

DENSITY
N-GEN MATH® GEOMETRY



One of the most important ideas in science and other fields is that of **density**. Density can be thought of as simply a measurement of **how much of something lies within a unit area or unit volume**. That “something” could be mass, weight, or really anything else. Often, it can be people or animals or some other type of **population**.

Exercise #1: The approximate population for Chicago, Miami, and Seattle in 2023 is shown in the table below along with the area of each city in square miles.

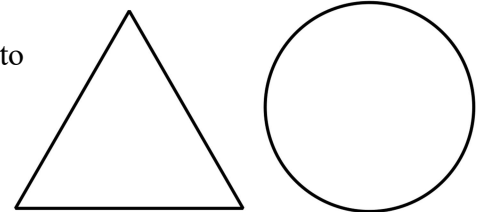
- (a) Calculate how many people per square mile live in each city. This is called its **population density**. Round to the nearest whole number.

| | Population | Area (mi ²) |
|---------|------------|-------------------------|
| Chicago | 2,694,000 | 227 |
| Miami | 462,000 | 36 |
| Seattle | 745,000 | 84 |

- (b) How many more people live in a square mile in the most densely populated city compared to the least densely populated?

Exercise #2: Shana is planting 400 sunflower plants in a garden. She is surrounding the garden with 30 meters of fencing. Shana is debating between a garden that is an equilateral triangle and one that is a circle. Assuming Shana uses all the fencing to surround each type of garden:

- (a) Calculate the maximum area Shana can enclose for each shape. Round to the nearest tenth of a square meter.



- (b) Using (a), calculate the sunflower density for each garden. Round your answers to the nearest tenth of a sunflower per square meter.

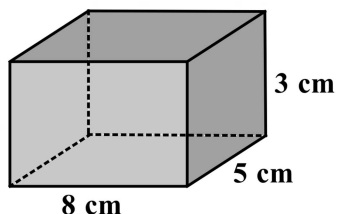
- (c) Which shape allows Shana to spread the sunflowers out more? Explain.



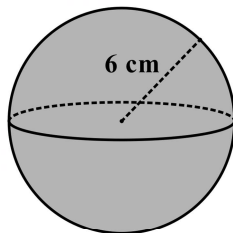
Density is often used on a **volume basis** rather than an **area basis**. We have seen densities arise in many volume problems already. One of the most common volumetric densities is **mass density**.

Exercise #3: Given each object below along with its mass, calculate its mass density to the nearest hundredth of a gram per cubic centimeter.

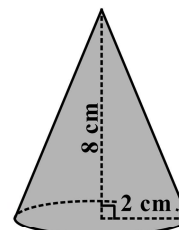
(a) mass = 785 grams



(b) mass = 751 grams



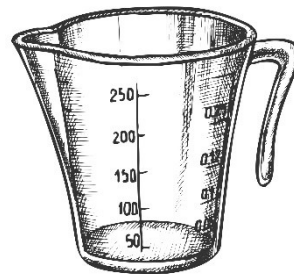
(c) mass = 93 grams



Exercise #4: Water has a density of 1 gram per cubic centimeter. An object will float in water if it has a mass density less than that of water. Would any of the objects in *Exercise #3* float?

Exercise #5: A salad dressing recipe calls for 225 grams of vinegar and 150 grams of olive oil. Maria doesn't have a scale but does have a measuring cup marked in milliliters. Maria knows that the density of vinegar is 1.05 grams per milliliter and the density of olive oil is 0.92 grams per milliliter.

(a) Determine the volume of vinegar and olive oil Maria needs for the recipe. Round your answers to the nearest milliliter.



(b) Maria shakes the oil and vinegar together in a jar. Eventually, the oil rises to the top of the container. Why do you think this happens?



Name: _____

Date: _____

DENSITY
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FLUENCY

- The population of Boston in 2023 is around 690,000 people. If Boston covers an area of 48 square miles, which of the following is the population density of Boston?
(1) 12,845 people per square mile
(2) 14,375 people per square mile
(3) 15,013 people per square mile
(4) 15,766 people per square mile

- A cube of gold has edges that have a length of 3 centimeters each. If the cube weighs 520 grams, then which of the following is the gold's density, to the nearest tenth?
(1) 17.2 grams per cubic centimeter
(2) 18.1 grams per cubic centimeter
(3) 18.8 grams per cubic centimeter
(4) 19.3 grams per cubic centimeter

- A substance has a density of 2.85 grams per cubic centimeter. If the substance weighs 24 grams, which of the following is closest to its volume?
(1) 8.42 cm^3
(2) 8.67 cm^3
(3) 9.11 cm^3
(4) 9.39 cm^3

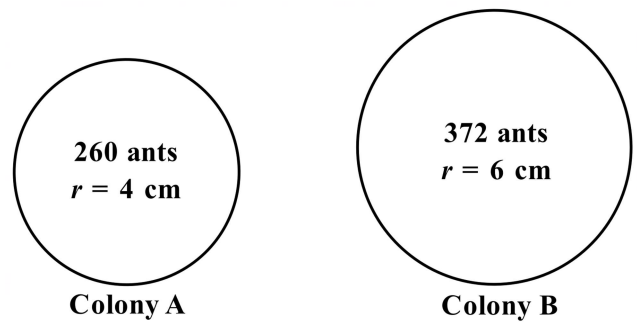
APPLICATIONS

- A can of refried beans has a mass of 437 grams. The can is in the shape of a cylinder whose diameter is 7.2 cm and whose height is 10 cm. Determine the mass density of the beans in the can to the nearest hundredth. Include appropriate units.
- If you dropped the can of refried beans in a sink full of water, would it float? Explain. (See Exercise #4)



6. Grady is studying populations of ants in two colonies. He takes pictures of the ants at the two colonies and draws circles around a grouping of ants in each picture. The sizes of the circles and number of ants in each is shown below.

Which colony had the larger population density?
Justify your answer.



7. A balloon in the shape of a sphere has a diameter of 30 centimeters. It is filled with 2,500 milligrams of helium. Determine the density of the helium in the balloon to the nearest hundredth of a milligram per cubic centimeter.

REASONING

8. A **torus** is the mathematical name we give to the solid that is generated by rotating a circle around a line that does not pass through the circle. It is essentially the shape we think of as a **donut**. The volume of a torus can be calculated using the formula:

$$V = (2\pi R)(\pi r^2)$$

where R is the distance from the center of the torus to the center of its circular cross section and r is the radius of the circular cross section.

An inner tube has the shape of a torus where $R = 50$ cm and $r = 20$ cm and is completely filled with 484 **grams** of normal air.

- (a) What is the volume of air to the nearest thousand cubic centimeters?
- (b) Find the density of the air in the tube in **milligrams** per cubic centimeter to the nearest hundredth.
- (c) Compare your answer in (b) to your answer to #7. Why do helium balloons float?

