Name: ____________________________________ Date: __________________

AVERAGE RATE OF CHANGE
N-GEN MATH® ALGEBRA I

*Functions are rules that give us **outputs** when we supply them with **inputs**. We often want to know how **fast** the outputs are changing compared to a change in the input values. This is referred to as the **average rate of change** of a function.*

***Exercise* #1:** The doors to a movie theater open 15 minutes prior to the movie starting. The table below shows the number of people who have entered the theater as a function of time, measured in minutes after the doors opened.

t	0	4	6	9	12	15
$n(t)$	0	32	54	102	132	150

(a) Find the value of $n(9)-n(4)$. What does this represent in the context of this problem?

(b) At what rate did people enter the theater from $t=4$ to $t=9$ minutes? Show how you found your answer and use appropriate "per" rate units.

(c) On average, were people entering the theater faster between 4 and 9 minutes or between 9 and 15 minutes? Justify your answer.

*What we found in Exercise #1 (b) and (c) is the **average rate of change** of the function over a specified **domain interval**. The formula for finding the average rate of change is important.*

AVERAGE RATE OF CHANGE

For the function $y=f(x)$, the average rate that $f(x)$ changes from $x=a$ to $x=b$ is given by:

$$\frac{f(b)-f(a)}{b-a}=\frac{\Delta y}{\Delta x}=\frac{\textbf{how much the } y\textbf{-values have changed}}{\textbf{how much the } x\textbf{-values have changed}}$$

***Exercise* #2:** For the function $f(x)=x^2+5$, find its average rate of change over the following two intervals.

(a) $-1\le x\le 5$

(b) $-6\le x\le 4$

***Exercise* #3:** The function $h(x)$ is given in the table below. Which of the following gives its average rate of change over the interval $2 \le x \le 6$?

x	$h(x)$
0	10
2	9
4	6
6	3

(1) $-\frac{3}{2}$ (3) $-\frac{7}{6}$

(2) $\frac{6}{4}$ (4) -1

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Average rate of change *often informs us about* ***how fast*** *something is happening over an interval of time. Keep in mind that the answers to all of these applied problems end up being a rate with* ***"per"*** *units attached.*

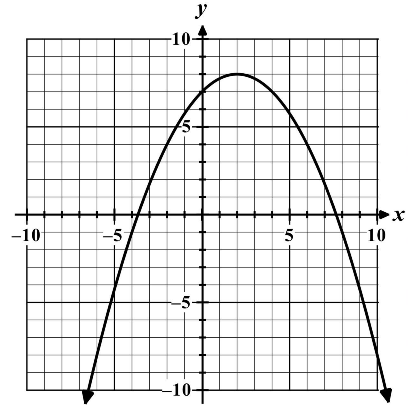
***Exercise* #4:** Skylar is selling glasses of lemonade. The function $g(t)=\frac{t^2+4}{2}$ models the number of glasses she has sold, *g*, after *t*-hours. What is the average rate at which she is selling lemonade between $t=2$ and $t=8$ hours? Include proper units in your answer.

***Exercise* #5:** The temperature of a liquid that is being heated has an average rate of change of 12.8 °F per minute from $t=3$ minutes to $t=8$ minutes. If the liquid's temperature at $t=3$ minutes is 72 °F, what is its temperature at $t=8$ minutes?

We should be able to calculate average rate of change if we are given functions in different forms, such as equations, graphs, and tables.

***Exercise* #6:** Given the two functions shown below, which has the greater average rate of change from $x=-4$ to $x=6$? Show calculations that lead to your answer.

$f(x)$



x	$g(x)$
–5	7
–4	9
–1	0
3	8
6	15
8	18

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AVERAGE RATE OF CHANGE
N-GEN MATH® ALGEBRA I HOMEWORK

FLUENCY

1. For the function shown in the table below, which of the following is its average rate of change over the interval $2 \le x \le 10$?

(1) $\frac{3}{4}$

(2) -2

(3) $-\frac{1}{2}$

(4) $-\frac{3}{2}$

x	-4	2	5	7	10
$f(x)$	4	9	6	-1	-3

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2. What is the average rate of change of $g(x) = x^2$ on the interval $3 \le x \le 9$?

(1) 8 (3) 15

(2) 12 (4) 18

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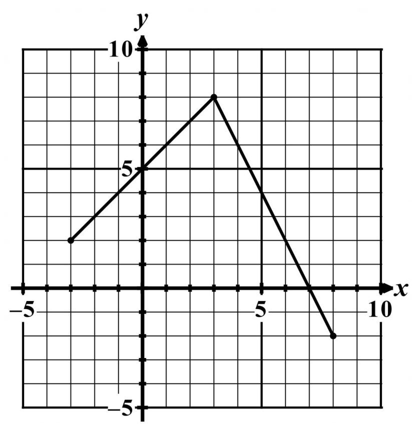
3. Given the function $f(x)$ shown graphed below, what is its average rate of change from $x = 1$ to $x = 7$?

(1) -1

(2) $-\frac{4}{3}$

(3) 8

(4) $\frac{3}{5}$



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4. A function $h(x)$ has an average rate of change equal to 7 on the interval $5 \le x \le 9$. If $h(5) = 12$, then which of the following must be the value of $h(9)$?

(1) 28

(2) 36

(3) 40

(4) 52

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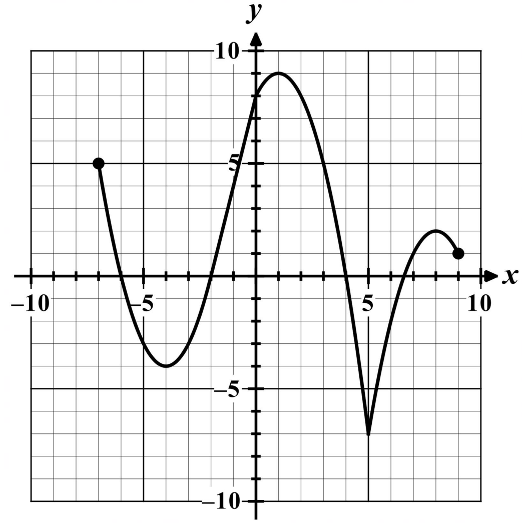
5. Given the graph of the function $f(x)$ shown below, find the average rate of change over each of the following intervals. Express in simplest form. Show the calculations you use to find your answers.

(a) $-3 \le x \le 1$

(b) $-1 \le x \le 8$

(c) $1 \le x \le 9$

(d) $-6 \le x \le 2$



APPLICATIONS

6. A pump is filling up a swimming pool. The volume of water in the pool is a function of time since it has been filling. The table below shows volumes of water, in gallons, at various times during filling.

t, in minutes	0	16	36	53
$V(t)$, in gallons	0	896	1,872	2,561

On average, is the water entering the pool faster over the interval $0 \le t \le 36$ minutes or $16 \le t \le 53$ minutes? Justify your answer with appropriate calculations.

REASONING

7. Consider the function given by $f(x) = 6x + 5$. Find its average rate of change over the following intervals.

(a) $1 \le x \le 3$

(b) $2 \le x \le 7$

8. The average rate of change should have been the same for both (a) and (b) in the last problem. What is special about this function that ensures its average rate of change will always be the same?