

Name: _____

Date: _____

AVERAGE RATE OF CHANGE AND MOTION
N-GEN MATH® ALGEBRA I



Previously, we learned about the **average rate of change** of a function, or how quickly a function's output changes compared to its input. There is a special type of average rate of change known as the **average speed**.

AVERAGE SPEED

$$\text{average speed} = \frac{\Delta d}{\Delta t} = \frac{\text{change in the distance traveled}}{\text{change in time}}$$

Exercise #1: Mia is going for a run and has an app tracking the distance she has traveled as a function of the length of time she has been running. The data is shown below for selected times.

- (a) What is Mia's average speed for the entire run? Include proper units in your answer and round to the nearest *hundredth*.

Time (min)	0	10	25	40	60	70	75
Distance Traveled (miles)	0	0.9	2.6	4.5	7.3	8.4	8.8

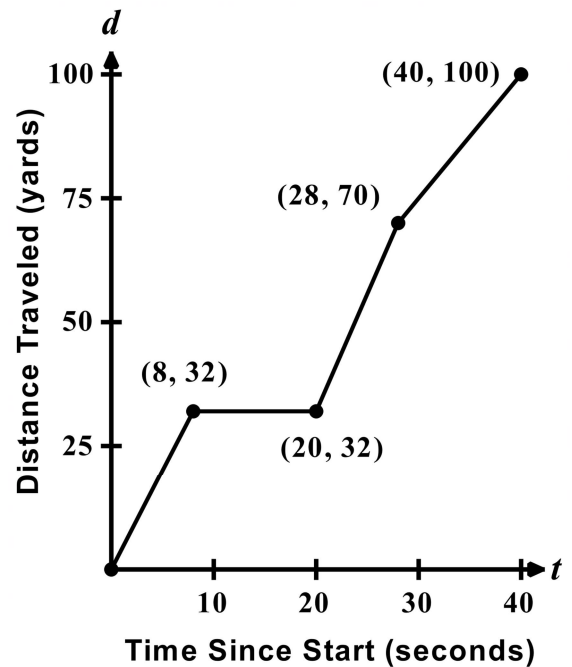
- (b) Is Mia running faster from 10 to 25 minutes or from 40 to 60 minutes? Support your answer with proper calculations.
- (c) Mia warms up for the first 10 minutes of her run and cools down for the last 5 minutes. On which of these intervals is she moving faster?
- (d) Is there evidence in the table that Mia stopped during her run? Explain.
- (e) Mia is training for a marathon whose total distance is 26 miles. At the average speed that you found in (a), would Mia finish the marathon in less than 4 hours?



Motion is often depicted in graphs that we can analyze. The **average speed** can be thought of as the **slope of the graph**. This can be helpful in quickly comparing speeds and analyzing motion.

Exercise #2: A motion detector is placed on Hazel the Bassett hound while she moves across a 100-yard football field. Her progress is shown in the graph below.

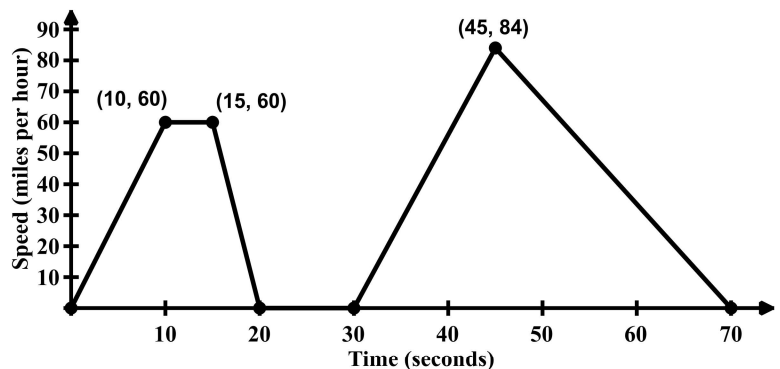
- (a) On average, what is Hazel's speed for the entire trip?
- (b) Is there any interval of time during which Hazel stops moving? How can you tell?
- (c) Is Hazel moving faster in the first 8 seconds or in the time between 20 and 28 seconds? Justify.



