Name: _____ Date: _____

TRIGONOMETRIC IDENTITIES – DAY #2
ALGEBRA 2 WITH TRIGONOMETRY

In this lesson we will develop and use identities that relate the sine and cosine of an angle to that of twice its measure. These identities, which are useful in calculus and equation solving, are known as the **Double Angle Identities**.

THE DOUBLE ANGLE IDENTITIES

$$\sin(2A) = 2\sin A\cos A$$

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$\cos(2A) = 2\cos^2 A - 1$$

$$\cos(2A) = 1 - 2\sin^2 A$$

***Exercise* #1:** Using the sum identities shown below, derive the identity for (a) $\sin(2A)$ and (b) the first identity for $\cos(2A)$.

(a) $\sin(A+B) = \sin A\cos B + \sin B\cos A$ (b) $\cos(A+B) = \cos A\cos B - \sin A\sin B$

The other two identities for $\cos(2A)$ are found by substituting the **Pythagorean Identity** into the first identity for $\cos(2A)$. These derivations will be left for the homework.

***Exercise* #2:** Verify the identity for $\sin(2A)$ using $A = 30^\circ$ and $2A = 60^\circ$.

***Exercise* #3:** For an angle α it is known that $90^\circ < \alpha < 180^\circ$. If $\cos\alpha = -3/5$ then find:

(a) $\sin(2\alpha)$ (b) $\cos(2\alpha)$

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THE DOUBLE ANGLE IDENTITIES

$$\sin(2A) = 2\sin A\cos A$$

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$\cos(2A) = 2\cos^2 A - 1$$

$$\cos(2A) = 1 - 2\sin^2 A$$

Exercise #4: For the angle A it is known that $180^\circ < A < 270^\circ$ and $\sin A = -\sqrt{11}/6$. Find the following:

(a) $\sin 2A$

(b) $\cos 2A$

Exercise #5: Which of the following is equivalent to $2\sin 50^\circ \cos 50^\circ$?

(1) $\sin 25^\circ$

(2) $\sin 100^\circ$

(3) $\cos 100^\circ$

(4) $\cos 25^\circ$

_____

Exercise #6: Which of the following is *not* equal to $\cos 80^\circ$?

(1) $\sqrt{1-\sin^2 80^\circ}$

(2) $1 - 2\sin^2 40^\circ$

(3) $2\cos^2 160^\circ - 1$

(4) $\cos^2 40^\circ - \sin^2 40^\circ$

_____

Exercise #7: If $\cos\alpha = 0.28$ then $\cos 2\alpha$ is closest to

(1) 0.56

(2) 0.14

(3) –0.68

(4) –0.84

_____

Exercise #8: The value of $\cos 130^\circ$ is equal to

(1) $1 - \sin^2 130^\circ$

(2) $1 - 2\sin^2 65^\circ$

(3) $2\cos^2 260^\circ - 1$

(4) $2\cos 65^\circ$

_____

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TRIGONOMETRIC IDENTITIES – DAY #2
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

THE DOUBLE ANGLE IDENTITIES

$$\sin(2A) = 2\sin A\cos A$$

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$\cos(2A) = 2\cos^2 A - 1$$

$$\cos(2A) = 1 - 2\sin^2 A$$

SKILLS

1. Which of the following is equal to the expression $2\cos^2 60^\circ - 1$?

   (1) $\cos 30^\circ$ (3) $\cos 120^\circ$

   (2) $\sin 30^\circ$ (4) $\sin 120^\circ$ _____

2. Which of the following is equal to $\sin 10^\circ$?

   (1) $1 - 2\sin^2 20^\circ$ (3) $2\sin 5^\circ \cos 5^\circ$

   (2) $2\sin 20^\circ \cos 20^\circ$ (4) $\cos^2 5^\circ - \sin^2 5^\circ$ _____

3. Which of the following is *not* equal to $\cos 50^\circ$?

   (1) $\sqrt{1-\sin^2 50^\circ}$ (3) $\cos 310^\circ$

   (2) $2\cos^2 100^\circ - 1$ (4) $1 - 2\sin^2 25^\circ$ _____

4. If $\cos\alpha = \frac{1}{3}$ then $\cos(2\alpha) = ?$

   (1) $-\frac{7}{9}$ (3) $-\frac{2}{3}$

   (2) $-\frac{5}{9}$ (4) $-\frac{\sqrt{3}}{3}$ _____

5. If $90^\circ < A < 180^\circ$ and $\cos(2A) = \frac{5}{8}$ then $\sin A = ?$

   (1) $\frac{3}{8}$ (3) $\frac{\sqrt{3}}{4}$

   (2) $\frac{\sqrt{6}}{2}$ (4) $\frac{5}{7}$ _____

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THE DOUBLE ANGLE IDENTITIES

$$\sin(2A) = 2\sin A\cos A$$

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$\cos(2A) = 2\cos^2 A - 1$$

$$\cos(2A) = 1 - 2\sin^2 A$$

6. For an angle β it is known that $90^\circ < \beta < 180^\circ$ and $\cos\beta = -5/13$. Find in simplest form:

   (a) $\sin(2\beta)$ (b) $\cos(2\beta)$

7. For an angle θ it is known that $\cos\theta = \sqrt{2}/4$ and $270^\circ < \theta < 360^\circ$. Find in simplest form:

   (a) $\sin(2\theta)$ (b) $\cos(2\theta)$

REASONING

8. We saw in the lesson where the first identity for $\cos(2A)$ comes from. By combining this identity with the **Pythagorean Identity**, $\cos^2 A + \sin^2 A = 1$ the other two can be found.

   (a) By substituting the Pythagorean Identity $\sin^2 A = 1 - \cos^2 A$ into the first identity for $\cos(2A)$, derive the second identity for $\cos(2A)$.

   (b) By substituting the Pythagorean Identity $\cos^2 A = 1 - \sin^2 A$ into the first identity for $\cos(2A)$, derive the third identity for $\cos(2A)$.