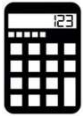


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AVERAGE RATE OF CHANGE

N-GEN MATH[®] 8



In the last lesson we looked at how the outputs of functions **increased** or **decreased** as their **input increased**. In this lesson we will investigate how **fast** this increase or decrease occurs. We generally measure this using what is known as **average rate of change**.

Exercise #1: The number of people who have entered a concert hall was measured as a function of time since the doors opened. The data is shown in the table below.

| | | | | | | |
|----------------|---|----|-----|-----|-----|-----|
| Time (minutes) | 0 | 4 | 10 | 15 | 18 | 25 |
| People | 0 | 48 | 144 | 239 | 275 | 338 |

- (a) At what rate, in people per minute, did the hall fill up in the first four minutes?
- (b) How many people entered the hall between 10 and 15 minutes?
- (c) At what rate did people enter the hall, in people per minute, between 10 and 15 minutes?
- (d) Was the rate in (c) greater or less than the rate that people entered the hall between 15 and 18 minutes?

To calculate the rate that the function is changing over any given **interval** of the **input** we simply divide the change in the function by the change in its input.

AVERAGE RATE OF CHANGE

The average rate of change can be calculated by $\frac{\text{change in the output}}{\text{change in the input}} = \frac{\Delta y}{\Delta x}$

Exercise #2: The formula to calculate the average rate of change of a function is identical to what formula that you have already learned?



Average rate of change finds the **slope** between any two points on the function. The **slope** has always measured how fast the y variable is changing compared to the x variable.

Exercise #3: A ball is thrown into the air such that its height is a function of time since it was thrown. The data is given in the table to the right.

(a) Find the average rate of change from 1 to 3 seconds in feet per second. Show your calculation.

(b) Find the average rate of change from 3 to 6 seconds in feet per second. Show your calculation.

| Time (sec) | Height (ft) |
|------------|-------------|
| 0 | 5 |
| 1 | 91 |
| 2 | 145 |
| 3 | 167 |
| 4 | 157 |
| 5 | 115 |
| 6 | 41 |

(c) Why is the average rate of change positive in (a) but negative in (b)?

So far, we have been looking at average rate of change using tables, which is very convenient because input and output values are easy to read. But, we can also find average rate of change when functions are represented in other forms.

Exercise #4: Two functions are shown below. Determine which function has the greater average rate of change from $x = -2$ to $x = 4$. Show the calculations that lead to your result.

Function A: $y = 10x + 3$

Function B: $y = 3x^2$



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AVERAGE RATE OF CHANGE

N-GEN MATH[®] 8 HOMEWORK

FLUENCY

1. The table below shows y as a function of x . Which of the following is the average rate of change of the function from $x = 2$ to $x = 6$?

(1) $\frac{13}{6}$

(3) $\frac{2}{5}$

| | | | | |
|-----|---|----|----|----|
| x | 0 | 2 | 6 | 9 |
| y | 6 | 11 | 13 | 20 |

(2) $\frac{1}{2}$

(4) $\frac{20}{9}$

2. Given the function defined by $y = x^2$, which of the following is its average rate of change from $x = 2$ to $x = 10$?

(1) $\frac{5}{2}$

(3) $\frac{8}{3}$

(2) 2

(4) 12

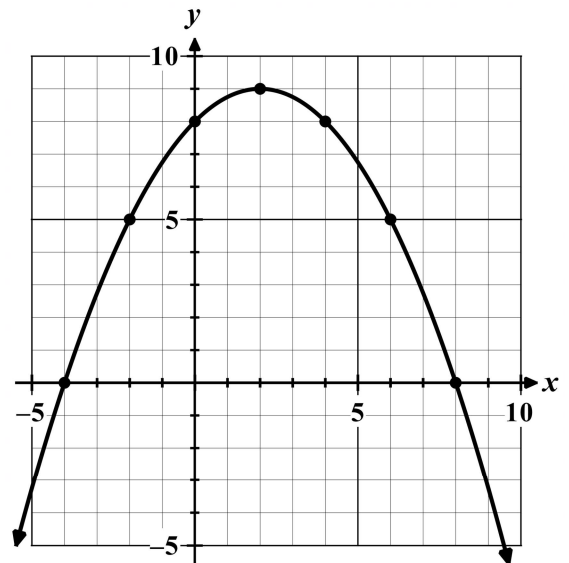
3. For the graph of the function shown, answer the following questions.

- (a) Find the outputs for each of the following inputs.

$$x = 0 \Rightarrow y = \underline{\hspace{2cm}} \quad x = 6 \Rightarrow y = \underline{\hspace{2cm}}$$

- (b) Find the average rate of change of the function between $x = 0$ and $x = 6$.

- (c) Why would the average rate of change be equal to zero from $x = -2$ to $x = 6$?



USING YOUR MATH

4. A semi-truck is being loaded with boxes of varying weights. The total weight of boxes loaded is shown below as a function of the number of boxes loaded into the truck.

| | | | | |
|-----------------------|----|----|----|-----|
| Number of Boxes | 5 | 11 | 20 | 25 |
| Total Weight (pounds) | 18 | 45 | 99 | 115 |

- (a) For each interval below, find the average rate of change. Do not round. Include units with your answers.

5 to 11 boxes:

11 to 20 boxes:

20 to 25 boxes:

- (b) Over which interval was the average weight per box the greatest?

5. A hot liquid was placed in a freezer and its temperature was recorded. The data for the first 8 minutes is shown below.

At what average rate is the temperature changing from 2 to 8 minutes? Use proper units.

| Time (min) | Temp. ($^{\circ}$ F) |
|------------|-----------------------|
| 0 | 190 |
| 2 | 135 |
| 4 | 93 |
| 6 | 77 |
| 8 | 63 |

REASONING

6. Given the function $y = 4x + 3$ do the following. Find its average rate of change:

(a) from $x = 2$ to $x = 5$

(b) from $x = 4$ to $x = 10$.

- (c) Why do you think these two answers were equal?

