentsrelatetoscientiic notation.

Stepone: Rewriteinstandardform.

| **Scientiicnotationform** | **standardform** |
| --- | --- |
| $3x10^{-3}$ | . 003 |

Always written as a number
greater than 1 and less than 10.

Step2: Showthemathematicalprocesstoprovetheyareequivalent.(hint:substitute afractioninfor$10^{-3}$.)

$3x10^{-3}$

$$3 \times \frac{1}{10^3}$$

$$3 \times \frac{1}{1000} = \frac{3}{1000} = .003$$

## Solve equations using the distributive property

$$-5(4x - 2) = -2(3 + 6x)$$

$$-20x + 10 = -6 - 12x$$ distributive property

$$\underline{+12x \qquad\qquad +12x}$$ put the "x" variable on one side using additive inverse

$$-8x + 10 = -6$$

$$\underline{\quad -10 \qquad -10}$$

$$\frac{-8x}{-8} = \frac{-16}{-8}$$

$$x = 2$$

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**CHECK YOUR SOLUTION**

$$-5(4x - 2) = -2(3 + 6x)$$ Substitute 2 in for each x.

$$-5(4(2) - 2) = -2(3 + 6(2))$$

$$-5(8-2) = -2(3+12)$$

$$-5(6) = -2\,(15)$$

$$-30 = -30$$ Check.

## Study Guide Examples