Name: ____________________________________ Date: _________________

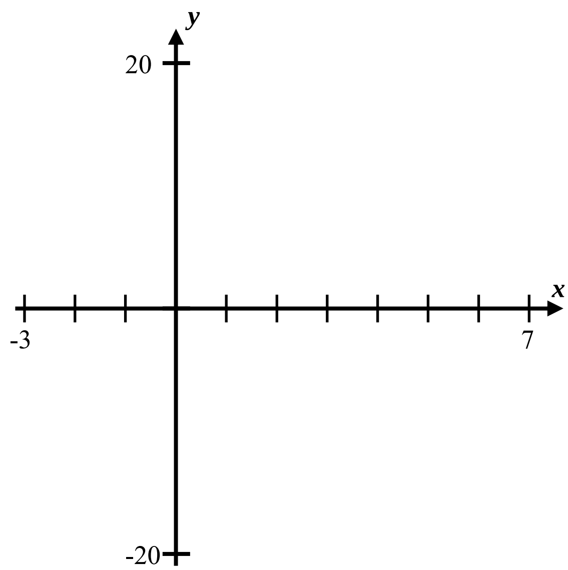
VERTICAL STRETCHING OF FUNCTIONS
COMMON CORE ALGEBRA II

We have now seen how to shift and reflect functions, specifically in the context of parabolas. In this lesson we will see how to stretch or compress a function in the vertical direction. The first exercise will illustrate this concept with three related parabolas.

***Exercise* #1:** Consider the quadratic function $f(x)=x^2-4x-5$. The quadratic functions g and h are defined by the formulas $g(x)=2f(x)$ and $h(x)=\frac{1}{2}f(x)$.

(a) Determine formulas for both g and h in simplest trinomial form.

(b) Using your calculator, sketch and label each curve on the set of axes below. Use the window indicated by the axes.



(c) Using the **MINIMUM** command on your calculator, determine the minimum value for each function.

$f_{\min}=$

$g_{\min}=$

$h_{\min}=$

(d) What points did not vary when f was vertically dilated by factors of 2 and $1/2$? Explain why this happened.

VERTICAL DILATIONS OF FUNCTIONS

The function $h(x)=k\cdot f(x)$ represents a vertical stretch of the function $f(x)$ if $k>1$ and a vertical compression of the function $f(x)$ if $0<k<1$.

Exercise #2: If the point $(-3, 12)$ lies on the graph of the function $y = f(x)$, which of the following points *must* lie on the graph of $y = 3f(x)$?

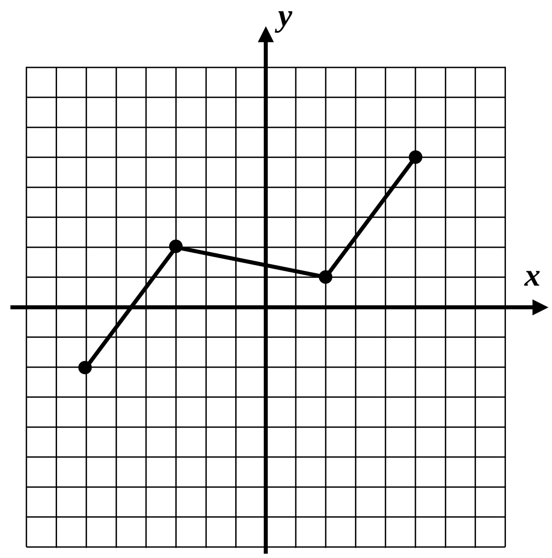
(1) $(-9, 36)$ (3) $(-3, 4)$

(2) $(-3, 36)$ (4) $(-9, 12)$

____

Exercise #3: The graph of $y = f(x)$ is shown below. Consider the function $y = g(x)$ defined by $g(x) = 2f(x) - 3$.

(a) What two transformations have occurred to the graph of *f* in order to produce the graph of *g*? Specify both the transformations and their order.



(b) Graph and label $y = g(x)$

Exercise #4: The function $h(x)$ has a range given by the interval $[2, 10]$. The function $f(x)$ is defined by $f(x) = \frac{3}{2}h(x) + 8$. Which of the following gives the range of $f(x)$?

(1) $[11, 23]$ (3) $[15, 27]$

(2) $[8, 12]$ (4) $[6, 32]$

____

Exercise #5: If the quadratic function $g(x)$ has a *y*-intercept of 12, which of the following is true about the function $h(x) = 3g(x) - 5$?

