

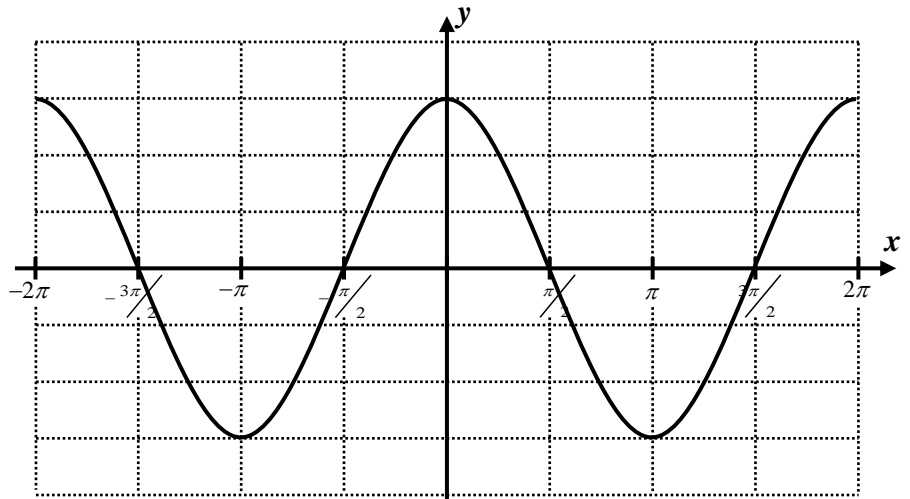
PHASE SHIFT
ALGEBRA 2 WITH TRIGONOMETRY

The last important transformation of sinusoidal graphs that we need to examine is known as the **phase shift** and represents the horizontal translation of the sinusoidal graph. The phase shift is important in electrical engineering (three phase power). The first exercise will illustrate the basic concept.

Exercise #1: As in the last lesson, the graph of $y = 3\cos x$ is shown below.

(a) Using this graph as a guide, sketch the graph of $y = 3\cos\left(x - \frac{\pi}{2}\right)$.

(b) State the phase shift of this new graph.



Exercise #2: Identify the phase shift of each of the following sinusoidal functions.

(a) $y = 2\sin(5(x - \pi)) + 1$ (b) $f(x) = -4\cos\left(\frac{\pi}{3}(x + 4)\right) - 1$ (c) $y = 8\sin\left(\frac{3}{4}\left(x - \frac{\pi}{3}\right)\right) + 6$

If written in the form $y = A\sin(B(x - C)) + D$, the phase shift is relatively easy to identify. It is more difficult if the frequency has not been factored first.

Exercise #3: Determine the phase shift for each of the following sinusoidal equations.

(a) $y = \sin(2x - 8)$ (b) $y = \cos(4x - 2\pi)$ (c) $y = \sin\left(\frac{\pi}{3}x + 2\pi\right)$ (d) $y = \cos\left(\frac{1}{2}x + \frac{\pi}{12}\right)$



We now have all of the essential transformations and terms associated with sinusoidal graphs. They are reviewed below:

SINUSOIDAL TERMINOLOGY

Amplitude – The distance between the midline and the maximum or minimum.

Midline – The average y -value of the sinusoidal curve.

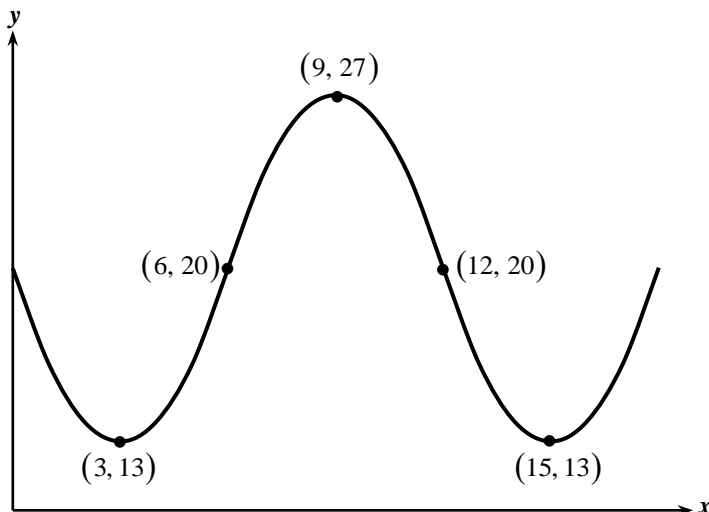
Period – The minimum horizontal distance before the graph starts repeating itself.

Frequency – The number of times a sinusoidal graph will repeat in 2π radians.

Phase Shift – The horizontal shift of a sinusoidal curve.

Exercise #4: For the sinusoidal curve $y = 5\sin(2x - 3\pi) + 7$, identify and label the amplitude, midline, frequency, period, and phase shift. Also, state the range of the sinusoidal curve. Show work where necessary.

Exercise #5: A graph is shown below that is a transformation of $y = \sin x$. Determine the period, frequency, amplitude, midline, and phase shift for this graph. Label each and show work where necessary.



Exercise #6: If the graph in *Exercise #5* had been a transformation of $y = \cos x$, what would be its phase shift?



Name: _____

Date: _____

PHASE SHIFT
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

SKILLS

1. State the phase shift for each of the following sinusoidal curves.

(a) $y = 3\sin\left(5\left(x - \frac{\pi}{4}\right)\right) + 2$ (b) $y = -2\cos\left(\frac{1}{3}(x + 8)\right) + 1$ (c) $y = 3.5\sin\left(\frac{2\pi}{365}(x - 90)\right) + 12$

2. Determine the phase shift for each of the following sinusoidal curves. Show necessary work.

(a) $y = 5\cos(2x - 20)$ (b) $y = -8\sin(6x + 18)$ (c) $y = \frac{1}{2}\cos(3x - \pi)$

(d) $y = 2\sin\left(\frac{\pi}{4}x - \frac{\pi}{2}\right)$ (e) $y = -3\cos(6x + 3\pi)$ (f) $y = \frac{3}{2}\sin\left(\frac{1}{2}x - \frac{\pi}{12}\right)$

3. Which of the following represents the phase shift of $y = 8\sin\left(2\left(x - \frac{\pi}{8}\right)\right) + 6$?

(1) $\frac{\pi}{4}$ (3) 8

(2) 2 (4) $\frac{\pi}{8}$

4. Which of the following represents the phase shift of $y = -5\cos\left(\frac{1}{2}x - \frac{\pi}{2}\right)$?

(1) π (3) $-\frac{\pi}{2}$

(2) $\frac{\pi}{2}$ (4) $\frac{1}{2}$



5. Given the sinusoidal function whose equation is $y = 8\sin(4x - \pi) - 2$, determine and clearly label its amplitude, midline, frequency, period, phase shift, and range. Show calculations where necessary.

6. Given the sinusoidal function whose equation is $y = -3\cos\left(\frac{\pi}{2}x + 3\pi\right) + 10$, determine and clearly label its amplitude, midline, frequency, period, phase shift, and range. Show calculations where necessary.

APPLICATIONS

7. Melinda is modeling her height on a Ferris wheel using a transformation of $y = \sin x$, where x measures the time she has been riding in seconds and y measures her height above the ground in feet. The graph of one full trip around the Ferris wheel is shown below. Determine and clearly label this graph's amplitude, midline, period, frequency, and phase shift.

