

## JUSTIFYING THE STEPS IN SOLVING AN EQUATION (EVEN STRANGE ONES)

### COMMON CORE ALGEBRA I



Now that we have reviewed how to solve linear equations involving variables on both sides, it is time to take it to another level. The Common Core asks us not only to know the how but also the why. Generally, we justify the steps we take in solving linear equations using the commutative, associative, and distributive properties of real numbers along with the following two **properties of equality**.

#### PROPERTIES OF EQUALITY

- (1) **ADDITIVE PROPERTY OF EQUALITY:** If  $a = b$  then  $a + c = b + c$  (you can add or subtract the same quantity from both sides and retain the equality).
- (2) **MULTIPLICATIVE PROPERTY OF EQUALITY:** If  $a = b$  then  $c \cdot a = c \cdot b$  (you can multiply or divide by the same quantity on both sides and retain the equality).

**Exercise #1:** Consider the equation  $2x + 9 = 21$ . The steps in solving the equation are shown below. Justify each step.

Step 1:  $2x + 9 - 9 = 21 - 9$

Justification: \_\_\_\_\_

Step 2:  $\frac{1}{2} \cdot 2x = \frac{1}{2} \cdot 12$   
 $x = 6$

Justification: \_\_\_\_\_

O.k. That was a reasonably simple two-step equation. Now, let's go for the full experience.

**Exercise #2:** Consider the equation  $3(x + 2) - 2(x + 7) = 4x + 7$ . As in the last problem, each step of the solution is shown. Justify each with either a property of equality or a property of real numbers.

Step 1:  $3x + 6 - 2x - 14 = 4x + 7$

Justification: \_\_\_\_\_

Step 2:  $3x + -2x + 6 + -14 = 4x + 7$

Justification: \_\_\_\_\_

Step 3:  $x(3 - 2) + -8 = 4x + 7$   
 $x - 8 = 4x + 7$

Justification: \_\_\_\_\_

Step 4:  $x - 8 - 4x + 8 = 4x + 7 - 4x + 8$

Justification: \_\_\_\_\_

Step 5:  $x - 4x - 8 + 8 = 4x - 4x + 7 + 8$

Justification: \_\_\_\_\_

Step 6:  $x(1 - 4) = 15$   
 $-3x = 15$

Justification: \_\_\_\_\_

Step 7:  $\frac{-3x}{-3} = \frac{15}{-3}$   
 $x = -5$

Justification: \_\_\_\_\_



Strange things can sometimes happen when you solve an equation. Even if every step is justified, results can turn out confusing.

**Exercise #3:** Consider the equation  $5x - 3(x + 1) = 2(x + 4)$ .

(a) Fill in this missing justifications in the solution of this equation below.

Step #1:  $5x - 3x - 3 = 2x + 8$

Justification: The Distributive Property

Step #2:  $5x - 3x - 3 - 8 = 2x + 8 - 8$

Justification: \_\_\_\_\_

Step #3:  $x(5 - 3) - 11 = 2x$

Justification: \_\_\_\_\_

$2x - 11 = 2x$

Step #4:  $2x - 11 - 2x = 2x - 2x$

Justification: Additive Property of Equality

Step #5:  $2x - 2x - 11 = 0$

Justification: \_\_\_\_\_

$-11 = 0$

(b) The final line of this set of manipulations is a very strange statement:  $-11 = 0$ . Is this a true statement? Could any value of  $x$  make it a true statement?

(c) What do you think this tells you about the solutions to this equation (i.e. the values of  $x$  that make it true)?

**Exercise #4:** Consider the equation  $7x + 2(x + 5) = 9x + 10$ .

(a) Show that  $x = -5$  and  $x = 2$  are both solutions to this equation.

(b) Solve this equation by manipulating each side of the equation as we did above. What does its final “strange” result tell you?

(c) Test your conclusion in (b) by picking a random integer (or really any number) and showing that it is a solution to the equation.



Name: \_\_\_\_\_

Date: \_\_\_\_\_

## JUSTIFYING STEPS IN SOLVING AN EQUATIONS COMMON CORE ALGEBRA I HOMEWORK

### FLUENCY

1. Which property justifies the second line in the following solution?

(1) Multiplicative Property of Equality      (3) Distributive

$$3x + 2 = 8$$

(2) Associative

(4) Additive Property of Equality

$$3x + 2 - 2 = 8 - 2$$

2. What is the solution to the following equation? Show all work.

$$3(x + 2) - 2x = -2(x - 3) + 3x$$

(1) No Solutions

(3)  $x = 2$

(2) Infinite Solutions

(4)  $x = -3$

3. Give a property of real numbers (associative, commutative, or distributive) or a property of equality (addition or multiplication) that justifies each step in the following equation:

$$3x + 1 + 2x - 7 = x + 22$$

(1)  $3x + 2x + 1 - 7 = x + 22$

(1) \_\_\_\_\_

(2)  $x(3 + 2) - 6 = x + 22$

(2) \_\_\_\_\_

$$5x - 6 = x + 22$$

(3)  $5x - 6 + 6 = x + 22 + 6$

(3) \_\_\_\_\_

$$5x = x + 28$$

(4)  $5x - x = x + 28 - x$

(4) \_\_\_\_\_

(5)  $x(5 - 1) = 28$

(5) \_\_\_\_\_

$$4x = 28$$

(6)  $\frac{1}{4} \cdot 4x = \frac{1}{4} \cdot 28$

(6) \_\_\_\_\_

$$x = 7$$



## APPLICATIONS

4. Antonio just signed up for new phone plan and is comparing his fees to that of his friend Marcus. They both create equations so that they could compare their fees with each other.

Antonio's plan: Monthly cost =  $3(.75m + 10) + 2.50m - 15$  where  $m$  is the number of minutes used

Marcus's Plan: Monthly cost =  $2(1.75m + 12.50) - .75m + 4$  where  $m$  is the number of minutes used

- (a) By setting their monthly cost equal, decide after how many minutes the two plans will cost the same.

- (b) Antonio compares his plan to another friend, Brielle's. Given that both Antonio and Brielle will only be charged for full minutes, is there an amount of time when their two plans cost the same? Explain.

Brielle's plan: Monthly cost =  $2(1.50m + 12) + m - 4$  where  $m$  is the number of minutes used

## REASONING

5. Without solving the following equations decide whether there will be **one solution**, **no solutions** or **infinitely many solutions** and explain why you think so.

$3x - 2 = 3x - 2$	$2x - 4 = 2x - 7$	$3x - 5 = 6x - 5$
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