Date:

MARGIN OF ERROR Common Core Algebra II

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In inferential statistics we attempt to infer characteristics about a **population** from sample statistics. But, these inferences always have a degree of uncertainty in them. Many times, this uncertainty itself is quantified by a **margin of error**, which is a measurement of how accurate we believe our sample statistic to be relative to the population.

Exercise #1: A recent poll found that 36% of all respondents would vote for Candidate A in an election. The poll reported a **margin of error** of 4%. Give an interpretation of what this margin of error means in terms of the 36% support for Candidate A.

The **margin of error** allows us to give a range of values that are reasonable for a population parameter based on a sample statistic. We will only consider what is known as the **95% margin of error**. Although, there are many others based on **confidence level**.

THE 95% MARGIN OF ERROR

The **uncertainty level** that would guarantee a 95% chance that the population parameter falls within a certain range of values.

Exercise #2: In any normal distribution, how much of the data falls within two standard deviations of the mean? Do this quickly using your calculator.

Since roughly 95% of all normally distributed data fall within two standard deviations of the mean, we use two **standard deviations** to develop the **margin of error**. The next exercise will illustrate how this is done for a population proportion.

Exercise #3: In a sample survey, 50 people were randomly sampled about their favorite soda. If 38% of them listed Soda A as their favorite, then answer the following questions.

- (a) Based on a proportion of p = 0.38, what is the standard deviation of sample proportions of this size, $\sigma_{\hat{p}}$?
- (b) What is the margin of error for this survey? What would be an acceptable range of values for the population proportion?





Margins of error are commonly found in surveys and other types of studies that are trying to determine a population proportion based on a sample proportion.

Exercise #4: In a poll of 500 potential voters, Candidate A led Candidate B by a 46% to 39% margin. Could these two candidates actually be tied in the population as a whole? Justify your response.

The margin of error can also be useful in working with sample means, which will also have a normal distribution.

Exercise **#5:** If a sample of three dozen jumbo eggs had a mean weight of 69.7 grams and a sample standard deviation of 3.2 grams, answer the following question.

- (a) What would be a reasonable estimate for the standard deviation of the sample means, i.e. $\sigma_{\bar{x}}$?
- (b) Based on (a), what would the margin of error for the population mean weight be?

(c) Jumbo eggs are considered to be eggs with weights at or above 70 grams. Is this within the margin of error for this sample? Explain.

Exercise #6: In 2015, a survey of fifty 20 to 24 year olds was done to determine their mean weekly earnings. The survey found a sample mean of \$495 with a standard deviation of \$48. If the *World Almanac* reported the 2014 mean weekly earnings of this age range to be \$472, do the results of this survey conclusively imply an increase in the mean weekly earning from 2014 to 2015? Explain.





MARGIN OF ERROR Common Core Algebra II Homework

FLUENCY

- 1. Use the two standard deviation rule to determine the margin of error, to the nearest thousandth, for each of the following proportions with the given sample size. Show the work that leads to your answer.
 - (a) p = 0.35 and n = 40 (b) p = 0.72 and n = 100

(c) p = 0.5 and n = 50 (d) p = 0.25 and n = 30

2. Assuming a population characteristic has a standard deviation, σ , of 38. Calculate the margin of error on the population mean given a sample of each of the sizes given below. Show how you calculate your answer.

(a) n = 30 (b) n = 100

(c) n = 1000

- (d) Generally, as the sample size increases, what happens to the margin of error? Why do you think this occurs?
- (e) What is the minimum sample size needed for the margin of error to be 2 or less? Show or explain how you determined your solution.





APPLICATIONS

- 3. In an election poll, 200 people were surveyed and 45% expressed their likelihood to vote for a particular candidate. The margin of error on this estimated support is closest to
 - (1) 2% (3) 7%
 - (2) 3% (4) 12%
- 4. In a survey of 125 students, 64% of them preferred to start the school day an hour later.
 - (a) Calculate the margin of error for this survey to the nearest tenth of a percent. Show your work.
 - (b) The administration of the school will only continue to study the feasibility of starting the day later if there is at least 70% support amongst students. Does this fall within the margin of error of the survey?

- 5. A consumer group is trying to determine the mean amount that a family of four spends on food per week. They perform a phone survey of 300 random families of four and find a sample mean of \$241.50 with a standard deviation of \$46.72.
 - (a) Determine an estimate for the standard deviation of the sample means, $\sigma_{\bar{x}}$. Show your calculation.
- (b) What is the margin of error for the mean amount spent on food per week?

(c) If the World Almanac found that the mean amount spent by all four person families in 2015 was \$244.90, was this within the margin of error you found in (b)? Explain or show how you arrived at your conclusion.



