

Name: \_\_\_\_\_

Date: \_\_\_\_\_



**PERPENDICULAR BISECTORS**  
**N-GEN MATH® GEOMETRY**



The **perpendicular bisector** of a segment is a surprisingly important idea. We have now seen it in connection with **line reflections** and with **isosceles triangles**.

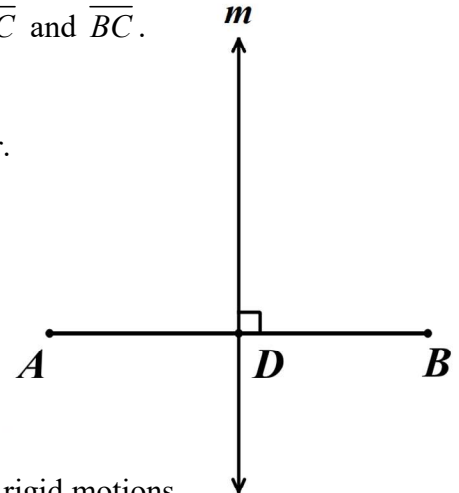
**Exercise #1:** In the diagram below, line  $m$  is the perpendicular bisector of  $\overline{AB}$ . Do the following:

(a) Choose a point above  $\overline{AB}$  on line  $m$  and mark it as point  $C$ . Draw in  $\overline{AC}$  and  $\overline{BC}$ .

(b) Measure the lengths of  $\overline{AC}$  and  $\overline{BC}$  to the nearest tenth of a centimeter.

$AC =$  \_\_\_\_\_       $BC =$  \_\_\_\_\_

(c) Based on your measurements, what can you say about the distance point  $C$  is away from the endpoints of  $\overline{AB}$ ?



**Exercise #2:** We should prove your observation in (c). We can do this using rigid motions.

(a) Explain why reflecting  $\triangle ACD$  across line  $m$  would have to map it on top of  $\triangle BCD$ .

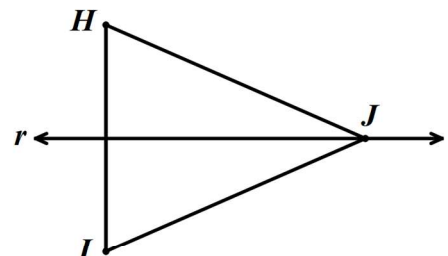
(b) What does this tell you about  $\triangle ACD$  and  $\triangle BCD$ ?

(c) What can you conclude about  $\overline{AC}$  and  $\overline{BC}$  based on (b)? Explain.

**Perpendicular Bisector Fact #1**

If a **point** lies on the **perpendicular bisector** of a segment, say  $\overline{AB}$ , then it will be the **same distance** (equidistant) **from the segment's two endpoints**,  $A$  and  $B$ .

**Exercise #3:** In the diagram below, line  $r$  is the perpendicular bisector of  $\overline{HI}$ . Point  $J$  lies on line  $r$ . Explain why  $\triangle HIJ$  must be isosceles.



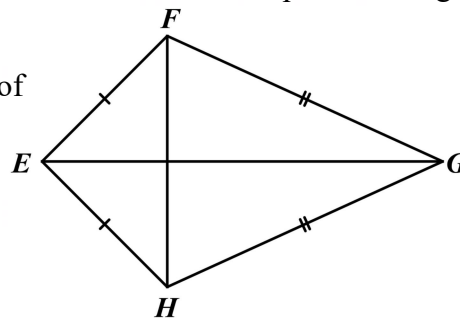
In our lesson on isosceles triangles, we saw that the **vertex point** of an isosceles triangle lies on the perpendicular bisector of the base of the isosceles triangle. This leads to a further observation about perpendicular bisectors.

**Perpendicular Bisector Fact #2**

If a **point is the same distance** (equidistant) **from the two endpoints** of a line segment, then **it must lie** on the **perpendicular bisector** of that line segment.

**Exercise #4:** A kite is a figure that has two pairs of congruent, adjacent sides that do not overlap. In the diagram below, kite  $EFGH$  is shown with  $\overline{EF} \cong \overline{EH}$  and  $\overline{GF} \cong \overline{GH}$ .

