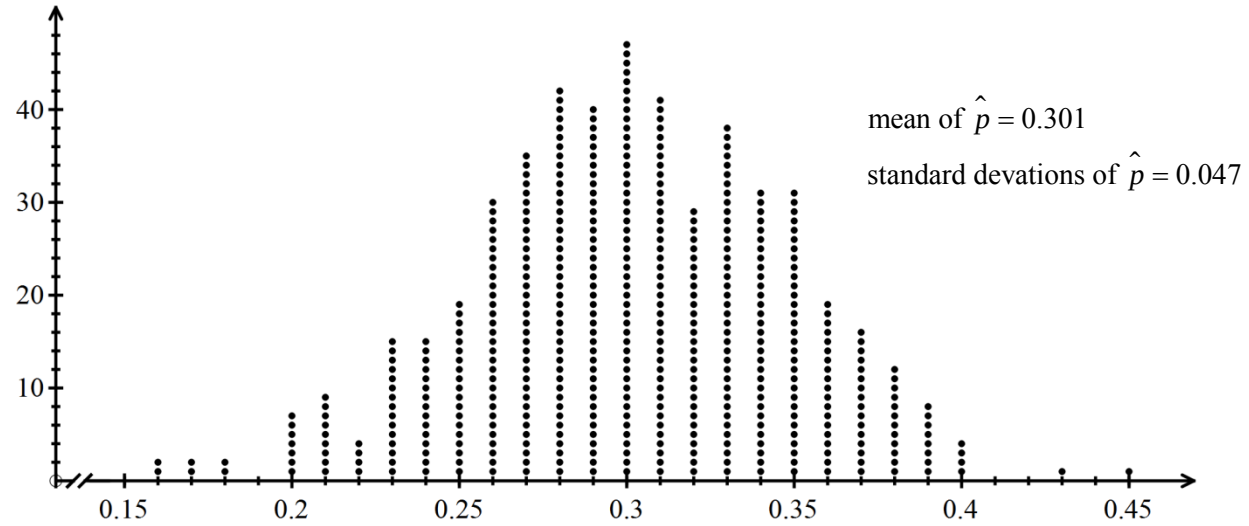
Name: _____ Date: _____

THE DISTRIBUTION OF SAMPLE PROPORTIONS
COMMON CORE ALGEBRA II

In the last lesson we saw how the **distribution of sample means** was **normal**. The **Central Limit Theorem** allowed us to find the standard deviation of these sample means. In this lesson, we will look at the same phenomena with sample proportions.

***Exercise* #1:** A simulation of samples taken from a population with a proportion, p, of 0.3 was created. The simulation had a sample size of 100 and 500 simulations were run. The sample proportions, $\hat{p}$, were calculated and their distribution is shown below:



(a) What does the shape of this distribution resemble? Explain.

(b) What is true about the mean of the sample proportions?

The distribution of sample proportions is governed by a very similar phenomena to the distribution of sample means via **The Central Limit Theorem**. The characteristics of the distribution are given below.

THE DISTRIBUTION OF SAMPLE PROPORTIONS

The distribution of sample proportions, $\hat{p}$, from a population with a proportion p and a sample size of n will:

1. Approximate a normal distribution
2. Have a mean of the population proportion, p.
3. Have a standard deviation given by $\sqrt{\frac{p(1-p)}{n}}$

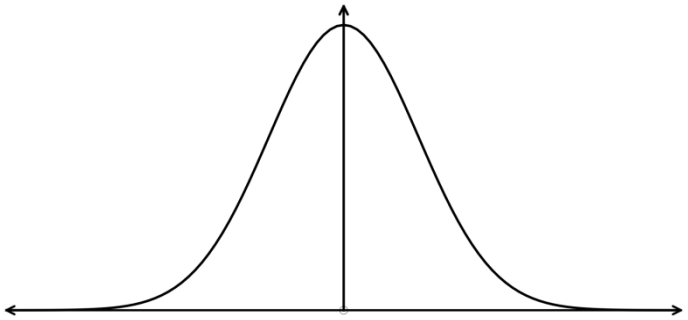
***Exercise* #2:** Does the standard deviation from the simulation agree with that predicted with the above formula?

Since sample proportions will be **normally distributed**, we can perform calculations similar to those done for **sample means**. In other words, we can see how likely a range of sample proportions would be given a particular population proportion.

***Exercise* #3:** Suppose the percent of seniors in high school that own a cell phone is 82%. If a random sample of 50 high school seniors was taken, determine the following:

(a) The standard deviation of sample proportions for this population proportion given this sample size. Show the calculation that leads to your answer.

(b) The probability that the sample proportion will be within 3% of the 82% proportion. Illustrate your work on the general normal curve below.



(c) Find each of the following probabilities. Round each answer to the nearest tenth of a percent.

