Name: ____________________________________ Date: ___________________

LOGARITHM LAWS
COMMON CORE ALGEBRA II

Logarithms have properties, just as exponents do, that are important to learn because they allow us to solve a variety of problems where logarithms are involved. Keep in mind that since logarithms give exponents, the laws that govern them should be similar to those that govern exponents. Below is a summary of these laws.

EXPONENT AND LOGARITHM LAWS

LAW	EXPONENT VERSION	LOGARITHM VERSION
Product	$b^x \cdot b^y = b^{x+y}$	$\log_b(x \cdot y) = \log_b x + \log_b y$
Quotient	$\dfrac{b^x}{b^y} = b^{x-y}$	$\log_b\left(\dfrac{x}{y}\right) = \log_b x - \log_b y$
Power	$\left(b^x\right)^y = b^{x \cdot y}$	$\log_b\left(x^y\right) = y \cdot \log_b x$

Exercise #1: Which of the following is equal to $\log_3(9x)$?

(1) $\log_3 2 + \log_3 x$ (3) $2 + \log_3 x$

(2) $2\log_3 x$ (4) $x + \log_3 2$

____

Exercise #2: The expression $\log\left(x^2/1000\right)$ can be written in equivalent form as

(1) $2\log x - 3$ (3) $2\log x - 6$

(2) $\log 2x - 3$ (4) $\log 2x - 6$

____

Exercise #3: If $a = \log 3$ and $b = \log 2$ then which of the following correctly expresses the value of $\log 12$ in terms of a and b?

(1) $a^2 + b$ (3) $2a + b$

(2) $a + b^2$ (4) $a + 2b$

____

Exercise #4: Which of the following is equivalent to $\log_2\left(\sqrt{x}/y^5\right)$?

(1) $\sqrt{\log_2 x} - 5\log_2 y$ (3) $\dfrac{1}{2}\log_2 x - 5\log_2 y$

(2) $2\log_2 x + 5\log_2 y$ (4) $2\log_2 x - 5\log_2 y$

____

***Exercise* #5:** The value of $\log_3\left(\sqrt{5}/27\right)$ is equal to

(1) $\dfrac{\log_3 5-6}{2}$ (3) $\dfrac{\log_3 5-3}{2}$

(2) $2\log_3 5+3$ (4) $2\log_3 5-3$ ________

***Exercise* #6:** If $f(x)=\log(x)$ and $g(x)=100x^3$ then $f(g(x))=$

(1) $100\log x$ (3) $300\log x$

(2) $6+\log x$ (4) $2+3\log x$

____

***Exercise* #7:** The logarithmic expression $\log_2\sqrt{32x^7}$ can be rewritten as

(1) $\sqrt{\log_2 35x}$ (3) $\sqrt{5+7\log_2 x}$

(2) $\dfrac{5+7\log_2 x}{2}$ (4) $\dfrac{35+\log_2 x}{2}$

____

***Exercise* #8:** If $\log 7=k$ then $\log(4900)$ can be written in terms of k as

(1) $2(k+1)$ (3) $2(k-3)$

(2) $2k-1$ (4) $2k+1$ ________

The logarithm laws are important for future study in mathematics and science. Being fluent with them is essential. Arguably, the most important of the three laws is the power law. In the next exercise, we will examine it more closely.

***Exercise* #9:** Consider the expression $\log_2\left(8^x\right)$.

(a) Using the third logarithm law (the Power Law), rewrite this as an equivalent product and simplify.

(b) Test the equivalency of these two expressions for $x=0, 1,$ and 2.

(c) Show that $\log_2\left(8^x\right)=3x$ by rewriting 8 as 2^3.

Name: ____________________________________ Date: __________________

LOGARITHM LAWS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Which of the following is not equivalent to $\log 36$?

   (1) $\log 2 + \log 18$ (3) $\log 30 + \log 6$

   (2) $2\log 6$ (4) $\log 4 + \log 9$ __________

2. The $\log_3 20$ can be written as

   (1) $2\log_3 2 + \log_3 5$ (3) $\log_3 15 + \log_3 5$

   (2) $2\log_3 10$ (4) $2\log_3 4 + 3\log_3 4$ __________

3. Which of the following is equivalent to $\log\left(x^3 / \sqrt[3]{y}\right)$?

   (1) $\log x - \log y$ (3) $3\log x - \frac{1}{3}\log y$

   (2) $9\log(x-y)$ (4) $\log(3x) - \log\left(y/3\right)$ __________

4. The difference $\log_2(3) - \log_2(12)$ is equal to

   (1) -2 (3) $\frac{1}{4}$

   (2) $-\frac{1}{2}$ (4) 4 __________

5. If $\log 5 = p$ and $\log 2 = q$ then $\log 200$ can be written in terms of p and q as

   (1) $4p + q$ (3) $2(p+q)$

   (2) $2p + 3q$ (4) $3p + 2q$

____

6. When rounded to the nearest hundredth, $\log_3 7 = 1.77$. Which of the following represents the value of $\log_3 63$ to the nearest *hundredth*? Hint: write 63 as a product involving 7.

(1) 3.54 (3) 3.77

(2) 8.77 (4) 15.93 ________

7. The expression $4\log x - \frac{1}{2}\log y + 3\log z$ can be rewritten equivalently as

(1) $\log\left(\frac{x^4z^3}{\sqrt{y}}\right)$ (3) $\log\left(\frac{x^4z^3}{2y}\right)$

(2) $\log\left(\frac{6xz}{y}\right)$ (4) $\log\left(\frac{6x^4z^3}{y}\right)$ ________

8. If $k = \log_2 3$ then $\log_2 48 =$

(1) $2k+3$ (3) $k+8$

(2) $3k+1$ (4) $k+4$ ________

9. If $g(x) = 8x^6$ and $f(x) = \log_4(2x)$ then $f(g(x)) = ?$

(1) $4\log_4 x + 1$ (3) $2(3\log_4 x + 1)$

(2) $3(\log_4 x + 2)$ (4) $6\log_4 x + 4$ ________

Reasoning

10. Consider the exponential equation $4^x = 30$.

(a) Between what two consecutive integers must the solution to this equation lie? Explain your reasoning.

(b) Write $\log\left(4^x\right)$ as an equivalent product using the third logarithm law.

(c) The solution to the original equation is $x = \frac{\log(30)}{\log(4)}$, can you see why based on (b)? Evaluate this expression and check to see it is correct.