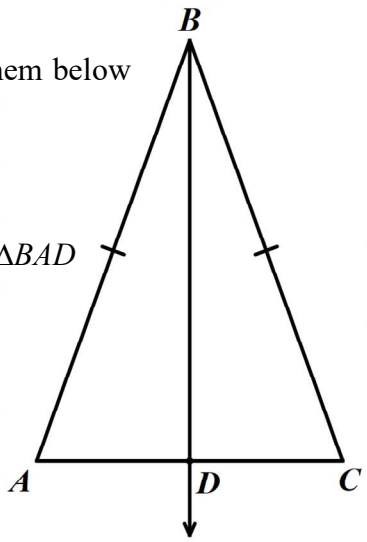
Name: _____ Date: _____

ISOSCELES TRIANGLES
N-GEN MATH® GEOMETRY

*Isosceles triangles are surprisingly important in geometry. An **isosceles triangle** is a triangle with **two sides** of **equal length**. In our first exercise, we will be exploring important properties of these triangles.*

***Exercise* #1:** In the diagram shown below, ΔABC is **isosceles** with $\overline{AB} \cong \overline{CB}$ (shown with dashes). $\overrightarrow{BD}$ is the angle bisector of $\angle ABC$ (known as the **vertex angle** of the isosceles triangle).



(a) What two angles must be congruent based on $\overrightarrow{BD}$ bisecting $\angle ABC$? State them below and verify using tracing paper.

(b) Based on properties of rigid motions, explain why ΔBCD would map on top of ΔBAD if reflected across $\overrightarrow{BD}$. (Use tracing paper to verify this mapping.)

(c) What does (b) tell you about ΔBCD and ΔBAD?

(d) Given (c), write down segments or angles that must be congruent to the ones given:

$\overline{AD} \cong$ _____ $\angle A \cong$ _____ $\angle ADB \cong$ _____

(This is known in geometry as **corresponding parts of congruent triangles are congruent**.)

(e) Explain why $\angle ADB$ and $\angle CDB$ must both be right angles. Base your explanation on facts that you know, not how the angles "look."

(f) Give one term that would describe the relationship between $\overrightarrow{BD}$ and **base** $\overline{AC}$.

Isosceles Triangle Facts

1. The **base angles** of an **isosceles triangle** are **congruent** (have equal measures). Another way to state this is that **angles opposite congruent sides are congruent**.
2. The bisector of the **vertex angle** in an **isosceles triangle** is also the **perpendicular bisector** of the **base** of the **isosceles triangle** (i.e., the **side opposite** the vertex angle).

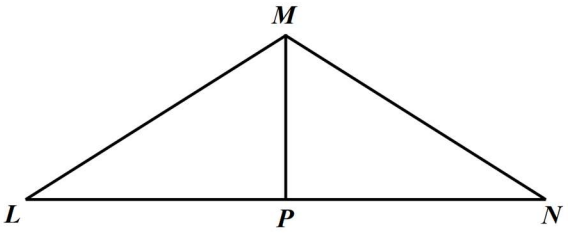
Recall that the sum of the angles of any triangle is 180°. We will prove this result later in the course. For now, we will just use this fact about triangles to solve problems.

***Exercise* #2:** In ΔLMN below, $\overline{ML} \cong \overline{MN}$ and $m\angle LMN = 114°$. P lies on $\overline{LN}$ such that $\overline{MP}$ bisects $\angle LMN$.

(a) Mark the two congruent sides on ΔLMN.

(b) Mark the two congruent base angles of ΔLMN.

(c) Use the angle sum fact above to determine the measures of the base angles of ΔLMN. Mark these on the diagram.

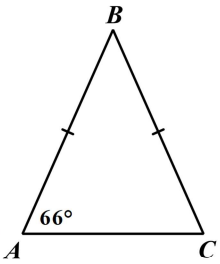


(d) What is the measure of $\angle LMP$? Mark this measure in the diagram.

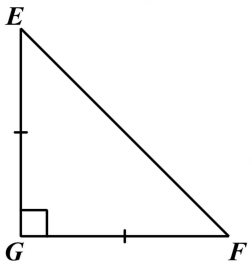
(e) Use the angle sum fact above to verify that $\angle MPL$ must be a right angle.

***Exercise* #3:** For each isosceles triangle shown below, find the missing two angle measures.

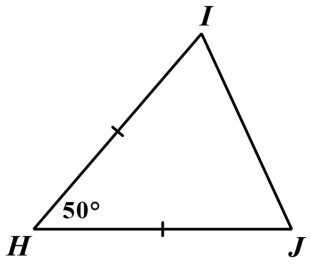
(a)



(b)



(c)



***Exercise* #4:** The ratio of the measure of either base angle to the measure of the vertex angle in an isosceles triangle is 3:2. Determine the measure of the vertex angle of this isosceles triangle.