- (a) Explain why points  $E$  and  $G$  must lie on the perpendicular bisector of diagonal  $\overline{FH}$ .

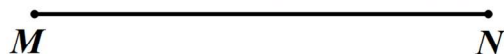


- (b) Based on (a), why can you conclude that  $\overline{EG}$  is the perpendicular bisector of  $\overline{FH}$ ?

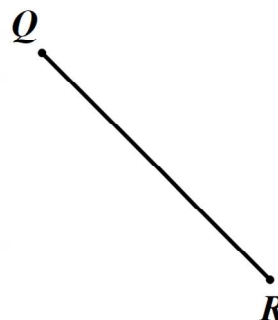
We can use what we just saw with the kite in the last exercise to **construct the perpendicular bisector**.

**Exercise #5:** For each segment below, use only a compass and straightedge to construct its perpendicular bisector.

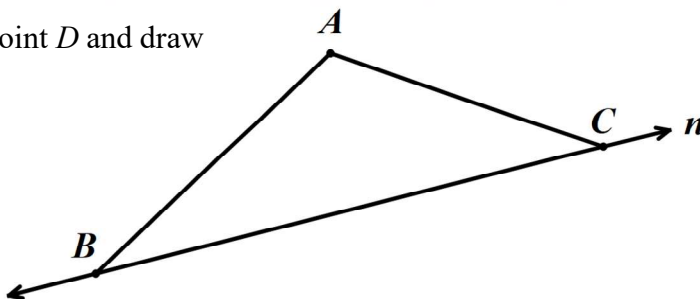
(a)



(b)

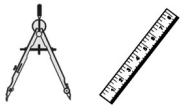


**Exercise #6:** Using a compass and straightedge, locate point  $D$  and draw  $\overline{BD}$  and  $\overline{CD}$  so that  $ABDC$  is a kite.



**Exercise #7:** What is the special relationship between points  $A$  and  $D$ ?



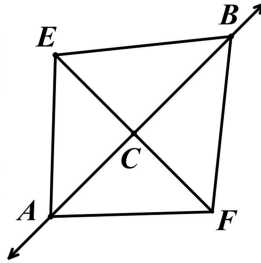


## PERPENDICULAR BISECTORS N-GEN MATH® GEOMETRY HOMEWORK

### FLUENCY

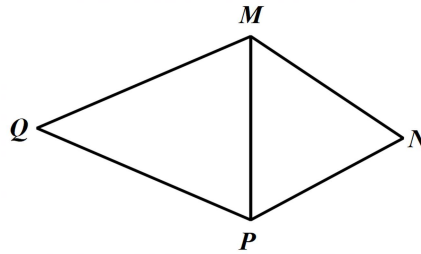
1. In the diagram shown,  $\overline{AB}$  is the perpendicular bisector of  $\overline{EF}$ . Which of the following does not have to be true?

- (1)  $\overline{AC} \cong \overline{BC}$
- (2)  $\overline{BF} \cong \overline{BE}$
- (3)  $\overline{EC} \cong \overline{FC}$
- (4)  $\overline{AF} \cong \overline{AE}$



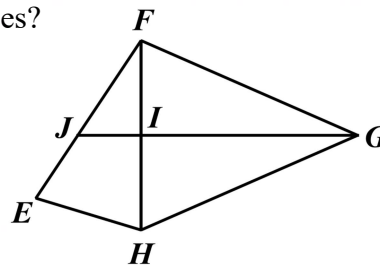
2. In the diagram below, it is known that  $\overline{QM} \cong \overline{QP}$  and  $\overline{MP} \cong \overline{MN}$ . Which of the following points must lie on the perpendicular bisector of  $\overline{MP}$ ?

- (1)  $M$
- (2)  $N$
- (3)  $P$
- (4)  $Q$



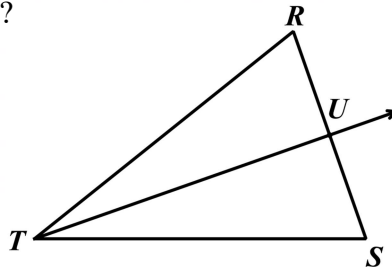
3. In the diagram below,  $\overline{GJ}$  is the perpendicular bisector of  $\overline{FH}$ . Point  $J$  is located on  $\overline{EF}$ . Based on this information, which triangle below must be isosceles?

