Name: _____ Date: _____

THE QUADRATIC FORMULA
N-GEN MATH® ALGEBRA I

*Our final topic in this unit looks at one of the most famous formulas in mathematics, the **quadratic formula**. The quadratic formula stems directly from the method of **completing the square**. Its proof or derivation is beyond the scope of this course. First, a review of completing the square.*

***Exercise* #1:** Consider the equation $x^2 + 8x + 3 = 0$.

(a) Solve this equation by completing the square.

(b) Are your solutions rational or irrational? Explain.

*Because of how **algorithmic** this process of completing the square is, it can be placed in a formula:*

THE QUADRATIC FORMULA

For the quadratic equation $\boldsymbol{ax^2 + bx + c = 0}$, the zeros can be found by $\boldsymbol{x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

***Exercise* #2:** For the previous quadratic $x^2 + 8x + 3 = 0$ identify the following.

(a) The values of *a*, *b*, and *c* in the quadratic formula.

(b) Substitute these values in the quadratic formula and simplify your expression. Compare your result to *Exercise* #1.

*Students often prefer the quadratic formula to either **factoring** or **completing the square** to find the zeros of a quadratic because it is very **mechanical** in nature (just plug in numbers and calculate the answers).*

***Exercise* #3:** Consider the quadratic equation $x^2 - 3x - 10 = 0$.

(a) Find the solutions to this equation by factoring.

(b) Find the solutions to this equation using the Quadratic Formula.

*The Quadratic Formula is particularly nice when the solutions are **irrational numbers** and thus **cannot be found by factoring**. Sometimes, we have to place the answers to these equations in **simplest radical form** and sometimes we just need decimal approximations.*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

***Exercise* #4:** For each of the following quadratic equations, find the solutions using the quadratic formula and express your answers in **simplest radical form**.

(a) $x^2 + 6x - 9 = 0$

(b) $3x^2 + 4x - 1 = 0$

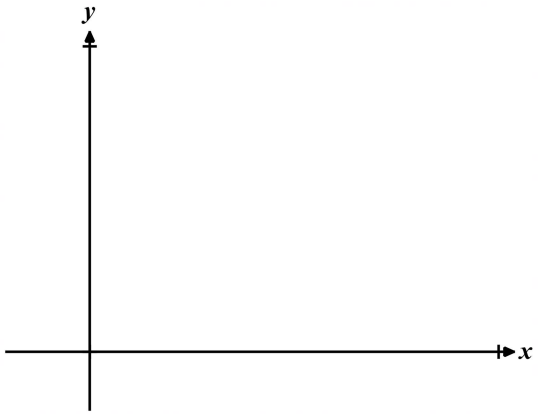
*In **applied problems**, it often makes greater sense to express the answers, if irrational, as rounded decimals.*

***Exercise* #5:** A projectile is fired vertically from the top of a 60-foot-tall building. Its height in feet above the ground after t-seconds is given by the formula

$$h = -16t^2 + 20t + 60$$

(a) Sketch a graph that shows the projectile's height over time for $t \geq 0$. Use your graph to determine the time when the projectile hits the ground. Round to the nearest tenth.

(b) Use the quadratic formula to determine when the projectile hits the ground. Also round to the nearest tenth of a second.



Name: _____ Date: _____

THE QUADRATIC FORMULA
N-GEN MATH® ALGEBRA I HOMEWORK

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

FLUENCY

1. Solve the equation $x^2 - 4x - 12 = 0$ two ways:

   (a) by factoring

   (b) by the using the quadratic formula

2. Solve the equation $x^2 + 6x + 3 = 0$ two ways. Express your answers both times in simplest radical form.

   (a) by completing the square

   (b) by using the quadratic formula

3. Solve the equation $x^2 - 11x + 24 = 0$ two ways:

   (a) by factoring

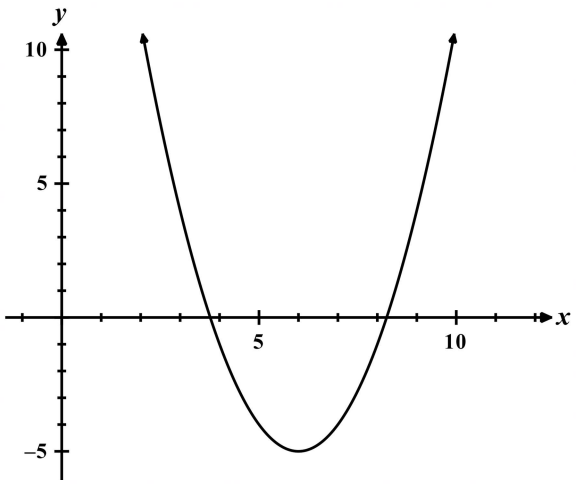
   (b) by using the quadratic formula

4. If the quadratic formula is used to solve the equation $x^2 - 4x - 41 = 0$, the correct roots are

   (1) $4 \pm 3\sqrt{10}$ (3) $-4 \pm 3\sqrt{10}$

   (2) $2 \pm 3\sqrt{5}$ (4) $-2 \pm 3\sqrt{5}$

5. The quadratic function $f(x) = x^2 - 12x + 31$ is shown below.



(a) Find the zeros of this function in simplest radical form by using the quadratic formula.

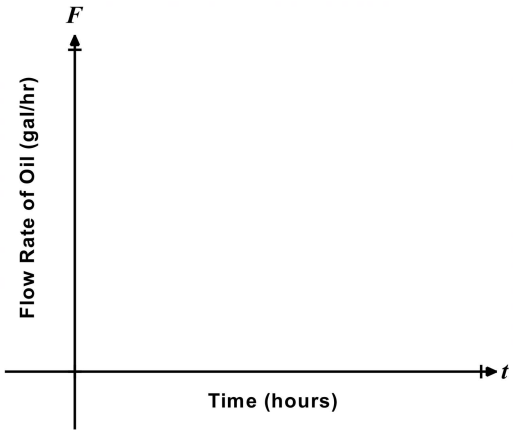
(b) Convert your answers in (a) to decimal form. Round to the nearest tenth. Then, mark your answers on the graph.

APPLICATIONS

6. The flow rate of oil in a pipe, in gallons per hour, can be modeled using the function $F(t) = -2t^2 + 20t + 11$

(a) Using your calculator, graph the function on the axes provided, for $t \geq 0$. Make sure your window is large enough to show the turning point and positive zero. Mark your window.

(b) Using the quadratic formula, find, to the nearest tenth of an hour, the time when the flow stops (is zero). Show your work.



(c) Use the axis of symmetry formula to find the location of the turning point of the function and mark it on your graph.

(d) What does your answer from (c) tell you about the flow rate of oil?