Name: ___________________________________ Date: __________________

INVERSES OF LINEAR FUNCTIONS
COMMON CORE ALGEBRA II

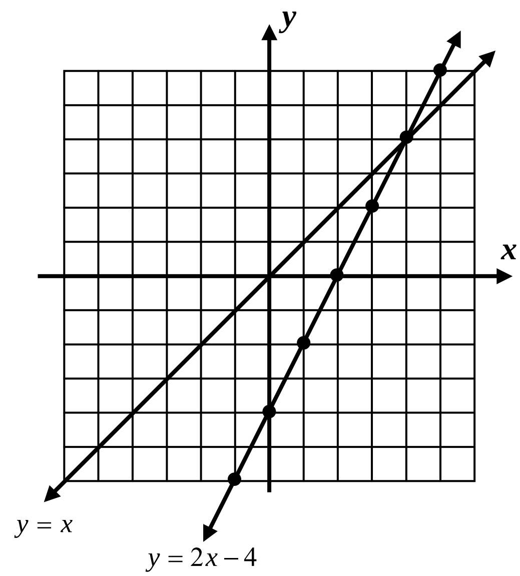
Recall that functions have inverses that are also functions if they are one-to-one. With the exception of horizontal lines, all linear functions are one-to-one and thus have inverses that are also functions. In this lesson we will investigate these inverses and how to find their equations.

***Exercise* #1:** On the grid below the linear function $y = 2x - 4$ is graphed along with the line $y = x$.

(a) How can you quickly tell that $y = 2x - 4$ is a one-to-one function?

(b) Graph the inverse of $y = 2x - 4$ on the same grid. Recall that this is easily done by switching the x and y coordinates of the original line.

(c) What can be said about the graphs of $y = 2x - 4$ and its inverse with respect to the line $y = x$?



(d) Find the equation of the inverse in $y = mx + b$ form.

(e) Find the equation of the inverse in $y = \dfrac{x+b}{a}$ form.

As we can see from part (e) in *Exercise* #1, inverses of linear functions include the inverse operations of the original function but in reverse order. This gives rise to a simple method of finding the equation of any inverse. **Simply switch the *x* and *y* variables in the original equation and solve for *y*.**

***Exercise* #2:** Which of the following represents the equation of the inverse of $y = 5x - 20$?

(1) $y = -\dfrac{1}{5}x + 20$

(3) $y = \dfrac{1}{5}x - 4$

(2) $y = \dfrac{1}{5}x - 20$

(4) $y = \dfrac{1}{5}x + 4$

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Although this is a simple enough procedure, certain problems can lead to common errors when solving for *y*. Care should be taken with each algebraic step.

***Exercise* #3:** Which of the following represents the inverse of the linear function $y = \frac{2}{3}x + 8$?

(1) $y = \frac{3}{2}x - 8$ (3) $y = -\frac{3}{2}x + 8$

(2) $y = \frac{3}{2}x - 12$ (4) $y = -\frac{3}{2}x + 12$

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***Exercise* #4:** What is the *y*-intercept of the inverse of $y = \frac{3}{5}x - 9$?

(1) $y = 15$ (3) $y = 9$

(2) $y = \frac{1}{9}$ (4) $y = -\frac{5}{3}$

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Sometimes we are asked to work with linear functions in their point-slope form. The method of finding the inverse and plotting it, though, do not change just because the linear equation is written in a different form.

***Exercise* #5:** Which of the following would be an equation for the inverse of $y + 6 = 4(x - 2)$?

(1) $y - 2 = \frac{1}{4}(x + 6)$ (3) $y - 6 = -4(x + 2)$

(2) $y - 2 = -\frac{1}{4}(x + 6)$ (4) $y + 2 = -4(x - 6)$

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***Exercise* #6:** Which of the following points lies on the graph of the inverse of $y - 8 = 5(x + 2)$? Explain your choice.

(1) $(8, -2)$ (3) $(-10, 40)$

(2) $(-8, 2)$ (4) $(-2, 8)$

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***Exercise* #7:** Which of the following linear functions would *not* have an inverse that is also a function? Explain how you made your choice.

(1) $y = x$ (3) $y = 2$

(2) $2y = x$ (4) $y = 5x - 1$

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Name: ______________________________________ Date: ___________________

INVERSES OF LINEAR FUNCTIONS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. The graph of a function and its inverse are always symmetric across which of the following lines?

(1) $y=0$ (3) $y=x$

(2) $x=0$ (4) $y=1$

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2. Which of the following represents the inverse of the linear function $y=3x-24$?

(1) $y=\frac{1}{3}x+8$ (3) $y=-\frac{1}{3}x+24$

(2) $y=-\frac{1}{3}x-8$ (4) $y=\frac{1}{3}x-\frac{1}{24}$

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3. If the *y*-intercept of a linear function is 8, then we know which of the following about its inverse?

(1) Its *y*-intercept is -8. (3) Its *y*-intercept is $\frac{1}{8}$.

(2) Its *x*-intercept is 8. (4) Its *x*-intercept is -8.

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4. If both were plotted, which of the following linear functions would be parallel to its inverse? Explain your thinking.

(1) $y=2x$ (3) $y=5x-1$

(2) $y=\frac{2}{3}x-4$ (4) $y=x+6$

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5. Which of the following represents the equation of the inverse of $y=\frac{4}{3}x+24$?

(1) $y=-\frac{4}{3}x-24$ (3) $y=\frac{3}{4}x-18$

(2) $y=-\frac{3}{4}x+18$ (4) $y=\frac{4}{3}x-24$

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6. Which of the following points lies on the inverse of $y+2=4(x-1)$?

(1) $(2,-1)$ (3) $\left(\frac{1}{2},1\right)$

(2) $(-1,2)$ (4) $(-2,1)$

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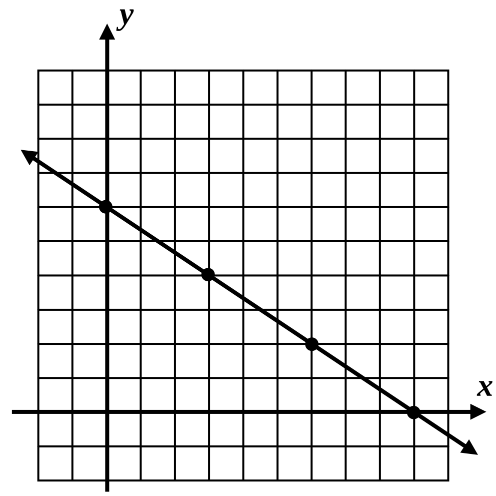
7. A linear function is graphed below. Answer the following questions based on this graph.

(a) Write the equation of this linear function in $y = mx + b$ form.



(b) Sketch a graph of the inverse of this function on the same grid.

(c) Write the equation of the inverse in $y = mx + b$ form.

(d) What is the intersection point of this line with its inverse?

APPLICATIONS

8. A car traveling at a constant speed of 58 miles per hour has a distance of *y*-miles from Poughkeepsie, NY, given by the equation $y = 58x + 24$, where *x* represents the time in hours that the car has been traveling.

(a) Find the equation of the inverse of this linear function in $y = \frac{x-a}{b}$ form.

(b) Evaluate the function you found in part (a) for an input of $x = 227$.

(c) Give a physical interpretation of the answer you found in part (b). Consider what the input and output of the inverse represent in order to answer this question.

REASONING

9. Given the general linear function $y = mx + b$, find an equation for its inverse in terms of *m* and *b*.