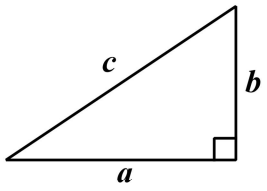
Name: _____ Date: _____

THE PYTHAGOREAN THEOREM AND ITS CONVERSE
N-GEN MATH® 8

In the last lesson we saw the **Pythagorean Theorem**, which applies to the side lengths of a right triangle. Specifically:

THE PYTHAGOREAN THEOREM

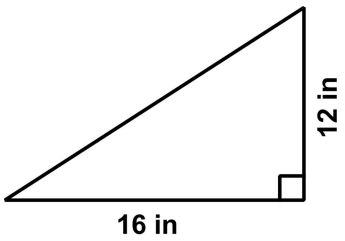
If you have a **right triangle** with legs of length a and b and the length of its hypotenuse is given by c then the following equation is always true:



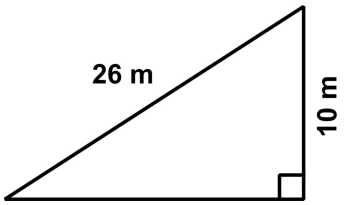
$$a^2 + b^2 = c^2 \text{ or } (\text{leg 1})^2 + (\text{leg 2})^2 = (\text{hypotenuse})^2$$

***Exercise* #1:** Given each of the following right triangles, use the Pythagorean Theorem to find the missing side. Both will be rational numbers.

(a)



(b)



The **Pythagorean Theorem** states that **if a triangle is a right triangle** then **the sum of the squares of the legs** equals the **square of the hypotenuse**. But, the **converse** of this statement is also true.

THE CONVERSE OF THE PYTHAGOREAN THEOREM

If the lengths of the three sides of a triangle are given by a, b, and c, where c is the longest side and $a^2 + b^2 = c^2$ then it must be a right triangle.

***Exercise* #2:** Given each set of numbers below, could they be the sides of a right triangle? Justify using the converse of the Pythagorean Theorem.

(a) $28, 45, 53$ (b) $17, 22, 29$ (c) $15, 36, 39$

When a **set of three integers** could be the sides of a right triangle, we call this set a **Pythagorean triple**. To see if a set of three numbers is a Pythagorean triple we only have to see if it makes the Pythagorean Theorem true. These triples are often written like **coordinate points**.

***Exercise* #3:** Determine whether each of the following is a **Pythagorean triple**. Justify.

(a) $(3, 4, 5)$ (b) $(14, 22, 30)$ (c) $(5, 12, 13)$

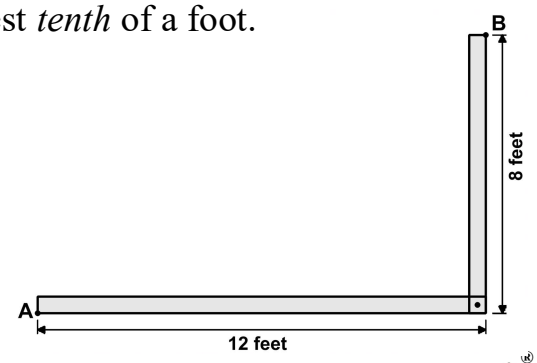
(d) $(48, 55, 73)$ (e) $(18, 24, 30)$ (f) $(32, 47, 57)$

***Exercise* #4:** One of the most common Pythagorean triples is $(3, 4, 5)$, see *Exercise* #3(a). Any multiple of this triple will also be a Pythagorean triple. Each of the triples below is a multiple of the $(3, 4, 5)$ triple. Show that they all could be side lengths of a right triangle.

(a) $(6, 8, 10)$ (b) $(9, 12, 15)$ (c) $(33, 44, 55)$

The converse of the Pythagorean Theorem is useful in many areas of construction to ensure that right angles are created.

***Exercise* #5:** On a construction job, Christoph wants to join a 12-foot-long board with an 8-foot long board so they make a right angle with each other. He can do this by making sure the diagonal from point A to point B is what length? Round to the nearest *tenth* of a foot.



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THE PYTHAGOREAN THEOREM AND ITS CONVERSE
N-GEN MATH® 8 HOMEWORK

FLUENCY

1. Determine if each set of lengths below could be the side lengths of a right triangle. Show the work that leads to your yes/no answer.

   (a) 16, 30, 34 (b) 12, 18, 30 (c) 9, 15, 20

   (d) 12, 35, 37 (e) 33, 56, 65 (f) 27, 34, 43

2. Which of the sets of numbers below could *not* be the sides of a right triangle?

   (1) 3, 4, 5

   (2) 5, 12, 13

   (3) 12, 16, 20

   (4) 7, 24, 27 _____

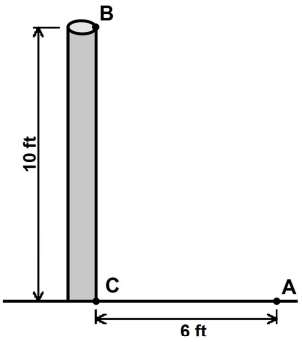
3. Given that $(5, 12, 13)$ is a Pythagorean triple, which of the following isn't a Pythagorean triple?

   (1) $(10, 24, 26)$ (3) $(50, 120, 130)$

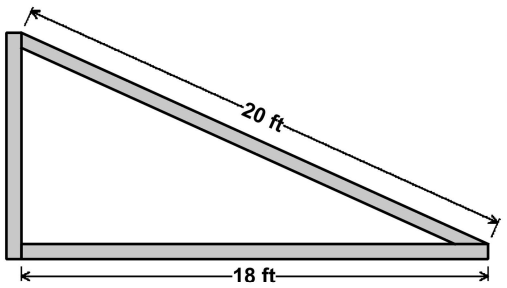
   (2) $(10, 17, 18)$ (4) $(15, 36, 39)$ _____

USING YOUR MATH

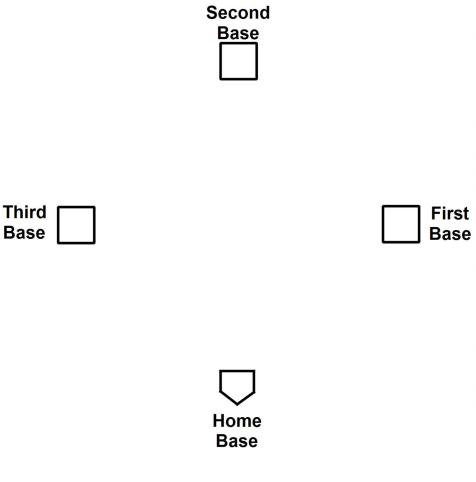
4. Jalen is trying to set a pole that is 10 feet long into the ground so that it is perpendicular with the horizontal surface. Jalen locates a point 6 feet from the base of the pole, at C, and labels it A. He then measures the distance from point A to point B. What distance, to the nearest tenth of a foot, does Jalen need for the pole to be perpendicular? Justify.



5. A ramp is to be built using a 20 foot long board and an 18 foot long board. To make the ramp stable, the builders would like to add a third board to create a right triangle. How long should that board be, to the nearest hundredth of a foot? Justify.



6. Irina is trying to lay out the bases for a game of kickball such that the infield is a square as shown. She would like the bases to be 25 feet apart. She first places home base and then places first and third base 25 feet from home base.



   (a) How far, to the nearest tenth of a foot, should first base and third base be from each other? Justify.

   (b) After finding the distance in (a), Irina places second base this distance away from home. How could she now determine if it is in the correct place to form a square?