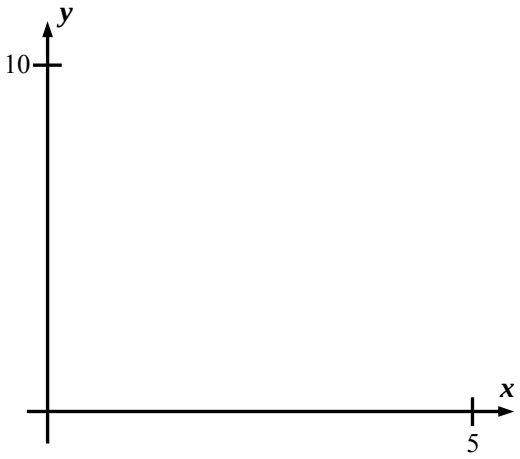
Name: _____ Date: _____

NEWTON'S LAW OF COOLING
COMMON CORE ALGEBRA II

The temperature of a cooling liquid in a large room with a steady temperature is a great example of a type of a **transformed** exponential function. We will explore this today to see how a simple exponential function can be used to build a more complex one.

***Exercise* #1:** Consider the decreasing exponential function $f(x)=8\left(\frac{1}{2}\right)^x$.

(a) Use your calculator to sketch the graph using the window indicated.

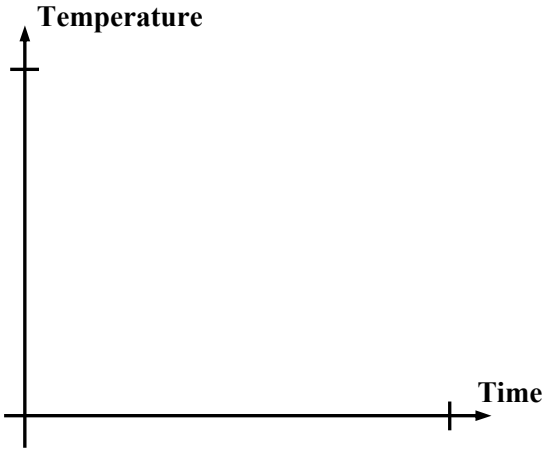


(b) Clearly the value of y gets smaller as x gets larger. Does it ever reach zero? Why or why not?

O.k. So, now let's try to model a liquid's temperature that is cooling in a large room.

***Exercise* #2:** Assume a liquid starts at a temperature of $200\ ^\circ\text{F}$ and begins to cool in a room that is at a steady temperature of $70\ ^\circ\text{F}$.

(a) Draw a rough sketch of what you believe the liquid's temperature function looks like as time increases.



(b) Based on your graph from (a) and your work in Exercise #1, why would an equation of the form $y=a(b)^t$ **not** model this cooling well (assuming that $0<b<1$)? How could we modify this equation to make it model the situation more realistically?

Exercise #3: Let's stick with the same cooling liquid that we had before, i.e. one that starts at $200\ ^\circ\text{F}$ and cools down in a room that is held at a constant temperature of $70\ ^\circ\text{F}$. We will now model this cooling Fahrenheit temperature using the equation $F(t) = a(b)^t + c$, where *a*, *b*, and *c* are all parameters (constants) in the model and *t* is time in minutes.

(a) Which of these constants is equal to 70 and why? Think about the last problem.

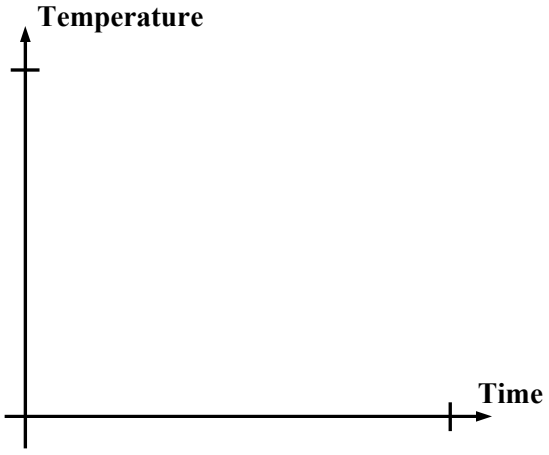
(b) None of these constants is equal to 200, but $T(0) = 200$. What constant does this let you solve for? Find its value.

(c) In order to find the value of *b* what additional information would we need?

(d) Determine the value of *b* if the temperature, after 5 minutes, is $153\ ^\circ\text{F}$. Round to the nearest *hundredth*.

(e) What is the temperature of the liquid after half an hour? An hour? Two hours?

(f) Using your calculator, sketch the graph of the liquid's temperature. Decide on an appropriate window and label it on your axes.



(g) Algebraically determine, to the nearest tenth of a minute, when the temperature reaches $100\ ^\circ\text{F}$.

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NEWTON'S LAW OF COOLING
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FLUENCY

1. For the function $f(x)=a(b)^x$, where $0<b<1$, what value does the function's output approach as x gets very large?

2. For the function $g(x)=a(b)^x+k$, where $0<b<1$, what value does the function's output approach as x gets very large?

3. Given $f(t)=a(b)^t+c$, which of the following represents the y-intercept of this function? Show how you arrived at your choice.

   (1) a (3) c

   (2) $a+c$ (4) $b+c$

_____

APPLICATIONS

4. A liquid starts at an initial temperature of 175 °C and cools down in a room held at a constant temperature of 16 °C. It's temperature can be modeled, as a function of time cooling, by the equation $y=a(b)^x+c$. Which of the following statements is true?

   (1) $a=159$ and $c=16$ (3) $a=175$ and $c=16$

   (2) $a=16$ and $c=159$ (4) $a=16$ and $c=175$

_____

5. A cooling liquid has a temperature given by the function $T(m)=132(.83)^m+40$, where m is the number of minutes it has been cooling. Which of the following temperatures did the liquid start at?

   (1) 40 (3) 172

   (2) 92 (4) 132

_____

6. A liquid starts at a temperature of 190 °F and cools down in a room held at a constant 65 °F. After 10 minutes of cooling, it is at a temperature of 92 °F. The Fahrenheit temperature, *F*, can be modeled as a function of time in minutes, *t*, by the equation:

$$F(t) = a(b)^t + c$$

(a) Determine the values of the parameters *a*, *b*, and *c*. Round the value of *b* to the nearest *hundredth*. State the equation of your final model. Show the work that leads to each of your answers.

(b) Algebraically, determine the number of minutes it will take for the temperature to reach 70 °F. Round to the nearest *tenth* of a minute.

REASONING

7. When we model the temperature of a cooling liquid using the equation $T = a(b)^x + c$, we have learned that the value of *c* represents the steady temperature of the room. The quantity $a(b)^x$ does model something physically. Can you determine what it is?

8. **A Warming Liquid** - A liquid is taken out of a refrigerator and placed in a warmer room, where its temperature, in F, increases over time. It can be modeled using the equation $T(m) = 74 - 39(0.87)^m$.

(a) What temperature did the liquid start at? Show the work that leads to your answer.

(b) What is the temperature of the room?