Name: ______________________________ Date: ______________

EXPONENTIAL GROWTH AND DECAY
COMMON CORE ALGEBRA I

There are many things in the real world that grow faster as they grow larger or decrease slower as they get smaller. These types of phenomena, loosely speaking, are known as **exponential growth (and decay in the case of decreasing).** In today's lesson, we will look at both growth and decay.

***Exercise* #1:** The number of people who have heard a rumor often grows exponentially. Consider a rumor that starts with 3 people and where the number of people who have heard it doubles each day that it spreads.

(a) Why does it make sense that the number of people who have heard a rumor would grow exponentially?

(b) Fill in the table below for the number of people, N, who knew the rumor after it has spread a certain number of days, d.

d	0	1	2	3	4	5
N	3	6				

***Exercise* #2:** We'd like to determine the number of people who know the rumor after 20 days, but to do that, we need to develop a formula to predict N (the number knowing the rumor) if we know d (the number of days it has been spreading).

(a) For the following number of days, fill in how you calculated your values based on extended products using the number 2.

$d=0 \quad N=3$

$d=1 \quad N=3\cdot 2$

$d=2 \quad N=(3\cdot 2)\cdot 2=3\cdot 2\cdot 2$

$d=3 \quad N=$

$d=4 \quad N=$

$d=5 \quad N=$

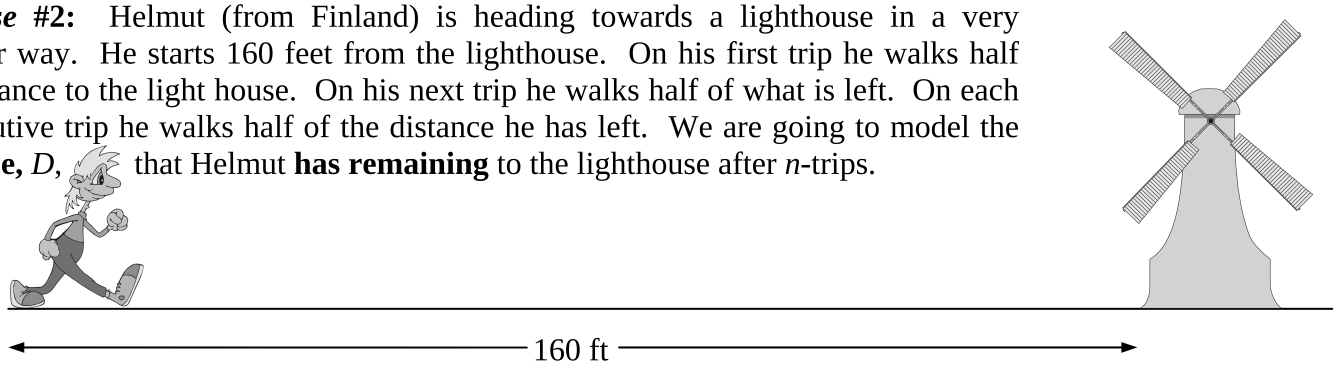
(b) Using the pattern you developed in (a), write a formula giving the number of people who know the rumor, N, if you know the number of days, d, it has been spreading.

(c) How many people would know the rumor after 20 days?

(d) Exponential growth can be very fast. Assuming our equation from (b) holds, how many days will it take for the number of people knowing the rumor to surpass the population of the United States, which is approximately 315 million people? Show calculations that support your answer.

Let's now look at developing a fairly simple exponential decay problem.

***Exercise* #2:** Helmut (from Finland) is heading towards a lighthouse in a very peculiar way. He starts 160 feet from the lighthouse. On his first trip he walks half the distance to the light house. On his next trip he walks half of what is left. On each consecutive trip he walks half of the distance he has left. We are going to model the **distance,** D, that Helmut **has remaining** to the lighthouse after n-trips.



(a) Fill in the table below for the amount of distance that Helmut has left after n-trips.

n	0	1	2	3	4
D (ft)	160	80			

(b) Each entry in the table could be found by **multiplying** the previous by what number? This is important because we **always** want to think about exponential functions in terms of **multiplying**

(c) Like in Exercise #2(a), we want to see this process as repeated multiplication by $\frac{1}{2}$. Fill out each of the following pattern:

$n=0 \quad D=160$

$n=1 \quad D=160\cdot\frac{1}{2}=80$

$n=2 \quad D=80\cdot\frac{1}{2}=\left(160\cdot\frac{1}{2}\right)\cdot\frac{1}{2}=$

$n=3 \quad D=$

$n=4 \quad D=$

(d) Based on (c), give a formula that predicts the distance, D, that Helmut has left after n-trips.

