

QUADRATIC EQUATIONS WITH IRRATIONAL SOLUTIONS

N-GEN MATH[®] ALGEBRA I



In the last unit we solved quadratic equations by using inverses, completing the square, and factoring. In every case, our solutions were both rational numbers. In this lesson we will begin to explore quadratic equations that have irrational numbers as their solutions.

Exercise #1: Solve each of the following simple quadratic equations using inverse operations. Write any irrational answers in simplest radical form.

(a) $x^2 = 100$

(b) $x^2 = 7$

(c) $x^2 = 20$

(d) $x^2 = 48$

FUNDAMENTAL KEY TO SOLVING QUADRATICS

The solutions to any equation of the form $x^2 = a$ are $x = -\sqrt{a}$ and $x = \sqrt{a}$, or just $x = \pm\sqrt{a}$.

We may have to do multiple steps to solve a quadratic equation.

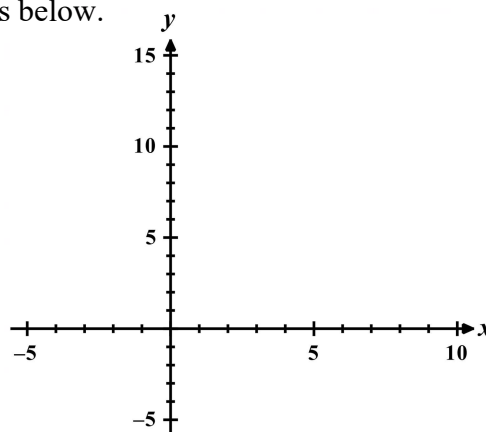
Exercise #2: Consider the equation $(x-3)^2 = 11$.

(a) Solve this equation using inverses.

(b) Why must both of these answers be irrational numbers?

(c) Write both answers as decimals accurate to three decimal places.

(d) Solve the equation graphically using your calculator. Produce a sketch of your graph on the axes below.



Of course, if our equation is not written in a convenient form where we can apply inverses, then we first might need to use the method of completing the square to place it in that form.

Exercise #3: Consider the equation $x^2 + 8x - 16 = 0$.

- (a) Solve this equation graphically. Produce a sketch of your graph in the space to the right. Indicate your window. Round your answers to the nearest thousandth.
- (b) Solve this equation by using completing the square as you did in the last unit. Write your answers in simplest radical form.

- (c) Verify that the answers you found in (b) match the ones you found in (a).

Exercise #4: Solve each of the following quadratic equations using the method of completing the square. Write all square roots in simplest radical form.

(a) $x^2 - 4x - 16 = 0$

(b) $x^2 + 14x + 34 = 0$

(c) $x^2 + 2x - 53 = 0$

(d) $x^2 - 10x + 13 = 0$



QUADRATIC EQUATIONS WITH IRRATIONAL SOLUTIONS
N-GEN MATH[®] ALGEBRA I HOMEWORK

FLUENCY

1. Which quadratic equation below has irrational roots?

(1) $x^2 = 1$

(3) $x^2 = 30$

(2) $x^2 = 49$

(4) $x^2 = 121$

2. Which of the following are the solutions to the equation $(x + 2)^2 = 10$?

(1) $x = 10 \pm \sqrt{2}$

(2) $x = -10 \pm \sqrt{2}$

(3) $x = 2 \pm \sqrt{10}$

(4) $x = -2 \pm \sqrt{10}$

3. The graph of $y = x^2 - 4x - 18$ is shown. Which of the following is closest to a solution of the equation below?

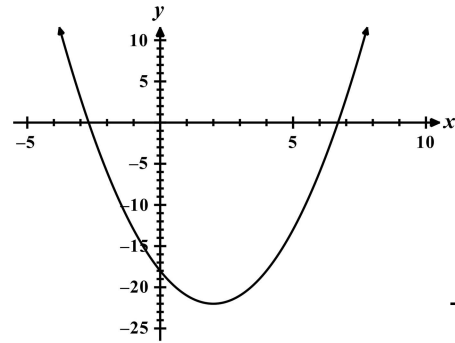
$$x^2 - 4x - 18 = 0$$

(1) 6.7

(3) -18

(2) 4.3

(4) -3.2



4. Which equation below can be solved to find the solutions to the equation $x^2 - 6x - 8 = 0$?

(1) $(x + 6)^2 = 15$

(2) $(x - 6)^2 = 23$

(3) $(x - 3)^2 = 17$

(4) $(x + 3)^2 = 13$

5. Which of the choices below are the solutions to $x^2 + 8x + 16 = 50$?

(1) $x = -8 \pm 2\sqrt{5}$

(2) $x = -4 \pm 5\sqrt{2}$

(3) $x = 4 \pm 7\sqrt{3}$

(4) $x = 8 \pm 4\sqrt{6}$



6. Use the method of completing the square to solve each of the following quadratic equations for all solutions. Express your solutions in simplest radical form. Do not convert to decimals.

(a) $x^2 + 16x + 40 = 0$

(b) $x^2 - 12x + 22 = 0$

(c) $x^2 - 22x + 90 = 0$

(d) $x^2 + 6x - 81 = 0$

7. Consider the equation $x^2 - 6x - 65 = 0$.

(a) Solve this equation graphically. Produce a sketch of your graph in the space provided to the right. Mark the zeros and your graphing window. Round answers to the nearest tenth.

(b) Solve the equation algebraically using completing the square. Leave your answers in exact form.

(c) Express your answer to (b) in decimal form. Round your answers to the nearest thousandth. Do they verify your answers from (a)?