- (1)  $\triangle EFH$
- (2)  $\triangle FIJ$
- (3)  $\triangle FGH$
- (4)  $\triangle FIG$



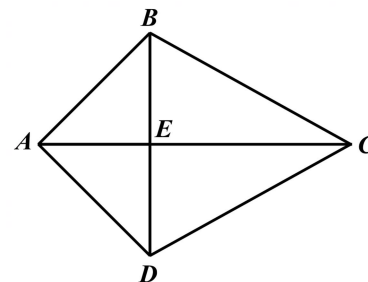
4. In the diagram shown,  $\overline{TU}$  is the perpendicular bisector of  $\overline{RS}$ . If  $m\angle URT = 71^\circ$ , then which of the following must be the measure of  $\angle STU$ ?

- (1)  $19^\circ$
- (2)  $38^\circ$
- (3)  $54.5^\circ$
- (4)  $62^\circ$



5. In the diagram below,  $\overline{AC}$  is the perpendicular bisector of  $\overline{BD}$ . Which of the following must be true?

- (1)  $C$  is the image of  $A$  after a reflection across  $\overline{BD}$
- (2)  $D$  is the image of  $B$  after a reflection across  $\overline{AC}$
- (3)  $A$  is the image of  $D$  after a clockwise rotation of  $90^\circ$  about  $E$
- (4)  $B$  is the image of  $A$  after a clockwise rotation of  $90^\circ$  about  $E$

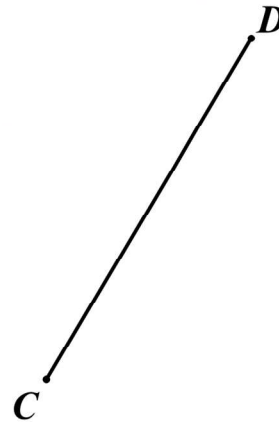


6. For each line segment shown below, use a compass and straightedge only to construct the perpendicular bisector. Leave all construction marks.

(a)

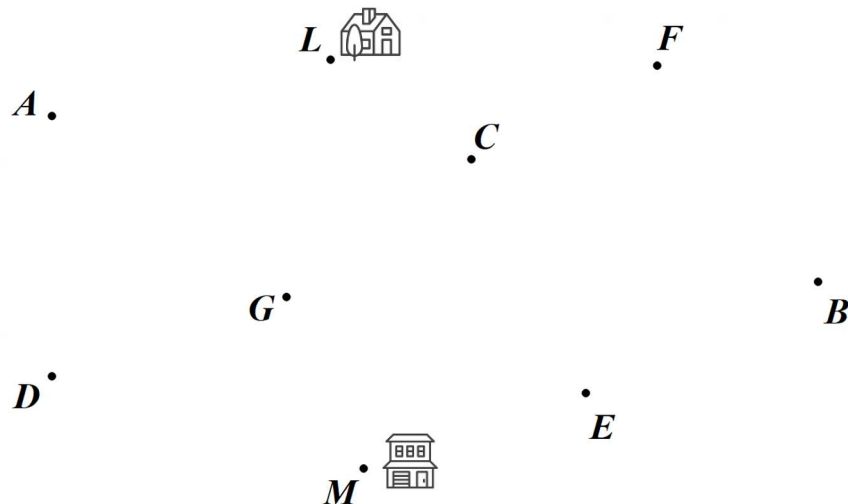


(b)



### APPLICATIONS

7. The Lopez and Miller houses are located at points  $L$  and  $M$  as shown in the map below. They are trying to determine which points on the map are closer to the Lopez house and which are closer to the Miller house. They know that points along the perpendicular bisector of the segment joining the two houses will be equidistant from both.



- (a) Using only a compass and ruler, draw in  $\overline{LM}$  and then construct its perpendicular bisector.
- (b) Given the line you drew in (a), list all points that are closer to the Lopez house and all that are closer to the Miller house.

Lopez House: \_\_\_\_\_

Miller House: \_\_\_\_\_