Sometimes you need to interpret a graph of the speed of an object. Use the units of the problem to help you think about what is happening physically.

Exercise #3: A company is testing the acceleration and braking power of a new car they have developed. They recorded the car's speed versus time in a trial with the results shown below.

- (a) Even though this graph is horizontal, the car is moving from 10 to 15 seconds. How can you tell?
- (b) Is there an interval of time when the car isn't moving? If so, when?



- (c) What is the average rate of change of the speed from 30 to 45 seconds? Using proper units, explain the physical significance of your answer.



AVERAGE RATE OF CHANGE AND MOTION
N-GEN MATH[®] ALGEBRA I HOMEWORK

FLUENCY

1. An object moves such that the total distance it has traveled is given as a function of time in the table below:

Time (seconds)	0	2	5	7	8	12	15
Distance (feet)	0	7	22	31	37	45	48

Calculate the average speed the object is moving over each of the following intervals. Include proper units in your answer.

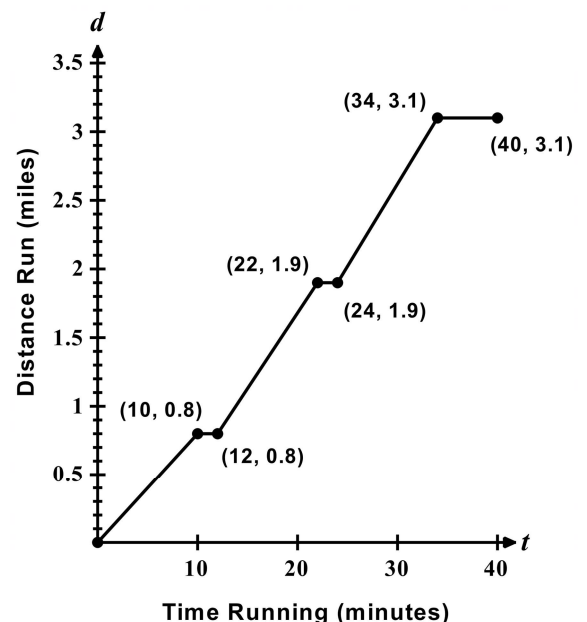
- (a) From 0 to 15 seconds (b) From 5 to 8 seconds (c) From 7 to 12 seconds

2. Using the table above, is the object moving faster on average from 2 to 5 seconds or from 7 to 8 seconds? Justify your answer with evidence.

APPLICATIONS

3. Nestor is trying a new running plan where he jogs for 10 minutes, then rests, then jogs for 10 minutes, then rests, etcetera. The graph of a recent run for Nestor is shown. Answer the following questions.

- (a) How long are both of the first two rest periods?
- (b) Nestor has three 10-minute run periods. On which one was he running the fastest? What was his speed in miles per minute?
- (c) Excluding the final rest period, what was Nestor's average speed for the entire run? Round to the nearest hundredth of a mile per minute.



4. An object is falling such that the distance it travels is given as a function by the formula $d(t) = 16t^2$, where t is the number of seconds the object has fallen and d is the distance it has traveled in feet.

(a) Fill out the following table for the distance the object has traveled each half-second. (Using tables on your calculator is a great way to do this quickly!)

t (seconds)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$d(t)$ (feet)									

(b) How much faster, in feet per second, is the object falling over the last two seconds than the first two seconds? Show your analysis.

(c) What is the object's average speed for the entire four seconds?

(d) There is a one-second long interval in the table where the average speed on that interval is the same as the speed that you found in (c). Which interval? How can you quickly tell?

5. While driving home on the Taconic State Highway, Ernesto hits some light traffic that makes him slow down. After getting out of the light traffic, he is able to speed up for a short time before he eventually has to slow to a complete stop.

(a) During the time that Ernesto is speeding up, what is the rate of change of his speed? Include units in your answer.

(b) Is Ernesto's speed changing faster when he slows down initially or when he comes to a complete stop? Justify your response.