(e) How far is Helmut from the windmill after 6 trips? Provide a calculation that justifies your answer and don't forget those units!

(f) Helmut believes he will reach the windmill after 10 trips. Is he correct?

(g) Explain why Helmut will never reach the windmill?

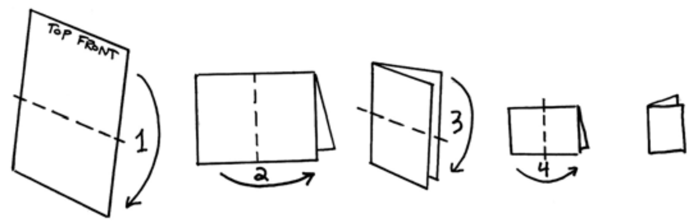
(h) Why is the domain of this function only the whole numbers, i.e. $\{0, 1, 2, 3, ...\}$?

Name: ____________________________________ Date: __________________

EXPONENTIAL GROWTH AND DECAY
COMMON CORE ALGEBRA I HOMEWORK

APPLICATIONS

1. A piece of paper is 0.01 centimeters (cm) thick. When you fold it once, it becomes 0.02 centimeters thick. If you fold it again, it doubles again to 0.04 centimeters thick. Each fold doubles the thickness of the paper.



(a) How thick is the paper after:

4 Folds:

5 Folds:

(b) For each of the following number of folds, *f*, show how you can calculate the thickness, *T*, based on repeatedly multiplying by 2.

$f = 0 \quad T = 0.01$

$f = 1 \quad T = 0.01(2)^1 = 0.02$

$f = 2 \quad T = 0.02(2) = 0.01(2)(2) = 0.01(2)^2$

$f = 3 \quad T =$

$f = 4 \quad T =$

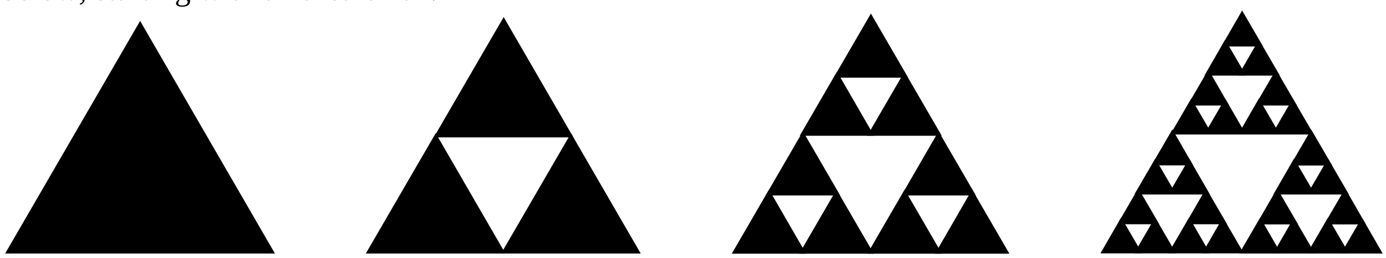
(c) Determine a formula, based on (b), for the thickness, *T*, based on the number of folds, *f*.

(d) How thick would the paper be if $f = 10$? Use proper units

(e) If there are 100 centimeters in a meter, how many *meters* thick is the paper after 20 folds? Show the work that leads to your answer.

(d) If there are 1000 meters in a kilometer and the Moon is 384,000 kilometers away from the Earth, will the paper reach the Moon after 40 folds? Show the calculations that lead to your answer.

2. The Sierpinski Triangle is a type of progression where an equilateral triangle has $\frac{1}{4}$ of its area removed to create a new shape. Then $\frac{1}{4}$ of its remaining area is taken away. A series of these triangles is shown below, starting with an area of 64.



$A_0 = 64$ $\qquad$ $A_1 = 48$ $\qquad$ $A_2 = ?$ $\qquad$ $A_3 = ?$

(a) If we remove $\frac{1}{4}$ of the area, what fraction of the area remains?

(b) Multiply 64 by the fraction you found in (a). What value do you get?

(c) Find the areas of the third and fourth pictures above by multiplying by the fraction you found in (a).

(d) Find a formula for the area, A, that remains after n removals of area.

(e) How much area remains after 10 removals?

(f) How much area remains after 20 removals?

(g) Will the area ever reach zero? Explain your thinking.

(h) If the Sierpinski triangle to the right had an original area of 15 square centimeters before any area was removed, what is the area of the figure shown to the right to the nearest tenth of a square centimeter? Show the calculation that leads to your answer.