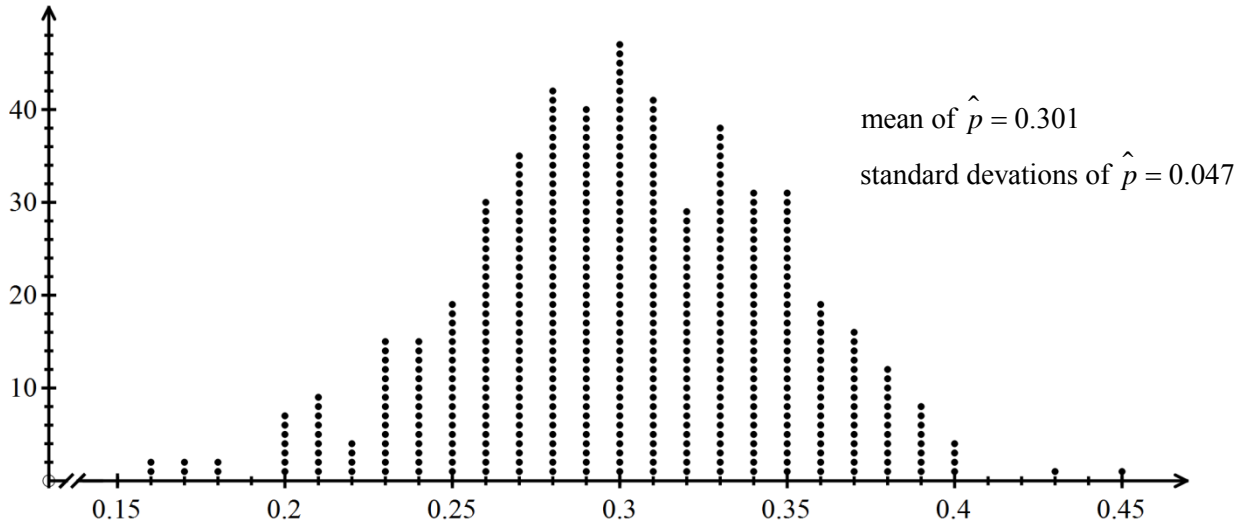


THE DISTRIBUTION OF SAMPLE PROPORTIONS COMMON CORE ALGEBRA II



In the last lesson we saw how the **distribution of sample means** was **normal**. The **Central Limit Theorem** allowed us to find the standard deviation of these sample means. In this lesson, we will look at the same phenomena with sample proportions.

Exercise #1: A simulation of samples taken from a population with a proportion, p , of 0.3 was created. The simulation had a sample size of 100 and 500 simulations were run. The sample proportions, \hat{p} , were calculated and their distribution is shown below:



- (a) What does the shape of this distribution resemble? Explain. (b) What is true about the mean of the sample proportions?

The distribution of sample proportions is governed by a very similar phenomena to the distribution of sample means via **The Central Limit Theorem**. The characteristics of the distribution are given below.

THE DISTRIBUTION OF SAMPLE PROPORTIONS

The distribution of sample proportions, \hat{p} , from a population with a proportion p and a sample size of n will:

1. Approximate a normal distribution
2. Have a mean of the population proportion, p .

3. Have a standard deviation given by $\sqrt{\frac{p(1-p)}{n}}$

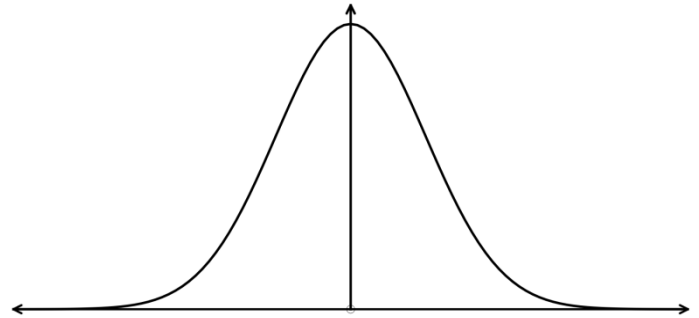
Exercise #2: Does the standard deviation from the simulation agree with that predicted with the above formula?



Since sample proportions will be **normally distributed**, we can perform calculations similar to those done for **sample means**. In other words, we can see how likely a range of sample proportions would be given a particular population proportion.

Exercise #3: Suppose the percent of seniors in high school that own a cell phone is 82%. If a random sample of 50 high school seniors was taken, determine the following:

- (a) The standard deviation of sample proportions for this population proportion given this sample size. Show the calculation that leads to your answer.
- (b) The probability that the sample proportion will be within 3% of the 82% proportion. Illustrate your work on the general normal curve below.



(c) Find each of the following probabilities. Round each answer to the nearest tenth of a percent.