(i) the sample proportion will be less than 75%

(ii) the sample proportion will be greater than 95%

***Exercise* #4:** Political polls can be tricky. Let's say that 47% of the public will vote for a particular candidate in the upcoming election. If a newspaper takes a random poll of 200 voters, what is the probability that this sample will have a proportion larger than 50%, thus predicting a win for this candidate?

Name: _____ Date: _____

THE DISTRIBUTION OF SAMPLE PROPORTIONS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

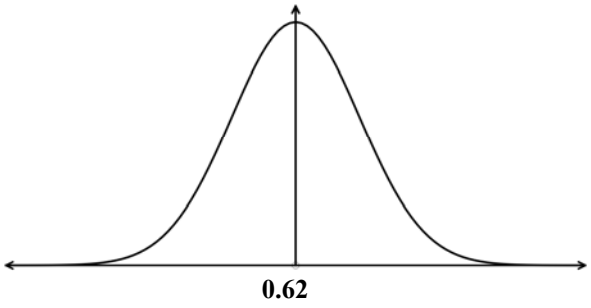
1. For each of the following population proportions, p, find the standard deviation of the sample proportions, $\hat{p}$, given the sample size n. Show your calculation. Round to three decimal place accuracy (nearest thousandth).

   (a) $p = 0.34$ and $n = 50$ (b) $p = 0.5$ and $n = 400$ (c) $p = 0.25$ and $n = 100$

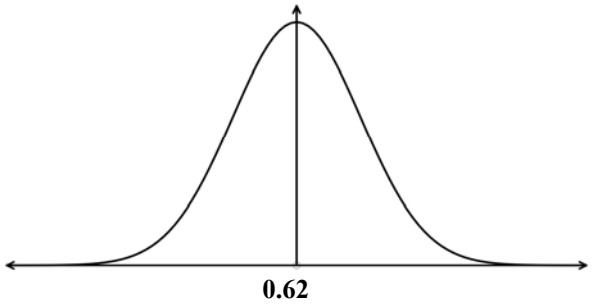
   $\sigma_{\hat{p}} =$ $\sigma_{\hat{p}} =$ $\sigma_{\hat{p}} =$

2. A population has a proportion of 0.62. A sample of size 40 was taken from this population. Determine the following probabilities. Illustrate each on the normal curve shown below each part.

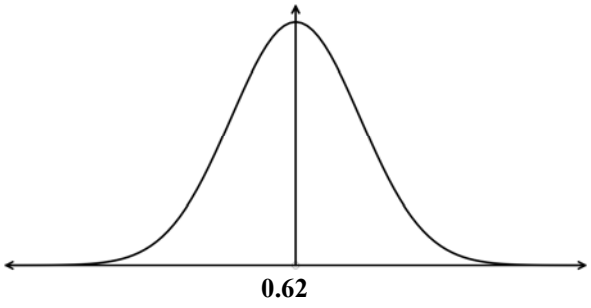
   (a) The probability the sample has a proportion between 0.5 and 0.7.

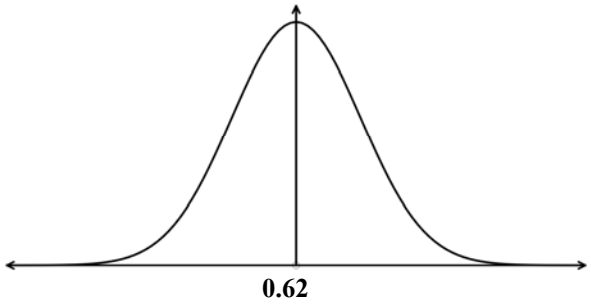
   (b) The probability the sample has a proportion within 5% of the population proportion.

   (c) The probability that the sample has a proportion less than 0.50.

   (d) The probability that the sample has a proportion greater than 0.80

APPLICATIONS

3. A candidate for political office has support from 40% of the public. If a random sample of 100 members of the public was taken, which of the following is closest to the probability that the sample had a proportion of 50% or greater support for this candidate?

   (1) 2% (3) 14%

   (2) 7% (4) 24%

4. A school will offer pizza on Friday's if at least 30% of the students will buy it. A sample of 50 students are asked if they would buy pizza on Friday and 10 respond that they would.

   (a) Determine the probability of getting a sample of this size with the proportion or lower given a population with a proportion of 0.30.

   (b) Should the school offer pizza on Fridays? Explain your choice by reflecting on what your answer from part (a) tells you.

5. If a 45% of a population likes a particular soda, then what range below shows all sample proportions within two standard deviations of the population proportion if the samples have a size of 70?

   (1) 38% to 52% (3) 33% to 57%

   (2) 20% to 70% (4) 28% to 62%

REASONING

6. Juniors at a high school own internet enabled devices at a rate of 71%. If 52 freshmen were sampled and only 58% of them owned internet enabled devices, is this enough proof to state that freshmen own these devices at a lower rate than juniors? Explain based on probability.