Name: _____ Date: _____

# ARITHMETIC SEQUENCES
# N-GEN MATH® ALGEBRA I

*There are many types of sequences, but there is one that is related to linear functions and in fact is a type of* ***discrete linear function****. These are known as* ***arithmetic sequences****.*

***Exercise* #1**: Laura is training for a marathon. In the first week of training, she runs 15 miles in total. She plans to then increase the amount she runs each week by the same amount. The following sequence shows the progression of how many miles she runs each week.

15, 20, 25, 30, ...

(a) By how many miles does Laura increase her distance each week?

(b) How many miles does Laura run in the fifth week of training?

(c) Laura would like to predict how many miles she runs in the $20^{th}$ week of training. How many **increases** occur between the first and $20^{th}$ week?

(d) How many miles does Laura run in the $20^{th}$ week? Show the calculation you use to find your answer.

*The previous example illustrates an* ***arithmetic sequence****. For an arithmetic sequence,* ***consecutive terms*** *always have a* ***constant difference****. That difference could be positive or negative.*

***Exercise* #2:** The first four terms of an arithmetic sequence are shown below. We would like to predict the $50^{th}$ and $100^{th}$ term of the sequence.

7, 15, 23, 31, ...

(a) What is the constant difference for this arithmetic sequence? Illustrate with multiple calculations.

(b) How many times would this difference need to be added to the first term to produce the $50^{th}$ term?

(c) What is the $50^{th}$ term of the sequence? Show your calculation.

(d) What is the $100^{th}$ term of the sequence? Show your calculation.

*The terms of an arithmetic sequence can increase, as we saw in Exercises #1 and #2. They can also* ***decrease*** *if the* ***constant difference*** *in the terms is* ***negative****.*

***Exercise* #3:** The first four terms of an arithmetic sequence are shown below.

$$116, 112, 108, 104, ...$$

(a) What is the constant difference for this arithmetic sequence?

(b) What is the value of the $20^{th}$ term of this sequence? Show the calculation you used.

*We can generalize the patterns we have seen so that we can write a formula to predict the $n^{th}$ term of any arithmetic sequence if we know its first term and the constant difference.*

***Exercise* #4:** Given an arithmetic sequence with a first term given by $a_1$ and a constant difference given by $d$ (which could be positive or negative), answer the following.

(a) To produce the $n^{th}$ term, or $a_n$, how many times would we need to add the constant difference $d$?

(b) Based on your answer to (a), write down a formula for the $a_n$ in terms of $a_1$, $d$, and $n$.

***Exercise* #5:** Given the arithmetic sequence $-10, -6, -2, 2, ...$, do the following.

(a) Determine a formula for $a_n$.

(b) Use the formula from (a) to determine the value of $a_{50}$.

***Exercise* #6:** The first two terms of an arithmetic sequence are $c_1 = 32$ and $c_2 = 27$. Determine the value of its $15^{th}$ term, or $c_{15}$. Show your work.

***Exercise* #7:** Seats in a small amphitheater follow a pattern where each row has a set number of seats more than the previous row. If the first row has 6 seats and the fourth row has 18, how many seats does the last row, which is the $20^{th}$, have in it? Show your work to justify your response.

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# ARITHMETIC SEQUENCES
# N-GEN MATH® ALGEBRA I HOMEWORK

## FLUENCY

1. The first two terms of an arithmetic sequence are $b_1 = 8$ and $b_2 = 6$. Which of the following is its fourth term, i.e. $b_4$? Show the work that leads to your answer.

   (1) 14 (3) 6

   (2) 2 (4) 4

   _____

2. The first four terms of various sequences are shown below. Which sequence could *not* be an arithmetic sequence?

   (1) 8, 12, 16, 20, …

   (2) 5, 10, 20, 40, …

   (3) 4, 0, $-4$, $-8$, …

   (4) $-6, 2, 10, 18$, …

   _____

3. The first three terms of an arithmetic sequence are 9, 16, and 23. Which of the following would be this sequence's $20^{th}$ term?

   (1) 128 (3) 142

   (2) 135 (4) 149

   _____

4. If the first three terms of an arithmetic sequence are 8, 20, and 32, which formula below gives the value of the $n^{th}$ term of the sequence?

   (1) $a_n = 8n + 12$

   (2) $a_n = 12n + 8$

   (3) $a_n = 12 + 8(n-1)$

   (4) $a_n = 8 + 12(n-1)$

   _____

5. The first two terms of arithmetic sequences are shown in each case below. Determine the value of the requested term. Show the work that leads to your answer.

   (a) $a_1 = 7$ and $a_2 = 20$, find $a_{25}$ (b) $b_1 = 9$ and $b_2 = 4$, find $b_{30}$

6. The second term of an arithmetic sequence has a value of 20 and the fifth term has a value of 38.

   (a) Determine the constant difference for this arithmetic sequence.

   (b) What is the first term of this arithmetic sequence?

   (c) Determine a formula for the $n^{th}$ term of this arithmetic sequence.

   (d) Determine the value of the hundredth term of this arithmetic sequence.

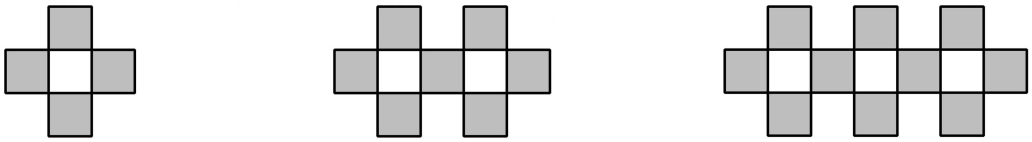
## APPLICATIONS

7. Marcus is building a tower out of paper cups. In each row (counting from the floor up), there are two less cups than the row below it. The first row has 26 cups in it.

   (a) State the number of cups in the second, third, and fourth rows.

   (b) How many cups are in the $11^{th}$ row?

8. The number of shaded squares shown in the patterns below forms the first three terms of an arithmetic sequence.



$n = 1$ $n = 2$ $n = 3$

Determine the number of squares that would be shaded in the $20^{th}$ pattern.

## REASONING

9. Kirk believes the formula $a_n = 3n + 4$ will predict the correct number of shaded squares in the pattern above. Explain why Kirk is incorrect.