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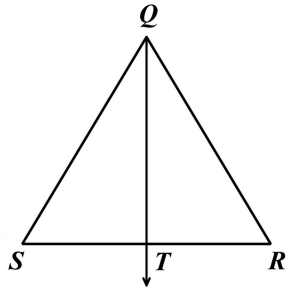
ISOSCELES TRIANGLES
N-GEN MATH® GEOMETRY HOMEWORK

FLUENCY

1. In ΔQRS shown, $\overline{QS} \cong \overline{QR}$ and point T is located on $\overline{RS}$ such that $\overrightarrow{QT}$ bisects $\angle SQR$. Which pair of angles below does *not* have to be congruent?



(1) $\angle QSR$ and $\angle QRS$

(2) $\angle RQT$ and $\angle QRT$

(3) $\angle QTS$ and $\angle QTR$

(4) $\angle SQT$ and $\angle RQT$

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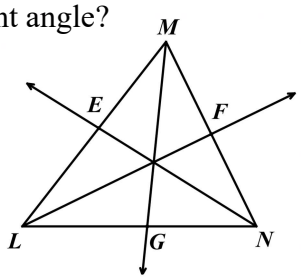
2. In ΔLMN below, an angle bisector for each of its angles has been drawn. It is known that $\overline{LM} \cong \overline{LN}$. Based on this information, which of the following must be a right angle?



(1) $\angle NEM$

(2) $\angle NMG$

(3) $\angle MGN$

(4) $\angle LFM$

_____

3. If the vertex angle of an isosceles triangle has a measure of $52°$, then which of the following is the measure of one of its base angles?

(1) $64°$

(2) $68°$

(3) $74°$

(4) $128°$

_____

4. An angle in an isosceles triangle lies opposite of one of the two congruent sides. It has a measure of $31°$. What is the measure of the angle that lies opposite of the non-congruent side?

(1) $31°$

(2) $62°$

(3) $118°$

(4) $149°$

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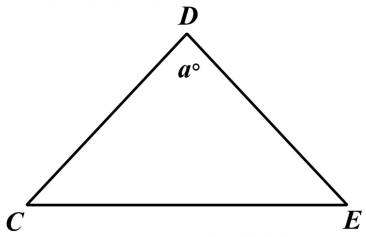
5. In ΔCDE, $\overline{CD} \cong \overline{ED}$. If $m\angle CDE = a°$, then which formula below would calculate the measure of $\angle E$ based on a?

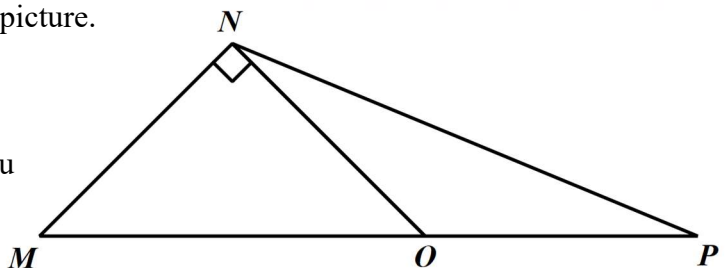


(1) $180 - 2a$ (3) $\frac{180 - a}{2}$

(2) $180 - \frac{a}{2}$ (4) $\frac{180 - 2a}{2}$

_____

6. In the diagram shown below, point O is located on $\overline{MP}$ such that $\overline{MN} \perp \overline{ON}$, $\overline{MN} \cong \overline{ON}$, and $\overline{ON} \cong \overline{OP}$.



(a) Name two different isosceles triangles in this picture.

(b) What is the measure of $\angle M$? Show how you found your answer.

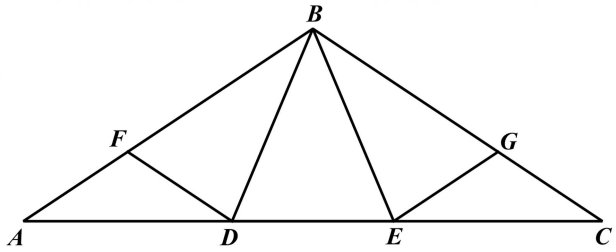
(c) What is the measure of $\angle NOP$? Show how you found your answer.

(d) What is the measure of $\angle P$? Show how you found your answer.

(e) What is the measure of $\angle MNP$. Show how you found your answer.

APPLICATIONS

7. A roof truss helps support the roof on a house. A common design of a roof truss is shown below. In this design, $\overline{BA} \cong \overline{BC}$, $\overline{BD} \cong \overline{BE}$, $\overline{AD} \cong \overline{BD}$, $\overline{CE} \cong \overline{BE}$, $\overline{FA} \cong \overline{FD}$, $\overline{GE} \cong \overline{GC}$, and $m\angle ABC = 114°$.



(a) Find the measure of $\angle A$.

(b) Find the measure of $\angle DBE$.

(c) Find the measure of $\angle BDE$.

(d) Find the measure of $\angle FDE$.

REASONING

8. In ΔABC shown, $\angle ABC$ is a right angle and $m\angle A = 34°$. Point D is located such that $\overline{AD} \cong \overline{BD}$. Show that $\angle DBC \cong \angle DCB$.