(1) It has a *y*-intercept of -5.
(2) It has a *y*-intercept of 21.
(3) It has a *y*-intercept of -15.
(4) It has a *y*-intercept of 31.

____

Name: ____________________________________ Date: _________________

VERTICAL STRETCHING AND COMPRESSING FUNCTIONS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. If the point $(-6,10)$ lies on the graph of $y=f(x)$ then which of the following points *must* lie on the graph of $y=\frac{1}{2}f(x)$?

(1) $(-3,5)$ (3) $(-6,5)$

(2) $(-3,10)$ (4) $(-12,20)$ __________

2. If the function $h(x)$ is defined as vertical stretch by a factor of 2 followed by a reflection in the *x*-axis of the function $f(x)$ then $h(x)=$

(1) $2f(-x)$ (3) $-\frac{1}{2}f(x)$

(2) $\frac{1}{2}f(x)$ (4) $-2f(x)$ __________

3. If the graph of $y=x^2$ is compressed by a factor of 3 in the *y*-direction and then shifted 4 units down, the resulting graph would have an equation of

(1) $y=\frac{1}{3}x^2-4$ (3) $y=-4x^2-3$

(2) $y=-3x^2-4$ (4) $y=-\frac{1}{3}x^2+4$ __________

4. The quadratic function $f(x)$ has a turning point at $(-3,6)$. The quadratic $y=\frac{2}{3}f(x)+3$ would have a turning point of

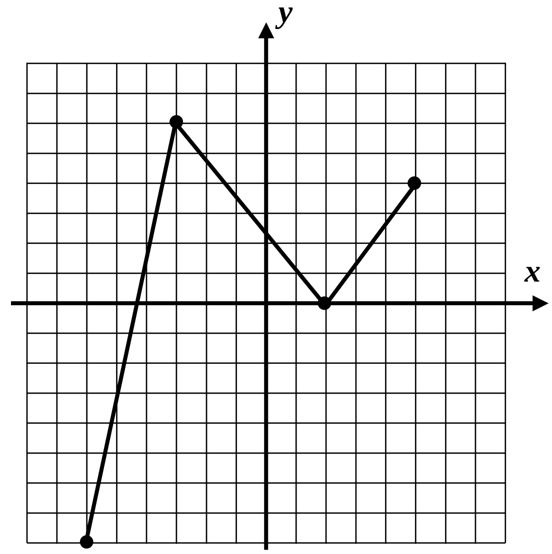
(1) $(-2,9)$ (3) $(-3,7)$

(2) $(1,7)$ (4) $(-1,9)$ __________

5. The function $g(x)$ is defined by $g(x)=-5f(x)+4$. What three transformations have occurred to the graph of *f* to produce the graph of *g*? Specify both the transformations and their order.

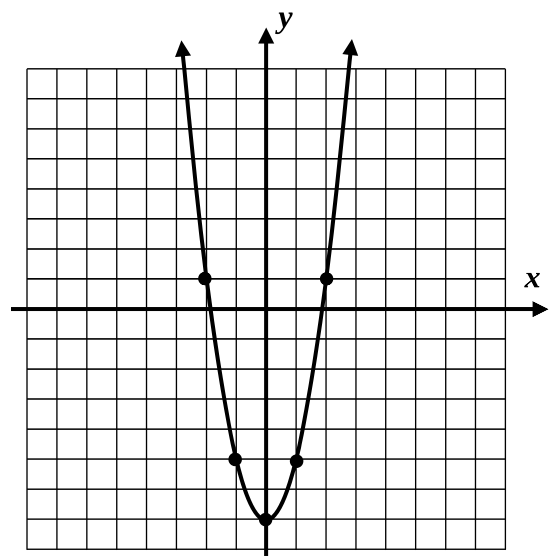
6. The graph of $y = h(x)$ is shown below. The function $f(x)$ is defined by $f(x) = -\frac{1}{2}h(x) + 3$.

(a) What three transformations have occurred to the graph of h to produce the graph of f? Specify the transformations and the order they occurred in.



(b) Graph and label the function $f(x)$ on the grid below that contains $h(x)$.

7. A parabola is shown graphed to the right that is a transformation of $y = x^2$. The transformation includes a vertical stretch and a vertical shift. What are the stretch and shift? Based on your answer, write an equation for this parabola.



Reasoning

8. The function $h(x)$ is defined by the equation $h(x) = 4f(x) - 12$. Determine two different *sets* of transformations that could produce the graph of $h(x)$ from the graph of $f(x)$. For each, specify two transformations and the order in which they occurred. As a hint, write $h(x)$ in its factored form.