- (i) the sample proportion will be less than 75% (ii) the sample proportion will be greater than 95%

Exercise #4: Political polls can be tricky. Let's say that 47% of the public will vote for a particular candidate in the upcoming election. If a newspaper takes a random poll of 200 voters, what is the probability that this sample will have a proportion larger than 50%, thus predicting a win for this candidate?



THE DISTRIBUTION OF SAMPLE PROPORTIONS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. For each of the following population proportions, p , find the standard deviation of the sample proportions, \hat{p} , given the sample size n . Show your calculation. Round to three decimal place accuracy (nearest thousandth).

(a) $p = 0.34$ and $n = 50$

$\sigma_{\hat{p}} =$

(b) $p = 0.5$ and $n = 400$

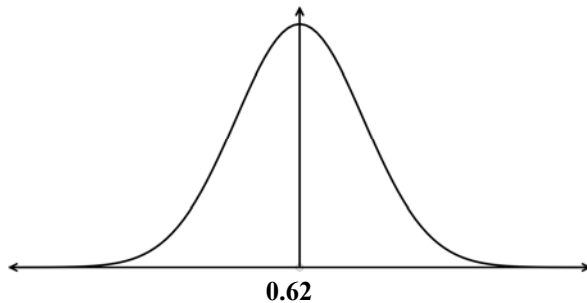
$\sigma_{\hat{p}} =$

(c) $p = 0.25$ and $n = 100$

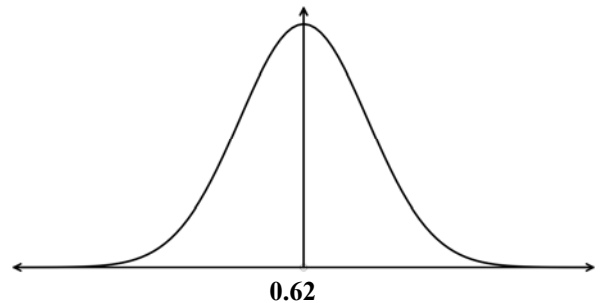
$\sigma_{\hat{p}} =$

2. A population has a proportion of 0.62. A sample of size 40 was taken from this population. Determine the following probabilities. Illustrate each on the normal curve shown below each part.

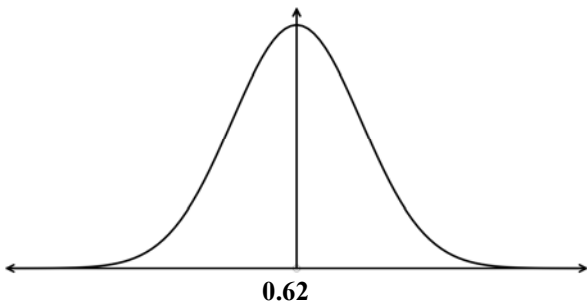
- (a) The probability the sample has a proportion between 0.5 and 0.7.



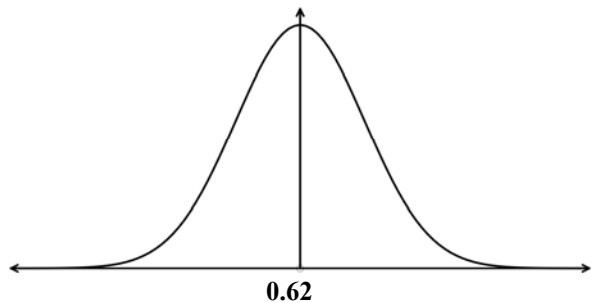
- (b) The probability the sample has a proportion within 5% of the population proportion.



- (c) The probability that the sample has a proportion less than 0.50.



- (d) The probability that the sample has a proportion greater than 0.80.



APPLICATIONS

3. A candidate for political office has support from 40% of the public. If a random sample of 100 members of the public was taken, which of the following is closest to the probability that the sample had a proportion of 50% or greater support for this candidate?

- (1) 2% (3) 14%
(2) 7% (4) 24%

-
4. A school will offer pizza on Friday's if at least 30% of the students will buy it. A sample of 50 students are asked if they would buy pizza on Friday and 10 respond that they would.

- (a) Determine the probability of getting a sample of this size with the proportion or lower given a population with a proportion of 0.30. (b) Should the school offer pizza on Fridays? Explain your choice by reflecting on what your answer from part (a) tells you.

5. If a 45% of a population likes a particular soda, then what range below shows all sample proportions within two standard deviations of the population proportion if the samples have a size of 70?

- (1) 38% to 52% (3) 33% to 57%
(2) 20% to 70% (4) 28% to 62%

REASONING

6. Juniors at a high school own internet enabled devices at a rate of 71%. If 52 freshmen were sampled and only 58% of them owned internet enabled devices, is this enough proof to state that freshmen own these devices at a lower rate than juniors? Explain based on probability.

